

Question	Mark
Part A/54	
Part B:	B1/9
	B2/12
	B3/12
	B4/13
Total Exam Mark / 100	

Multiple Choice Answer Sheet

NAME (LAST then FIRST, in print): _____

STUDENT NUMBER: _____

Please shade in or circle your answer to each question in Part A.

1. (a) (b) (c) (d) (e)

2. (a) (b) (c) (d) (e)

3. (a) (b) (c) (d) (e)

4. (a) (b) (c) (d) (e)

5. (a) (b) (c) (d) (e)

6. (a) (b) (c) (d) (e)

7. (a) (b) (c) (d) (e)

8. (a) (b) (c) (d) (e)

9. (a) (b) (c) (d) (e)

10. (a) (b) (c) (d) (e)

11. (a) (b) (c) (d) (e)

12. (a) (b) (c) (d) (e)

13. (a) (b) (c) (d) (e)

14. (a) (b) (c) (d) (e)

15. (a) (b) (c) (d) (e)

16. (a) (b) (c) (d) (e)

17. (a) (b) (c) (d) (e)

18. (a) (b) (c) (d) (e)

PART A - Multiple Choice Questions

INSTRUCTIONS: All answers must be given on the **Multiple Choice Answer Sheet** that is attached to this paper. Each question is worth **3** marks. No explanation required. For rough work, use the back side of a page.

1. The domain of the function $f(x) = \frac{1}{\sqrt{(x-1)(x-2)}}$ is
- a) $[1, 2]$, or equivalently, $\{x : 1 \leq x \leq 2\}$.
 - b) $(1, 2)$, or equivalently, $\{x : 1 < x < 2\}$.
 - c) $(-\infty, 1] \cup [2, +\infty)$, or equivalently, $\{x : x \leq 1 \text{ and } x \geq 2\}$.
 - d) $(-\infty, 1) \cup (2, +\infty)$, or equivalently, $\{x : x < 1 \text{ and } x > 2\}$.
 - e) None of the above.
2. Let $f(x) = 2x^3 - 1$ and $g(x) = \frac{1}{x}$. Then the value of the composition $f(g(1))$ is
- a) 1.
 - b) $-\frac{1}{3}$.
 - c) -1.
 - d) -3.
 - e) None of the above.
3. If $e^{x+2} = 3$, what is x ?
- a) $\ln 2$.
 - b) $3 - \ln 2$.
 - c) $\ln 3 - 2$.
 - d) $\frac{\ln 3}{2}$.
 - e) None of the above.

4. The expression $\frac{(x^{0.4})^2 \cdot x^{-3.8}}{x^2}$ simplifies to

- a) x^5 . b) x^{-5} . c) x . d) x^{-1} . (e) None of the above.

5. Which of the following is equal to $\log_{\frac{1}{2}} \frac{1}{8}$?

- a) $\frac{1}{3}$. b) $-\frac{1}{3}$. c) 3. d) -3. e) None of the above.

6. The statement $3 \ln(2x) - \ln(x^2) + \ln 5$ written as a single logarithm is

- a) $\ln x$. b) $\ln(40x)$. c) $\ln(8x^3 - x^2 + 5)$. d) $\ln(5 - x^2)$. e) None of the above.

7. What is $\lim_{x \rightarrow \infty} \frac{4x^3 + x^2 - 3}{1 + 2x - x^3}$?

- a) -4. b) -3. c) 0. d) ∞ . e) None of the above.

8. What is the slope of the curve $y = 3x^{2/3}$ at $x = \frac{1}{8}$?

- a) 1. b) 4. c) $\frac{1}{4}$. d) $\frac{4}{3}$. e) None of the above.

9. Given that the profit function for a company is

$$P(x) = x^2 + \frac{50}{x} + 10x - 7,$$

find the **marginal profit** at a production level of $x = 5$ units.

- a) 78. b) 87. c) 22. d) 18. e) None of the above.

10. The derivative f' of $f(x) = (3x^2 - e^x)^7$ is

- a) $7(3x^2 - e^x)^6 \cdot (6x - e^x)$. b) $7(3x^2 - e^x)^6 \cdot (6x - 1)$. c) $(3x^2 - e^x)^6 \cdot (6x - e^x)$.
d) $7(3x^2 - e^x) \cdot (6x - 1)$. e) None of the above.

