CARLETON UNIVERSITY

FINAL/DEFERRED
EXAMINATION
April 2012

DURATION: 3 HOURS

Department Name & Course Number: Mathematics and Statistics MATH 1009*ABCD
Course Instructor(s) : Dr Nikitina, Dr Devdariani, Dr Nguyen, Dr Mallick.

AUTHORIZED MEMORANDA

Non programmable, non graphing, non symbolic logic calculators are allowed.

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This exam consists of **11** pages, including this title page. There are two parts of the exam. Part A consists of 17 Multiple Choice questions worth 3 marks each. The answers to part A must be given on the **Multiple Choice Answer Sheet** which is attached to this examination paper.

This examination question paper **MAY NOT** be taken from the examination room.

This examination question paper **MAY NOT** be released to the library.

Family Name (print) : _____ **First Name :** _____

Student Number : _____ **Section:**
A (Dr. Nikitina, TR 8:30 am)
B (Dr. Devdariani, TR 1:00 pm)
C (Dr. Nguyen, TR 2:30 pm)
D (Dr. Mallick, MW 6:00 pm)

Question	**Mark**
Part A/51	
Part B: B1/16	
B2/10	
B3/10	
B4/13	
Total Exam Mark / 100	

Multiple Choice Answer Sheet

NAME (LAST then FIRST, in print): _____

STUDENT NUMBER: _____

Please shade in your answer to each question in Part A.

1. (a) (b) (c) (d)
2. (a) (b) (c) (d)
3. (a) (b) (c) (d)
4. (a) (b) (c) (d)
5. (a) (b) (c) (d)
6. (a) (b) (c) (d)
7. (a) (b) (c) (d)
8. (a) (b) (c) (d)
9. (a) (b) (c) (d)
10. (a) (b) (c) (d)
11. (a) (b) (c) (d)
12. (a) (b) (c) (d)
13. (a) (b) (c) (d)
14. (a) (b) (c) (d)
15. (a) (b) (c) (d)
16. (a) (b) (c) (d)
17. (a) (b) (c) (d)

PART A - Multiple Choice Questions

INSTRUCTIONS: All answers must be given on the **Multiple Choice Answer Sheet** that is attached to this paper. Each question is worth **3** marks. No explanation required.

1. The domain of the function $f(x) = \sqrt{(x+1)(x-4)}$ is

   (a) $[-1, 4]$, or equivalently, $\{x : -1 \leq x \leq 4\}$.

   (b) $(-1, 4)$, or equivalently, $\{x : -1 < x < 4\}$.

   (c) $(-\infty, -1) \bigcup (4, \infty)$, or equivalently, $\{x : x < -1 \text{ or } x > 4\}$.

   (d) $(-\infty, -1] \bigcup [4, \infty)$, or equivalently, $\{x : x \leq -1 \text{ or } x \geq 4\}$.

2. What is $\lim\limits_{x \to \infty} \dfrac{x^2 - x + 7}{x^3 + x - 5}$?

   (a) 0 (b) 1 (c) ∞ (d) Does not exist

3. If $e^{x+2} = 7$, what is x?

   (a) $\ln 2 - 7$ (b) $7 - \ln 2$ (c) $\ln 7 - 2$ (d) $2 - \ln 7$

4. Which of the following is equal to $\ln x - \ln(x^3) + \ln(x^2)$?

   (a) $\ln(x - x^3 + x^2)$ (b) $6 \ln x$ (c) $2 \ln x$ (d) 0

5. The expression $\dfrac{x^{2.4} \cdot (x^{-0.1})^4}{x^{-3} \cdot (x^2)^2}$ simplifies to

   (a) x (b) $x^{-0.5}$ (c) x^3 (d) 1

6. Which of the following is equal to $\log_{\frac{1}{2}} 16$?

   (a) $\dfrac{1}{4}$ (b) $-\dfrac{1}{4}$ (c) 4 (d) -4

7. What is the slope of the curve $y = 2^x$ at $x = 2$?

   (a) $2 \ln 2$ (b) $4 \ln 2$ (c) 4 (d) 0

8. Given that the cost function for a company is

$$C(x) = -0.01x^3 - 0.025x^2 + 60x + 2 \quad \text{for} \quad 0 < x < 40.$$

Find the **marginal cost** at a production level of $x = 20$ units.

