



**ENGR 213**

**Applied Ordinary Differential Equations**

**Second Mid Term Exam**

**March 12<sup>th</sup> 2017**

**Student Name:** \_\_\_\_\_

**Student ID Number:** \_\_\_\_\_

**ENGR 213 Section:** \_\_\_\_\_

**Course Given BY:** \_\_\_\_\_.

- Exam is closed book close notes.
- No use of any electronic devices.
- Use only the approved calculator.
- Write your answers in the provided space.

Q1	
Q2	
Q3	
Q4	
Q5	
Total	

**Problem#1**

Write the number  $(3+6i) + (4-i)(3+5i) + \frac{1}{2-i}$  in the form  $a+bi$ .

**Solution:**

$$(3 + 6i) + (17 + 17i) + \frac{1}{2-i} \frac{2+i}{2+i}$$

$$(20 + 23i) + \frac{2+i}{4+1} = (20 + 23i) + \frac{2}{5} + \frac{i}{5}$$

$$\frac{102}{5} + \frac{116}{5} i$$

### Problem#2

Verify that  $y_1 = e^{5x}$  and  $y_2 = e^{-7x}$  are form a fundamental set of solutions of the differential equation  $y'' + 2y' - 35y = 0$  on the interval  $(-\infty, \infty)$ . Write the form of the general solution.

Solution:

$$y_1 = e^{5x}$$

$$y_2 = e^{-7x}$$

$$y_1' = 5e^{5x}$$

$$y_2' = -7e^{-7x}$$

$$y_1'' = 25e^{5x}$$

$$y_2'' = 49e^{-7x}$$

For  $y_1$

$$y'' + 2y' - 35y$$

$$25e^{5x} + 10e^{5x} - 35e^{5x} = 0$$

o.k ←

$$\text{For } y_2 = 49e^{-7x} - 14e^{-7x} - 35e^{-7x} = 0 \quad \text{o.k.} \leftarrow$$

$$y = c_1 e^{5x} + c_2 e^{-7x}$$

### Problem#3

#### Solution:

(a) Write the characteristic equation and the general solution of the differential equation:

$$y^{(6)} - 13y^{(5)} + 70y^{(4)} - 198y^{(3)} + 308y'' - 248y' + 80y = 0.$$

$y^{(k)}$  is the  $k$ -th derivative of  $y$ . For Your convenience, the roots of the characteristic equation are: 1, 2, 2, 2,  $3 + i$ ,  $3 - i$ . You do not have to check this.

(b) Find the general solution of the differential equation:  $x^2 y'' - 7xy' + 16y = 0$ , given one solution  $y_1 = x^4$ .

#### Solution:

① characteristic Eq:

$$m^6 - 13m^5 + 70m^4 - 198m^3 + 308m^2 - 248m + 80 = 0$$

General solution:

$$y = C_1 e^x + C_2 e^{2x} + C_3 x e^{2x} + C_4 x^2 e^{2x} + C_5 e^{3x} \cos x + C_6 e^{3x} \sin x$$

②  $x^2 y'' - 7xy' + 16y = 0$  given  $y_1 = x^4$

$$y_2 = u x^4 \quad y_2' = u' x^4 + 4u x^3$$

$$y_2'' = u'' x^4 + 8u' x^3 + 12u x^2$$

$$u'' x^6 + 8u' x^5 + 12u x^4 - 7(u' x^5 + 4u x^4) + 16u x^4 = 0$$

$$u'' x^6 + u' x^5 = 0$$

Substitute:  $w = u'$

$$w' x^6 + w x^5 = 0$$

$$w' + \frac{1}{x} w = 0$$

$$\frac{dw}{dx} = -\frac{1}{x} w$$

$$\int \frac{dw}{w} = \int -\frac{1}{x} dx$$

$$\ln w = -\ln x \Rightarrow \ln w = \ln \frac{1}{x}$$

$$w = \frac{1}{x} \Rightarrow u = \int w = \int \frac{1}{x} dx = \ln x$$

$$y_2 = x^4 \ln x$$

$$\text{general solution } y = C_1 x^4 + C_2 x^4 \ln x$$

$$y = x^2 e^x$$

**Problem#4**

Using the method of undetermined coefficients, solve the initial value problem

$$y'' + y' - 2y = (6x+2)e^x; y(0)=0; y'(0)=0.$$

**Solution:**

$$m^2 + m - 2 = 0 \Rightarrow (m+2)(m-1) = 0$$

$$m = -2 \text{ or } m = 1$$

$$y_c = c_1 e^x + c_2 e^{-2x}$$

$$y_p = (Ax^2 + Bx)e^x$$

$$y_p' = (2Ax + B)e^x + (Ax^2 + Bx)e^x$$

$$y_p'' = (2A)e^x + (2Ax + B)e^x + (2Ax + B)e^x + (Ax^2 + Bx)e^x$$

$$= 2Ae^x + (4Ax + 2B)e^x + (Ax^2 + Bx)e^x$$

$x^2:$   $-2A + A + A = 0$

$x:$   $-2B + 2A + B + 4A + B = 0 \Rightarrow 6A = 6 \Rightarrow A = 1$

$C:$   $B + 2A + 2B = 2 \Rightarrow B = \frac{2 - 2A}{3} = 0$

$$y_p = x^2 e^x$$

$$\text{Solution} \Rightarrow y = c_1 e^x + c_2 e^{-2x} + x^2 e^x$$

$$y(0) = 0 \Rightarrow 0 = c_1 + c_2 \Rightarrow \underline{c_1 = -c_2}$$

$$y'(0) = 0 \Rightarrow 0 = c_1 e^x - 2c_2 e^{-2x} + (2x e^x + x^2 e^x)$$

$$0 = c_1 - 2c_2 \Rightarrow \underline{c_1 = 2c_2}$$

Problem#5

Solve the following differential Equation  $y'' + (3/x)y' + (5/x^2)y = 0$

Solution:

$$y'' + \frac{3}{x}y' + \frac{5}{x^2}y = 0$$

$$x^2 y'' + 3xy' + 5y = 0 \quad \text{Cauchy Euler Eq}$$

$$y = x^m$$

$$m(m-1) + 3(m) + 5 = 0$$

$$m^2 - m + 3m + 5 = 0$$

$$m^2 + 2m + 5 = 0$$

$$m = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm 4i}{2}$$

$$= -1 \pm 2i$$

$$y = \cancel{C_1 x^{-1} \cos(2 \ln x) + C_2 x^{-1} \sin(2 \ln x)}$$

$$= C_1 x^{-1} \cos(2 \ln x) + C_2 x^{-1} \sin(2 \ln x)$$

