FINAL SUMMARY ENGR-242 (ch. 1,2,3,4,5,6,7,8,9)

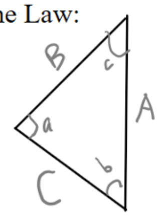
Review:

- Dot product:
  - Multiply each component then add all together.
    - Ex: u*v= (u1v1+u2v2+u3v3)=____(this will be a number not a vector).
- u*v = ||u||||v||cosθ if u,v≠0 and θ is the angle between the vectors.
  - θ=cos^{-1}(u*v/||u||||v||)
  - u*v >0, θ =acute angle
  - u*v <0,θ=obtuse angle
  - u*v=0 then u⊥v
  - u*v=1 then u is parallel to v
- $||u|| = \sqrt{(u_1{}^2 + u_2{}^2 + u_3{}^2)}$ this is the magnitude or norm of a vector

Cosine rule:

$$C = \sqrt{A^2 + B^2 - 2AB\cos(c)}$$
$$A = \sqrt{C^2 + B^2 - 2AC\cos(a)}$$
$$B = \sqrt{A^2 + C^2 - 2AC\cos(b)}$$

Sine Law:



$$\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)}$$

Finding a Force vector knowing only two points on its line and its magnitude:

F =||F||*λ where λ is $\frac{AB}{||AB||}$

AB is a vector along the same line of action as F

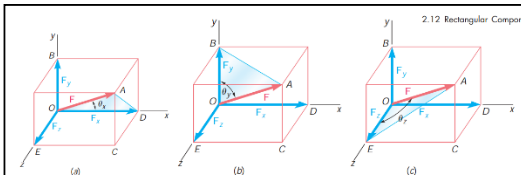


Fig. 2.31

$$F_x = F\cos\theta_x,\ \ F_y = F\cos\theta_y,\ \ F_z = F\cos\theta_z$$

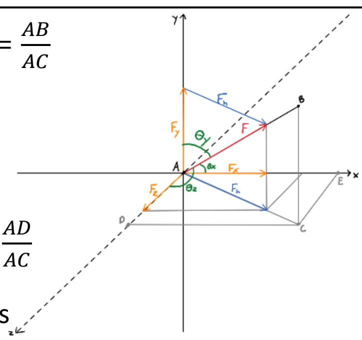
F_h=Fsinθ_y $\quad \frac{F}{F_h} = \frac{AB}{AC}$

F_x=$F_h$cosθ_x

F_z=$F_h$cosθ_z = F_h sinθ_x

$$\frac{F_x}{F_h} = \frac{AE}{AC}\ \ and\ \ \frac{F_z}{F_h} = \frac{AD}{AC}$$

Use similar triangles

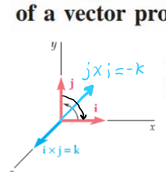


Cross product:

$$\mathbf{V} = (P_yQ_z - P_zQ_y)\mathbf{i} + (P_zQ_x - P_xQ_z)\mathbf{j} + (P_xQ_y - P_yQ_x)\mathbf{k}$$

Rectangular components of a vector product

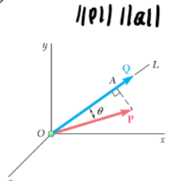
$$V_x = P_yQ_z - P_zQ_y$$
$$V_y = P_zQ_x - P_xQ_z$$
$$V_z = P_xQ_y - P_yQ_x$$



$\mathbf{i} \times \mathbf{i} = \mathbf{0}$	$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$	$\mathbf{k} \times \mathbf{i} = \mathbf{j}$
$\mathbf{i} \times \mathbf{j} = \mathbf{k}$	$\mathbf{j} \times \mathbf{j} = \mathbf{0}$	$\mathbf{k} \times \mathbf{j} = -\mathbf{i}$
$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$	$\mathbf{j} \times \mathbf{k} = \mathbf{i}$	$\mathbf{k} \times \mathbf{k} = \mathbf{0}$

Angle between two vectors:

$$\cos\theta = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{||P||\ ||Q||}$$



Projections:

Of a Force Vector on an axis OL: P_{OL}=Pλ

Of a Moment vector on an axis BL:

$$M_{BL} = \boldsymbol{\lambda} \cdot \mathbf{M}_B = \boldsymbol{\lambda} \cdot (\mathbf{r}_{A/B} \times \mathbf{F})$$

$$M_{BL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$

$$x_{A/B} = x_A - x_B \qquad y_{A/B} = y_A - y_B \qquad z_{A/B} = z_A - z_B$$

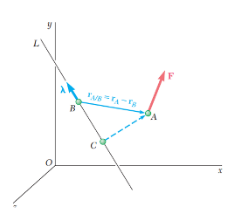


Fig. 3.24 The moment of a force about an axis or line L can be found by evaluating the mixed triple product at a point B on the line. The choice of B is arbitrary, since using any other point on the line, such as C, yields the same result.

