2.7 $1-P(A)-P(B)+P(A\cap B)=(a+b+c+d)-(a+b)-(a+c)+a=d$
$$=P(A'\cap B')$$

2.13 $P(A)+P(B)+P(C)+P(D)-P(A\cap B)-P(A\cap C)-P(A\cap D)-P(B\cap C)$
$$-P(B\cap D)-P(C\cap D)+P(A\cap B\cap C)+P(A\cap B\cap D)$$
$$+P(A\cap C\cap D)+P(B\cap C\cap D)-P(A\cap B\cap C\cap D)$$
$$=(a+b+d+g+i+j+l+o)+(a+b+c+e+i+j+k+m)$$
$$+(a+c+d+f+i+k+l+n)+(a+b+c+d+e+f+g+h)$$
$$-(a+b+i+j)-(a+d+i+l)-(a+b+d+g)$$
$$-(a+c+i+k)-(a+b+c+e)-(a+c+d+f)$$
$$+(a+i)+(a+b)+(a+d)+(a+c)-a$$
$$=a+b+c+d+e+f+g+h+i+j+k+l+m+n+o$$
$$=P(A\cup B\cup C\cup D)$$

2.16 **(a)** P:ostulate 1 $P(A)=\dfrac{a}{a+b}\geq 0$

(B) Postulate 2 $P(A)=\dfrac{a}{a+b},\quad P(A')=\dfrac{b}{a+b}$
$$P(A)+P(A')=\frac{a}{a+b}+\frac{b}{a+b}=1=P(S)$$

2.19 $P(A\cap B\cap C\cap D)=P(A\cap B\cap C)P(D|A\cap B\cap C)$
$$=P(A\cap B)P(C|A\cap B)P(D|A\cap B\cap C)$$
$$=P(A)P(B|A)P(C|A\cap B)P(D|A\cap B\cap C)$$

2.26 (Refer to 2.24 and 2.25) Already showed that A and B *independent*,.
A and C *independent*
$P[(A \cap (B \cap C)] = 0.54$, $P(A) = 0.60$, $P(B \cup C) = 0.92$, $(0.6)(0.92) = 0.552 \neq 0.54$

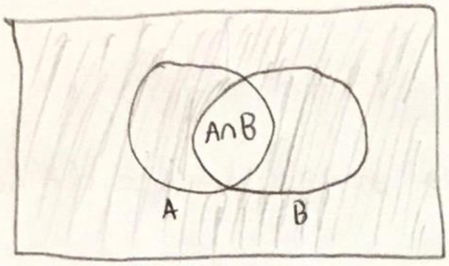
2.56 **(a)** $0.12 + 0.17 = 0.29$; **(b)** $0.17 + 0.34 + 0.29 = 0.80$
(c) $0.34 + 0.17 + 0.12 = 0.63$; **(d)** $0.34 + 0.29 + 0.08 + 0.71$

2.88 **(a)** $\frac{0.52}{0.74} = \frac{25}{37}$; **(b)** $\frac{0.34}{0.52} = \frac{17}{26}$; **(c)** $\frac{0.18 + 0.16 - 0.10}{0.70 + 0.62 - 0.44} = \frac{0.24}{0.88} = \frac{3}{11}$
(d) $\frac{0.46 - 0.34}{0.30} = \frac{0.12}{0.30} = \frac{2}{5}$

