

ADM2303 formula sheet - Fall 2015

Probability theory

Rule of sum of probabilities:

$$P(S) = 1$$

Subtraction rule (Let A^c be compliment of A , i.e., Not A):

$$P(A) = 1 - P(A^c)$$

Addition rule for two mutually exclusive events (where \cup connotes "or" aka union):

$$P(A \cup B) = P(A) + P(B)$$

Addition rule for two not mutually exclusive events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication rule for two independent events (where \cap connotes "and" aka intersection):

$$P(A \cap B) = P(A) \times P(B)$$

Multiplication rule for n independent events:

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) \times P(A_2) \times \dots \times P(A_n)$$

Multiplication rule for dependent events:

$$P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$$

Partition rule: for a partition B_1, B_2, \dots, B_k :

$$P(A) = \sum_{i=1}^k P(A \cap B_i) = \sum_{i=1}^k P(A|B_i)P(B_i)$$

Bayes' formula:

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)} = \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^k P(A|B_i)P(B_i)}$$

Checking for independence of event A and B :

$$\begin{aligned} \text{If } P(A|B) &= P(A) \text{ and } P(B|A) = P(B) \\ &\Rightarrow A \text{ and } B \text{ are independent} \end{aligned}$$

Random variables (RV)

Expected value of discrete RV X :

$$E(X) = \mu = \sum_{i=1}^n x_i P(X = x_i)$$

Variance of discrete RV X :

$$\begin{aligned} Var(X) &= \sigma^2 = \sum_{i=1}^n (x_i - \mu)^2 P(X = x_i) \\ &= \sum_{i=1}^n x_i^2 P(X = x_i) - \mu^2 \end{aligned}$$

Standard deviation of discrete RV X :

$$SD(X) = \sigma = \sqrt{Var(X)}$$

Coefficient of variation of discrete RV X :

$$CV(X) = \frac{SD(X)}{E(X)}$$

Combining random variables

Adding a constant c to random variable X :

$$\begin{aligned} E(X \pm c) &= E(X) \pm c \\ Var(X \pm c) &= Var(X) \end{aligned}$$

Multiplying random variable X by a constant a :

$$\begin{aligned} E(aX) &= aE(X) \\ Var(aX) &= a^2 Var(X) \end{aligned}$$

Expected value of linear combination of RVs:

$$E(aX \pm bY \pm c) = aE(X) \pm bE(Y) \pm c$$

Variance of linear combination of **independent** RVs:

$$Var(aX \pm bY \pm c) = a^2 Var(X) + b^2 Var(Y)$$

Variance of linear combination of **dependent** RVs:

$$\begin{aligned} Var(aX \pm bY) &= a^2 Var(X) + b^2 Var(Y) \\ &\quad + 2abCOV(X, Y) \end{aligned}$$

$$\text{Where: } COV(X, Y) = rSD(X)SD(Y)$$

Combining Normal random variables

$$\begin{aligned} \text{If } X &\sim N(\mu_x, \sigma_x), Y \sim N(\mu_y, \sigma_y) \text{ and independent} \\ &\Rightarrow aX \pm bY \sim N(a\mu_x \pm b\mu_y, \sqrt{a^2\sigma_x^2 + b^2\sigma_y^2}) \end{aligned}$$

† This can be extended to more than two random variables.

Discrete and continuous distributions

The Binomial probability distribution:

$$\begin{aligned} P(X = x) &= \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \text{ for } x = 0, 1, \dots, n \\ E(X) &= np, Var(X) = np(1-p) \end{aligned}$$

The Poisson probability distribution (if approx'n of binomial, $\lambda = np$):

$$\begin{aligned} P(X = x) &= \frac{e^{-\lambda} \lambda^x}{x!} \text{ for } x = 0, 1, 2, \dots \\ E(X) &= \lambda, Var(X) = \lambda, e = 2.718 \end{aligned}$$

The Geometric probability distribution:

$$\begin{aligned} P(X = x) &= (1-p)^{x-1} p \text{ for } x = 1, 2, \dots \\ E(X) &= \frac{1}{p}, Var(X) = \frac{1-p}{p^2} \end{aligned}$$

The Normal distribution:

$$\begin{aligned} X &\sim N(\mu, \sigma) \Rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \\ Z &= \frac{X-\mu}{\sigma} \sim N(0, 1) \Rightarrow P(Z \leq z) = \text{using normal table} \end{aligned}$$

The Exponential distribution :

$$\begin{aligned} X &\sim Expo(\lambda) \Rightarrow f(x) = \lambda e^{-\lambda x} \\ P(X \leq a) &= 1 - e^{-a\lambda} \\ E(X) &= \frac{1}{\lambda}, Var(X) = \left(\frac{1}{\lambda}\right)^2 \end{aligned}$$

The Uniform distribution:

$$\begin{aligned} X &\sim \text{Uniform}(a, b) \Rightarrow f(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b \\ P(x_1 \leq X \leq x_2) &= \frac{x_2 - x_1}{b - a} \\ E(X) &= \frac{a+b}{2}, SD(X) = \frac{b-a}{\sqrt{12}} \end{aligned}$$