Scalar triple product:

$\underline{S} * (\underline{P} \times \underline{Q}) = S_x(P_yQ_z - P_zQ_y) + S_y(P_zQ_x - P_xQ_z) + S_z(P_xQ_y - P_yQ_x)$

$$\mathbf{S} \cdot (\mathbf{P} \times \mathbf{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

S(PxQ)=-S(QxP)=Q(SxP)=P(QxS)

Moment:

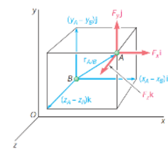
$$\mathbf{M}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$

where $x_{A/B}$, $y_{A/B}$, and $z_{A/B}$ denote the components of the vector $\mathbf{r}_{A/B}$:

$$x_{A/B} = x_A - x_B \qquad y_{A/B} = y_A - y_B \qquad z_{A/B} = z_A - z_B$$

M=r × F=d*F=F*r*sinθ

$$M_x = yF_z - zF_y$$
$$M_y = zF_x - xF_z$$
$$M_z = xF_y - yF_x$$



Moment of a couple

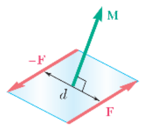


Fig. 3.27 The moment **M** of a couple equals the product of F and d, is perpendicular to the plane of the couple, and may be applied at any point of that plane.

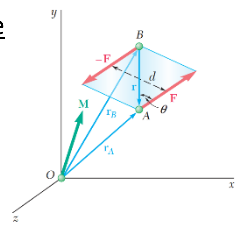


Fig. 3.26 The moment **M** of the couple about O is the sum of the moments of **F** and of −**F** about O.

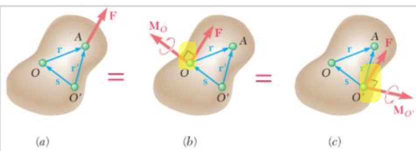


Fig. 3.35 Moving a force to different points. (a) Initial force **F** acting at A; (b) force **F** acting at O and a couple; (c) force **F** acting at O′ and a different couple.

$$\mathbf{M}_{O'} = \mathbf{r}' \times \mathbf{F} = (\mathbf{r} + \mathbf{s}) \times \mathbf{F} = \mathbf{r} \times \mathbf{F} + \mathbf{s} \times \mathbf{F}$$

$$\mathbf{M}_{O'} = \mathbf{M}_O + \mathbf{s} \times \mathbf{F}$$

This helps us with the next section (seen below)

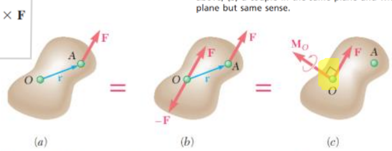


Fig. 3.34 Replacing a force with a force and a couple. (a) Initial force **F** acting at point A; (b) attaching equal and opposite forces at O; (c) force **F** acting at point O and a couple.

Equivalent couples

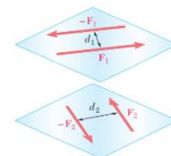


Fig. 3.28 Two couples have the same moment if they lie in parallel planes, have the same sense, and if $F_1d_1 = F_2d_2$.

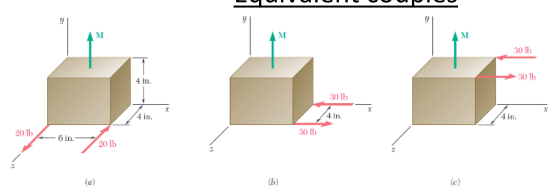


Fig. 3.29 Three equivalent couples. (a) A couple acting on the bottom of the box, acting counterclockwise viewed from above; (b) a couple in the same plane and with the same sense but larger forces than in (a); (c) a couple acting in a different plane but same sense.

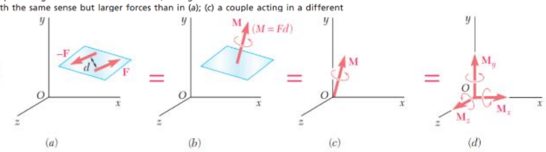


Fig. 3.33 (a) A couple formed by two forces can be represented by (b) a couple vector, oriented perpendicular to the plane of the couple. (c) The couple vector is a free vector and can be moved to other points of application, such as the origin. (d) A couple vector can be resolved into components along the coordinate axes.

