Faculty of Science »
Department of mathematics and statistics

MAT 1341A –DGD 2– Test 3, 2014

20-October, 2014.

Instructor: Barry Jessup.

Family Name:______________________

First Name:______________________

Student number:____________________

Multiple choice answers →	1	
	2	
	3	
	subtotal	
For the marker's use only →	4	
	5	
	6	
	[Bonus] 7	
	Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 4 to 7, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers in the space provided above.
3. Questions 4 and 5 are worth 6 points each, question 6 is worth 4 points, and part marks can be earned in each. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.** Question 7 is a bonus question and is worth only 2 points. It is much more difficult to earn points in the bonus question.
4. Where it is possible to check your work, do so.
5. Good luck! Bonne chance!

1. It is known that a subspace Y of $\mathbf{R}^9$ can be spanned by 8 vectors, and that Y has a linearly independent set with 6 vectors. Then it is always true that:

A. $\dim Y < 6$

B. $\dim Y > 6$

C. $6 < \dim Y \leq 8$

D. $6 \leq \dim Y < 8$

E. $6 \leq \dim Y \leq 8$

F. None of the above is true.

2. Suppose $\{u, v\}$ is a linearly **independent** set in vector space V, and that $w \in V$ is chosen so that $\{u, v, w\}$ is linearly **dependent**. Which of the following statements is **ALWAYS** true?

A. $\{u, w\}$ is linearly dependent.

B. $\{v, w\}$ is linearly dependent.

C. $\{v, u\}$ is linearly dependent.

D. $u \in \text{span}\{v, w\}$.

E. $v \in \text{span}\{u, w\}$.

F. $w \in \text{span}\{u, v\}$.

3. Which of the following statements is true for the linear system (in 4 variables)?

$$\begin{array}{ccccccccc} x_1 & + & 3x_2 & & & + & 6x_4 & = & 0 \\ & & & & x_3 & & & = & 2 \end{array}$$

A. The system has no solutions
B. $(-3s - 6t, s, 2, t)$ is a solution for any values of s and t
C. $(-9, 1, 2, 1)$ is the unique (only) solution of the system
D. $(3s + 6t, s, -2, t)$ is a solution for any values of s and t
E. $(-3s - 6t, s, 0, t)$ is a solution for any values of s and t
F. $(3s + 6t, s, 0, t)$ is a solution for any values of s and t

4. Let $U = \{(x, y, z) \in \mathbf{R}^3 \mid x + 2y + z = 0\}$.

a) Find a basis for U and hence determine $\dim U$.

b) Give a complete geometric description of U.

c) Extend your basis in (a) to a basis of $\mathbf{R}^3$.

5. Recall the vector space $\mathcal{P}_2 = \{a + bx + cx^2 \mid a, b, c \in \mathbf{R}\}$ of polynomial functions of degree at most 2, and define

$$W = \{p \in \mathcal{P}_2 \mid p(1) = 0\}.$$

a) Show that $W = \operatorname{span}\{x - 1, x^2 - x\}$. *(Hint: recall the Factor Theorem: if p is any polynomial and $p(a) = 0$ for some $a \in \mathbf{R}$, then $p(x) = (x - a)q$ for some polynomial q of degree one less than that of p.)*

b) Explain why W is a subspace of $\mathcal{P}_2$ *without using the subspace test.*

c) Find a basis for W.

d) Find $\dim W$.

6. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example - with numbers!
- If you say the statement is always true, you must give a clear explanation.

a) If V is a vector space and $\{v_1, v_2, v_3\} \subset V$ is linearly dependent, then $\{v_1, v_2\}$ is also linearly dependent.

ANSWER

b) If u_1, u_2 and u_3 are non-zero vectors in a vector space V, and $U = \text{span}\{u_1, u_2, u_3\}$ then $\dim U = 3$.

ANSWER

c) If W and X are subspaces of $\mathbf{R}^2$, then their intersection

$$W \cap X = \{v \in \mathbf{R}^2 \mid v \in W \text{ and } v \in X\}$$

is closed under addition.

d) Given that $\{(1,1,1),(0,1,0)\}$ is a basis for a subspace W of $\mathbf{R}^3$, the coordinate vector of $(2,3,2)$ with respect to this basis is $(2,1)$.

7. [Bonus] Suppose that u, v, w are non-zero vectors in $\mathbf{R}^{2014}$ such that $u \cdot v = u \cdot w = v \cdot w = 0$. Prove that $\{u, v, w\}$ is linearly independent. *(Hint: Use the definition. No 'geometric' argument - e.g. "they are not co-planar" - will suffice, and in any case is meaningless to low-dimensional beings like your instructor and marker.)*

(You may use this page for rough work or solutions that did not fit on previous pages.)