(a) 47 (b) 49 (c) 55.5 (d) 1112

9. The derivative $f'(x)$ of $f(x) = (3x^2 + 1)^{5/3}$ is

(a) $10(3x^2 + 1)^{8/3}$ (b) $\frac{5}{3}(3x^2 + 1)^{2/3}$ (c) $10x(3x + 1)^{2/3}$ (d) $5x(3x^2 + 1)^{8/3}$

10. The graph of the function $y = \dfrac{3x}{x - 1}$ has

(a) no horizontal asymptote and a vertical asymptote $x = 1$.

(b) a horizontal asymptote $y = 3$ and a vertical asymptote $x = 1$.

(c) a horizontal asymptote $y = 3$ and no vertical asymptote.

(d) neither horizontal nor vertical asymptote.

11. What are the critical numbers of the function $f(x) = 3x^{1/3}$?

    (a) $x = 0$ (b) $x = 3$ (c) $x = \dfrac{1}{3}$ (d) None

12. A company estimates that its daily cost function is $C(x) = x^3 - 6x^2 + 13x$ million dollars and its daily revenue function is $R(x) = 13x$ million dollars. What is the value of x that maximizes the profit?

    (a) 1 (b) 2 (c) 4 (d) 5

13. The second derivative $f''(x)$ of $f(x) = \log_3(x-5)$, $x > 5$, is

    (a) $\dfrac{-1}{(x-5)^2}$ (b) $\dfrac{1}{(x-5)}$ (c) $\dfrac{-1}{\ln 3 \cdot (x-5)^2}$ (d) $\dfrac{1}{\ln 3 \cdot (x-5)}$

14. Let $f(x, y) = e^{2x+y}$. What is $f_{xx}(0, 1)$?

    (a) e (b) $4e$ (c) 0 (d) None of the above

15. How long will it take an investment of $\$5,000$ to grow to $\$6,000$ if the investment earns interest at the rate of 4% compounded **continuously**?

    (a) 9.3 years (b) 7.2 years (c) 4.5 years (d) 0.2 years

16. Consider the Cobb-Douglas production function $f(x, y) = 8x^{1/4}y^{3/4}$, where x is the number of units of labour and y is the number of units of capital. What is the marginal productivity of **capital** at $x = 16,\ y = 81$?

    (a) 432 (b) $\frac{27}{4}$ (c) 81 (d) 4

17. The value of $\int_{-2}^{1} (2x + 1)\, dx$ is

    (a) 0 (b) 1 (c) 4 (d) 6

PART B - Written Questions

INSTRUCTIONS: For full marks, be sure to show all of your steps.

B1. [16 Marks] Consider the function $f(x) = x^3 - 3x + 2$.

[6] **(a)** Find the intervals where f is increasing and those where f is decreasing.

[3] **(b)** Find the relative extrema of f and the values of f at these points.

[3] **(c)** Determine the intervals where the graph of the function is concave up and those where it is concave down.

[2] **(d)** Find the inflection points of f. (Give both the x- and the y-coordinates).

[2] **(e)** Sketch the graph of f.

B2. **[10 Marks]** Use the method of Lagrange multipliers to minimize the function

$$f(x, y) = x^2 + 2y^2 - xy$$

subject to the constraint

$$2x + y = 22.$$

B3. [10 Marks]

[4] **(a)** Find all the critical points of the function $f(x, y) = x^2 - 6xy + y^2 - 16y + 10$. (Do NOT classify them).

[6] **(b)** Given that the function $f(x, y) = 2x^3 + y^2 - 6x + 4y$ has two critical points $(1, -2)$ and $(-1, -2)$. Use the Second Derivative Test to classify each point as local maximum, local minimum or saddle point.

B4. [13 Marks]

[9] **(a)** Evaluate the following integrals:

[2] (i) $\displaystyle\int \left(5x^4 + \frac{1}{x}\right) dx$ [4] (ii) $\displaystyle\int_0^1 (3x-2)^4\, dx$ [3] (iii) $\displaystyle\int 6x^2\, e^{x^3+1}\, dx.$

[4] **(b)** Consider the equation $xy + y^2 - x^2 = 10$, where $y = y(x)$ is defined implicitly as a function of x. Find $y'(x) = \dfrac{dy}{dx}$.