Reducing a System of Forces to a Force-Couple System

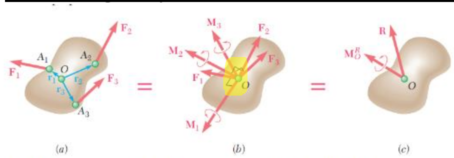
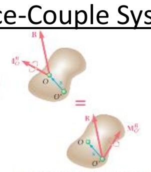


Fig. 3.36 Reducing a system of forces to a force-couple system. (a) Initial system of forces; (b) all the forces moved to act at point O, with couple vectors added; (c) all the forces reduced to a resultant force vector and all the couple vectors reduced to a resultant couple vector.

$$\mathbf{R} = \Sigma\mathbf{F} \qquad \mathbf{M}_O^R = \Sigma\mathbf{M}_O = \Sigma(\mathbf{r} \times \mathbf{F})$$



ig. 3.37 Once a system of forces has been educed to a force-couple system at one oint, we can replace it with an equivalent orce-couple system at another point. The orce resultant stays the same, but we have to dd the moment of the resultant force about he new point to the resultant couple vector.

$$\mathbf{M}_{O'}^R = \mathbf{M}_O^R + \mathbf{s} \times \mathbf{R}$$

Resultant of Parallel Forces

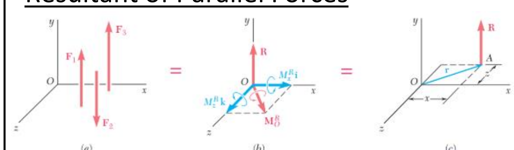


Fig. 3.40 Reducing a system of parallel forces. (a) Initial system of forces; (b) equivalent force-couple system at O, resolved into components; (c) moving **R** to point A, chosen so that the moment of **R** about O equals the resultant moment about O.

Reduction of Coplanar Forces

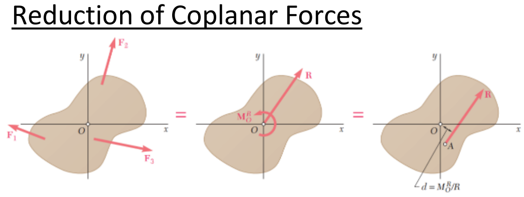


Fig. 3.38 Reducing a system of coplanar forces. (a) Initial system of forces; (b) equivalent force-couple system at O; (c) moving the resultant force to a point A such that the moment of **R** about O equals the couple vector.

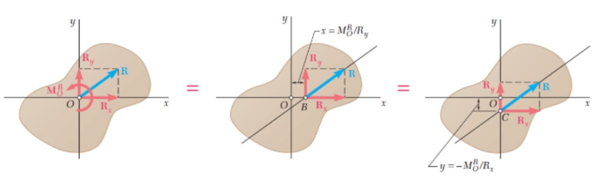


Fig. 3.39 Reducing a system of coplanar forces by using rectangular components. (a) From Fig. 3.38(b), resolve the resultant into components along the x and y axes; (b) determining the x intercept of the final line of action of the resultant; (c) determining the y intercept of the final line of action of the resultant.

The previous reductions are only possible if the moments and the forces are perpendicular. To solve when they aren't, split them into perpendicular components.

