ECON 4021 A Fall 2012

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Assignment 1
Marking solutions

1. In attempting to forecast the impact of changes in policy on economic variables, policymakers may assume that historical relationships between variables (that may have been estimated using econometric methods for example) will continue to hold in the future once policy has changed. Lucas argued that the relationships that we may see in the data are in fact the result of individual decisions that result partly from agents' expectations about policy at the time. If policy changes, or is expected to change, people may change their behaviour, causing the historical relationships to shift / break down. A classic example is the shift of the Phillips curve in the late 1960's/early 1970's as it became apparent central bankers were pursuing inflationary policies.

If indeed the estimated relations that we see in the data are the result of individual decision-making that is conditional on agents' expectations of policy etc., then ultimately we would like to model this individual decision-making behaviour and understand how decisions may change as policy changes. Modern macroeconomics attempts to model the behvaiour of individual agents - consumers, firms etc - in terms of "primitives" of the individual that should be invariant to (unaffected by) policy, yielding decision-rules that are a function of policy parameters that allow us to predict how agents' behaviour may change in response to an actual or expected change in policy.

3. From that data we see that time series deviations from trend in **real GDP** are "choppy", and there is no regularity in the amplitude or frequency of **real GDP** about trend. Nevertheless, given these fluctuations in real GDP, we can see regular and

predictable behaviour in the co-movements between economic variables - such as consumption, investment, hours-worked, employment, average labour productivity - over time, and even across geographies. In other words, these variables fluctuate together with strong regularity. These comovement regularities include (contemporaneous) correlations, lead-lag correlations, and relative volatility. When Lucas said "business cycles are all alike", he was referring to the regularity between economic variables, not the fluctuations in real GDP.

2. $d_t = (Y_t - Y_t^g) / Y_t^g$

$= \frac{Y_t}{Y_t^g} - 1$

Since d_t small, $\log(1+d_t) \approx d_t$

$\Rightarrow \log\left(Y_t / Y_t^g\right) \approx d_t$

$\Rightarrow d_t \approx \log Y_t - \log Y_t^g$

4. (a) $\mathcal{L} = u(c, e) + \lambda\left[w(h-e) - \pi - T - c\right]$

$$u_c - \lambda = 0$$

$$u_e - \lambda = 0$$

$$w(h-e) + \pi - T - C = 0$$

(b) $u_{cc} \cdot dc + u_{ce} \cdot de - d\lambda = 0$

$$u_{ec} \cdot dc + u_{ee} \cdot de - w\,d\lambda - \lambda\,dw = 0$$

$$(h-e)\,dw - w \cdot de + d\pi - dT - dc = 0$$

$$\begin{bmatrix} u_{cc} & u_{ce} & -1 \\ u_{ec} & u_{ee} & -w \\ -1 & -w & 0 \end{bmatrix} \begin{bmatrix} dc \\ de \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda\,dw \\ -(h-e)dw - d\pi + dT \end{bmatrix}$$

(c) "A" matrix looks like Bordered Hessian for problem, except that border-terms are negative. In fact, it is the Bordered Hessian $\Rightarrow$ can be

$\Rightarrow$ the bordered Hessians can be written either way $\Rightarrow$ determinant unaffected.

(d) $|A| = (-1)\left[-w \cdot u_{c\ell} + u_{\ell\ell}\right] + w\left[-w \cdot u_{cc} + u_{\ell c}\right]$

$= -u_{\ell\ell} + 2w \cdot u_{c\ell} - w^2 u_{cc}$

(e) $|\bar{H}| = |A|$ from above

(f) For this problem, since $u(\cdot)$ is s.q.c., then $\bar{H}$ is negative definite, which in this case implies $|\bar{H}| > 0$

(g) $$\frac{\partial c}{\partial d\pi} = \frac{\begin{vmatrix} 0 & u_{ce} & -1 \\ 0 & u_{ee} & -w \\ -1 & -w & 0 \end{vmatrix}}{|A|}$$

$$= \frac{-u_{ee} + w \cdot u_{ce}}{|A|}$$

$$\frac{\partial e}{\partial \pi} = \frac{\begin{vmatrix} u_{cc} & 0 & -1 \\ u_{ec} & 0 & -w \\ -1 & -1 & 0 \end{vmatrix}}{|A|} = \frac{u_{ec} - w \cdot u_{cc}}{|A|}$$