11. What are the critical numbers of the function $f(x) = \frac{1}{x+2}$?

- a) -2. b) 0. c) 2. d) No critical numbers. e) None of the above.

12. The second derivative f'' of $f(x) = \ln(3x - 1)$, $(x > \frac{1}{3})$ is

- a) $\frac{-9}{(3x - 1)^2}$ b) $\frac{3}{(3x - 1)^2}$ c) $\frac{-9x}{3x - 1}$ d) $\frac{3}{3x - 1}$ e) None of the above.

13. The graph of the function $y = \frac{x^2}{x^2 + 1}$ has

- a) no horizontal asymptote and a vertical asymptote $x = -1$.
b) a horizontal asymptote $y = 1$ and a vertical asymptote $x = -1$.
c) a horizontal asymptote $y = 1$ and no vertical asymptote.
d) neither horizontal no vertical asymptote.
e) none of the above.

14. Consider the function $f(x) = x^3 - 3x^2 + 1$. The function is increasing when:

- a) $x \in (0, 2)$.
b) $x \in (0, \infty)$.
c) $x \in (-\infty, 0) \cup (2, \infty)$.
d) $x \in \mathbb{R}$.
e) None of the above.

15. Let $f(x, y) = \frac{x^4}{6} - x^2 + \frac{5}{6}$. What are the inflection points of the function (if any)?
- a) $(0, -1)$ and $(0, 1)$. b) $(-1, 0)$ and $(1, 0)$. c) $(1, 0)$.
d) No inflection points. e) None of the above.
16. Let $f(x, y) = e^{x+3y}$. What is $f_{yy}(1, 0)$?
- a) 1. b) e . c) $3e$. d) $9e$. e) None of the above.
17. Consider the Cobb-Douglas production function $f(x, y) = 9x^{2/3}y^{1/3}$, where x is the number of units of labour and y is the number of units of capital. What is the marginal productivity of **labour** at $x = 27$, $y = 64$?
- a) 8. b) 16. c) 256. d) 432. e) None of the above.
18. The value of $\int_{-1}^2 (x + 1) dx$ is
- a) -2.5 . b) -1 . c) 3. d) 4.5. e) None of the above.

PART B - Long Answer Questions

INSTRUCTIONS: Answer all questions and show all appropriate steps in your work; otherwise only partial marks may be awarded.

B1. [9 Marks] An amount of \$10,000 is deposited in a bank that pays interest at the rate of 6% per year. Answer the questions below, rounding the answers to two decimal points.

[2] **(a)** If the money is invested for the period of 5 year compounded **quarterly**, what would be the **interest** earned?

[3] **(b)** How long would it take for the investment to grow to \$12,000 if the interest is compounded **semiannually**?

[1] **(c)** What would be the accumulated amount (or return) after 10 years from the investment if the interest is compounded **continuously**?

[3] **(d)** What should the interest rate be with **continuous compounding** in order to double the original amount of money after 10 years?

B2. [12 Marks] Use the method of Lagrange multipliers to find the maximum and the minimum values of the function

$$f(x, y) = xy$$

subject to the constraint

$$x^2 + y^2 = 8.$$

B3. [12 Marks]

[5] (a) Find all the critical points of the function $f(x, y) = x^2 - 4xy + 2y^2 + 4x + 8y - 1$. (Do NOT classify them).

[7] (b) The function $f(x, y) = 2x^3 - 2xy + y^2 + 7$ has two critical points $(0, 0)$ and $\left(\frac{1}{3}, \frac{1}{3}\right)$. Use the Second Derivative Test to classify each point as local maximum, local minimum or saddle point.

B4. [13 Marks]

(a) Evaluate the following integrals:

[2] (i) $x > 0$, $\int \left(2x^5 + \frac{1}{x} \right) dx$ [4] (ii) $\int_0^2 (3x - 2)^3 dx$ [3] (iii) $\int 2e^{1-6x} dx$.

[4] (b) Consider the equation $x^3 - 2xy + y^4 = 5$, where $y = y(x)$ is defined implicitly as a function of x . Find $y'(x) = \frac{dy}{dx}$.