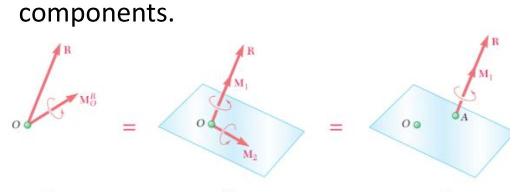
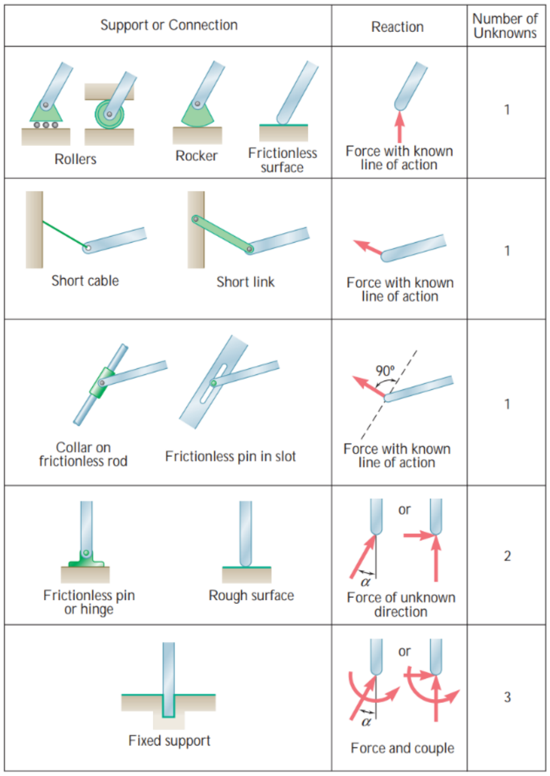
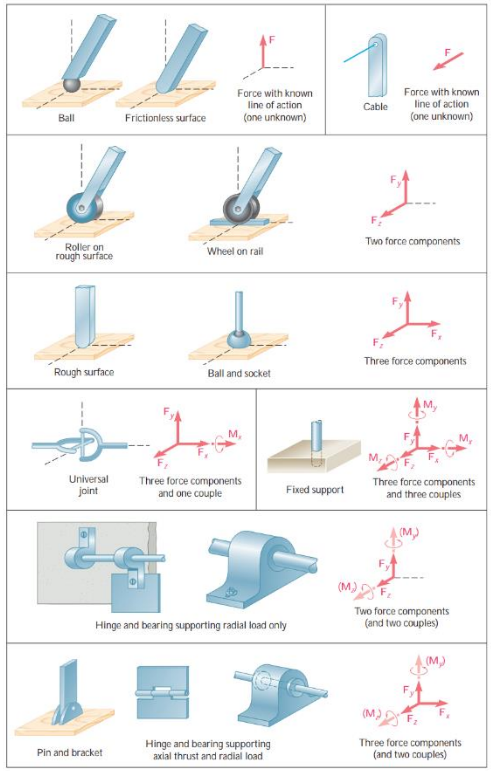


Fig. 3.41 Reducing a system of forces to a wrench. (a) General force system reduced to a single force and a couple vector, not perpendicular to each other; (b) resolving the couple vector into components along the line of action of the force and perpendicular to it; (c) moving the force and collinear couple vector (the wrench) to eliminate the couple vector perpendicular to the force.

2D Reactions



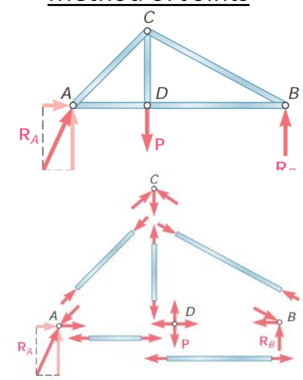
3D Reactions



To solve:

1) Draw free body diagram.
2) Equilibrium gives:
   - $F_{net}x=0$
   - $F_{net}y=0$
   - $F_{net}z=0$
   - M_{net}(about any point)=0
3) Solve for all unknowns.

Method of Joints



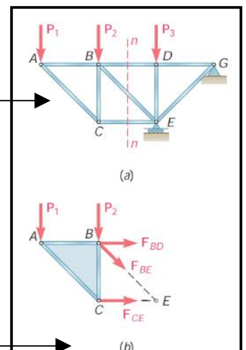
Method of joints:

1) Draw a free body diagram for entire truss (to find reaction forces).
   a. Use $F_{net}x$, $F_{net}y$, M_{net} all =0 to find forces
2) Draw a free body diagram for each joint (to find forces in each member). Start with the joints with the smallest number of unknown forces, preferably at one of the ends of the truss.
   a. Use $F_{net}x$, $F_{net}y$, M_{net} all =0 to find forces

   Note: use similar triangles and trig to help find ratios of forces.

Method of sections: (use to find forces in a specific truss member).

1) Draw a free body diagram for entire truss (to find reaction forces).
   a. Use $F_{net}x$, $F_{net}y$, M_{net} all =0 to find forces.
2) To find the force in member BE:
   a. Split the truss so that you split member BE.
   b. Pick the side of the truss with the least unknowns and external forces to simplify future work, then treat that section as its own truss and solve using $F_{net}x$, $F_{net}y$, M_{net} all =0.