(h) The assumption that c & ℓ are normal means that the demand for c & ℓ ↑'s as income ↑'s. The demand for c & ℓ is given by c^* & ℓ^*, & a Δ in income here is equivalent to $d\pi$, therefore, normality implies that

$$\frac{\partial c}{\partial \pi} > 0 \; ; \; \frac{\partial \ell}{\partial \pi} > 0$$

(5)

(a) let $m(\sigma) = [c\, v(e)]^{1-\sigma} - 1$; $n(\sigma) = 1-\sigma$

$m'(\sigma) = \ln[c \cdot v(e)]\,[c\, v(e)]^{1-\sigma}(-1)$

$n'(\sigma) = -1$

$$m'(\sigma)/n'(\sigma) = \ln[c\, v(e)]\,[c\, v(e)]^{1-\sigma}$$
$$= [\ln c + \ln v(e)] \cdot [c\, v(e)]^{1-\sigma}$$

$$\lim_{\sigma \to 1} \frac{m'(\sigma)}{n'(\sigma)}$$
$$= [\ln c + \ln v(e)][c\, v(e)]^{0}$$
$$= \ln c + \ln v(e)$$

(b) from class & williamson,

$$\frac{\partial \ell}{\partial w} = \frac{-u_c + (h-\ell)(u_{c\ell} - w\,u_{cc})}{\nabla} \qquad \circledast$$

$$u_c = [c \cdot v(\ell)]^{-\sigma} \cdot v(\ell)$$

$$u_{cc} = -\sigma \cdot [c\, v(\ell)]^{-\sigma-1} (v(\ell))^2$$

$$u_{c\ell} = -v(\ell) \cdot \sigma \cdot [c\, v(\ell)]^{-\sigma-1} c\, v'(\ell)$$

$$= -\sigma [c \cdot v(\ell)]^{-\sigma} v'(\ell) + [c\, v(\ell)]^{-\sigma} v'(\ell)$$

$$= (1-\sigma)[c\, v(\ell)]^{-\sigma} v'(\ell)$$

(don't need to simplify more than this for part (c), but....

- from FOC's, we know that

$$w = u_e / u_c$$

$$= \frac{[\cdot]^{-\sigma} \cdot c\, v'(e)}{[\cdot]^{-\sigma} \cdot v(e)} = \frac{c \cdot v'(e)}{v(e)} \quad (*A)$$

So $w \cdot u_{cc} =$

$$c \frac{v'(e)}{v(e)} (-\sigma) [\cdot]^{-\sigma-1} \cdot (v(e))^2$$

$$= -\sigma [c\, v(e)]^{-\sigma} \cdot v'(e)$$

So $u_{ce} - w \cdot u_{cc}$

$$= (1-\sigma)[\cdot]^{-\sigma} v'(e) + \sigma [\cdot]^{-\sigma} v'(e)\, v'(e)$$

$$= [c\, v(e)]^{-\sigma} \cdot v'(e)$$

So can simplify $\frac{\partial e}{\partial w}$ as

$$\frac{\partial e}{\partial w} = -\frac{[c\, v(e)]^{-\sigma}}{\nabla} v(e) + \frac{(h-e)}{\nabla} [c\, v(e)]^{-\sigma} v'(e) \quad (***)$$

(c) now in eq$\underline{m}$, if $c = w(h-e)$:

find expression for $h-e$

$$h-e = \frac{c}{w} = \frac{c}{c \cdot w'(e)/v(e)} = v(e)/w'(e)$$

Sub into (***)

$$\frac{\partial e}{\partial w} = \frac{-[\cdot]^{-\nabla} v(e) + v(e)/w'(e)\,[\cdot]^{-\nabla} w'(e)}{\nabla}$$

$$= 0$$

$$\boxed{\frac{\partial e}{\partial w} = 0}$$

(d) We can interpret this result of zero as the substitution effect cancelling out the income effect - ie neither dominates the other.

(e) Here we can either think about the single period being "a long time"- ie many years - or alternatively think about requiring that these equations must hold every period. Since the SE cancels the IE, that agent's optimal choice of leisure doesn't change as the real wage changes. We can think about this as a vertical labour supply curve on a plot of wage on the vertical axes and hours on the horizontal axes . As the wages rises over time due to say a shifting labour demand curve, the real wage will rise, yet hours-worked will remain constant, consistent with the evidence given in the question.