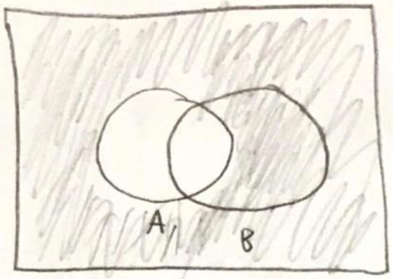
A1

2.2 pag 27

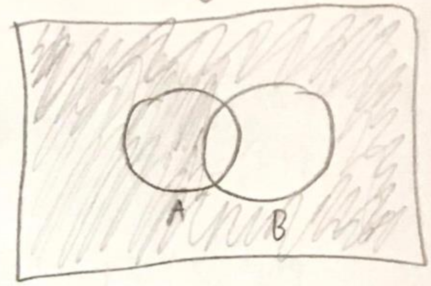
1 a) $(A \cap B)' = A' \cup B'$



A^c

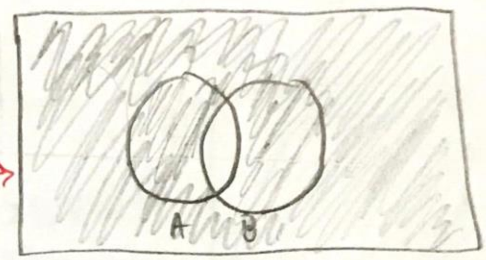


B^c



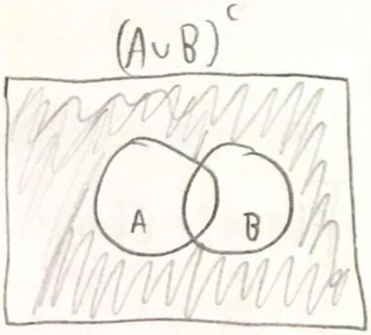
$A^c \cup B^c$

Solution →

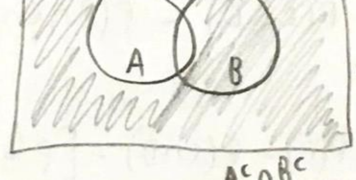


b) $(A \cup B)^c = A^c \cap B^c$

$(A \cup B)^c$

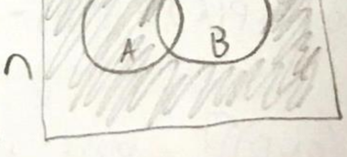


A^c



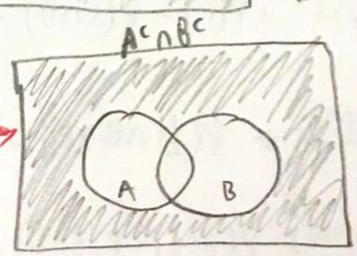
$\cap$

B^c



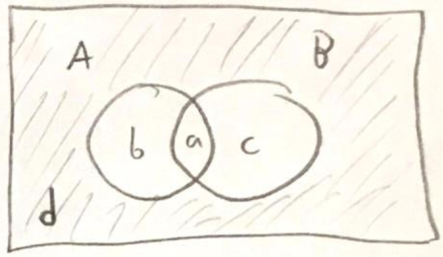
$A^c \cap B^c$

Solution →



2.6 page 36

$P(A^c \cap B^c) = d$



$P(A^c \cap B^c) = 1 - P(A) - P(B) + P(A \cap B)$

$P(A) = a + b$

$P(B) = a + c$

$P(A \cap B) = a$

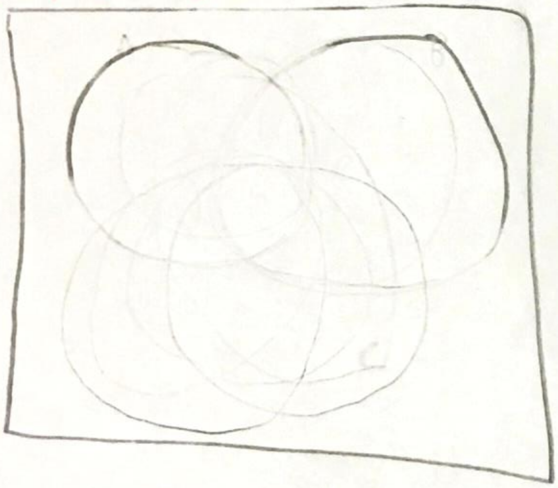
$P(\Omega) = 1 = d + a + b + c$

$\Rightarrow d = 1 - a - b - c = P(A^c \cap B^c)$

$P(A^c \cap B^c) = 1 - (a+b) - (a+c) + a$

$= 1 - 2a - b - c + a = 1 - a - b - c = d$

2.13 page 36



$P(A \cup B \cup C \cup D) = P(A \cup [B \cup C \cup D])$

$= P(A) + P(B \cup C \cup D) - P(A \cap [B \cup C \cup D])$

$= P(A) + [P(B) + P(C \cup D) - P(B \cap [C \cup D])]$

$- P((A \cap B) \cup (A \cap (C \cup D)))$

$= P(A) + P(B) + P(C) + P(D) - P(C \cap D) - P((B \cap C) \cup (B \cap D)) - [P(A \cap B) + P(A \cap (C \cup D))$

$- P((A \cap B) \cap (C \cup D))] = P(A) + \ldots + P(D) - P(C \cap D) - [P(B \cap C) + P(B \cap D) - P(B \cap C \cap D)]$

$- P(A \cap B) - P((A \cap C) \cup (A \cap D)) + P([A \cap B \cap C] \cup [A \cap B \cap D])$

$= P(A) + P(B) + P(C) + P(D) - P(A \cap B) - P(C \cap D) - P(B \cap C) - P(B \cap D) + P(B \cap C \cap D) - P(A \cap C) - P(A \cap D) + P(A \cap C \cap D)$

$+ P(A \cap B \cap C) + P(A \cap B \cap D) - P(A \cap B \cap C \cap D)$ ▨

⑤ 2.19 pag 48

$$P(A\cap B\cap C\cap D) = P(A)\cdot P(B|A)\, P(C|A\cap B)\cdot P(D|A\cap B\cap C)$$

$P(A\cap B\cap C)\neq 0$

$P(A\cap B) = P(A)\cdot P(B|A)$

$$= P[(A\cap B\cap C)\cap D] = P(A\cap B\cap C)\cdot P(D|A\cap B\cap C)$$

$$= P([A\cap B]\cap C)\cdot P(D|A\cap B\cap C) = P(A\cap B)\cdot P(C|A\cap B)\, P(D|A\cap B\cap C)$$

$$= P(A)\cdot P(B|A)\cdot P(C|A\cap B)\cdot P(D|A\cap B\cap C)$$

⑥ 2.26 Page 48

$P(A\cap B) = P(A)\cdot P(B)$

$P(A\cap C) = P(A)\cdot P(C) \Rightarrow$ A is not necessarily ind. of $B\cup C$ $[P(A\cap(B\cup C)) \neq P(A)\cdot P(B\cup C)$

$P(A\cap B) = .48 = P(A)\cdot P(B) = (0.6)(0.8)$

$P(A) = 0.6$

$P(A\cap C) = .3 = (0.6)(0.5) = P(A)\cdot P(C)$

$P(B) = 0.8$

$P(C) = 0.5$

$P(A\cap(B\cup C)) = .54$

$P(B\cup C) = 0.92$

$P(A)\cdot P(B\cup C) = 0.552 \neq P(A\cap(B\cup C))$

⑦ 2.56 page 54

$P(VD) = .12$ $P(A) = .39$ $P(VE) = .08$

$P(D) = .17$ $P(E) = .29$

a) $P(D\cup VD) = P(D) + P(VD) = .29$

b) $P((VD\cup VE)^c) = .17 + .39 + .29 = .80$

c) $P(A\cup D\cup VD) = .39 + .17 + .12 = .63$

d) $P(A\cup E\cup VE) = .75$

④ 2-16 page 36

a) $p = \frac{A}{A+B} = \frac{3}{3+2} = \frac{3}{5} = 0.6$

b) $P(S) = 1$

$P(A) \geq 0 \quad \forall A \in S$

People will always bet positive amounts against or in favor of a certain event, or if they think the event is not possible, they will bet 0\$.

bet in favor $= a \geq 0$ If the bet in favor is 0\$, that implies the bet against "b" must be greater than 0\$. $(b > 0)$

Let

$p = \text{subjective probability} = \frac{a}{a+b}$ of event in favor

$a \geq 0$ and $b > 0$

$\Rightarrow \quad a + b > 0$

If $\quad a \geq 0$

$p = \frac{a}{a+b} \geq \frac{0}{a+b} = 0 \quad \therefore \quad p \geq 0$ ■

b) We defined the probability of the event in favor as $\frac{a}{a+b}$,

then the subjective probability of the event against is $\frac{b}{a+b}$.

$$\frac{a}{a+b} + \frac{b}{a+b} = \frac{a+b}{a+b} = 1$$

Either the event happens with $p = \frac{a}{a+b}$ or it doesn't with $q = \frac{b}{a+b}$.

But any of the two things has to happen with $p = 1$ ■

(8) 2-88 page 57

$P(J.C.) = .74$ $P(J.C. \cap M) = .52$ $P(JC \cap M \cap MH) = .34$

$P(A \cap B) = P(A)\,P(B|A)$

$P(M) = .7$ $P(JC \cap MH) = .46$

$P(MH) = .62$ $P(M \cap MH) = .44$ $P(J.C.^c) = .26$

a) $P(M \mid JC) = \dfrac{P(JC \cap M)}{P(JC)} = \dfrac{.52}{.74} = .7027$

$P(JC^c \cap M) = P(M) - P(M \cap JC)$

$= .18$

b) $P(MH \mid JC \cap M) = \dfrac{P(JC \cap M \cap MH)}{P(JC \cap M)} = \dfrac{.34}{.52} = 0.6538$

$P(JC^c \cap MH) = .16$

c) $P(JC^c \mid M \cup MH) = \dfrac{P(JC^c \cap M \cup (JC^c \cap MH))}{P(M) + P(MH) - P(M \cap MH)} = \dfrac{.18 + .16 - \overbrace{(.44 - .34)}^{.10}}{.7 + .62 - .44} = \dfrac{.24}{.88} = .\overline{27} = \dfrac{3}{11}$

d) $P(MH \cap JC \mid M^c) = \dfrac{P(MH \cap JC \cap M^c)}{P(M^c)} = \dfrac{P(MH \cap JC) - P(JC \cap M \cap MH)}{.3} = 0.4$