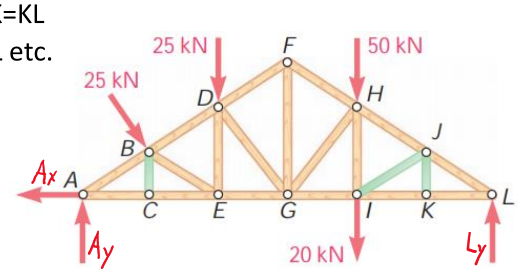
Members:

Zero-Force members:

- Forces in opposite members must be equal so any 3rd member joining them is a zero force member.
  - Ex: Members AC and CE are opposite, so BC is a zero-force member. Same for JK and IJ.

Also:

- AC=CE, HJ=JL, GI=IK=KL
- $JL_y=\pm L_y$, and $JL_x=\pm KL$ etc.



2. To determine whether any other truss is or is not completely constrained and determinate, you first count the number m of its members, the number n of its joints, and the number r of the reaction components at its supports. You then compare the sum $m + r$ representing the number of unknowns and the product $2n$ representing the number of available independent equilibrium equations.

a. If $m + r < 2n$, there are fewer unknowns than equations. Thus, some of the equations cannot be satisfied; the truss is only *partially constrained*.

b. If $m + r > 2n$, there are more unknowns than equations. Thus, some of the unknowns cannot be determined; the truss is *indeterminate*.

c. If $m + r = 2n$, there are as many unknowns as there are equations. This, however, does not mean that all the unknowns can be determined and that all the equations can be satisfied. To find out whether the truss is *completely* or *improperly constrained*, you should try to determine the reactions at its supports and the forces in its members. If all can be found, the truss is *completely constrained and determinate*.

Frames and Machines:

- Have more than just axial forces on each end, so may have other forces acting at 63° to the member.

Steps:

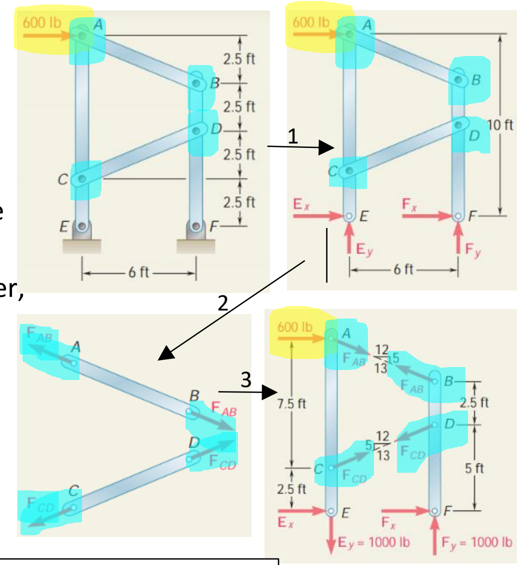
1) Free body for entire frame with equations $F_{net}x=0$, $F_{net}y=0$, $F_{net}z=0$, M_{net}(about any point)=0
2) Free body for each 2 force member.
3) Free body for all multi-force members.

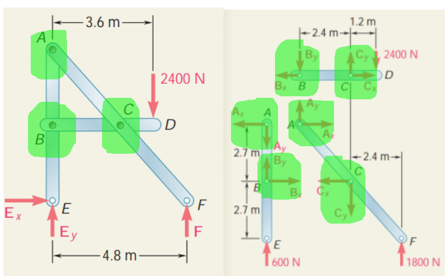
   *If there is an external force acting on a joint between two members, only use it in one of the free body diagrams.

   **This does not apply to reaction forces, those are shown on all free body diagrams but will be positive in one and negative in the other.

   ***Where a multi-force member connects to a two force member, draw a force equal and opposite to that shown on the two force member diagram.

   ****Where a multi-force member connects with another multi-force member, draw the vertical and horizontal components and draw the equal but opposite components on the 2nd attached mulri-force member.





Note: if you obtain a negative value for a force that you arbitrarily drew onto the diagram, it just mean the force should be in the other direction. Do not change it however because if you do then you need to rewrite all your equations.

Forces in 3D:

- A vector = $\frac{\text{(its magnitude)(a direction vector)}}{\text{(norm of the direction vector)}}$
- Find the moment about a point (using cross product)
  - You know in equilibrium that that moment =0, so that means
    - Sum of all i components=0
    - Sum of all j components=0
    - Sum of all k components=0
  - Use that to solve for unknowns.
- 6 Equilibrzium equations are:
  - $F_{net}x=0$
  - $F_{net}y=0$
  - $F_{net}z=0$
  - $M_{(point)}=0$
    - $\sum i_{coefficients} = 0$
    - $\sum j_{coefficients} = 0$
    - $\sum k_{coefficients} = 0$

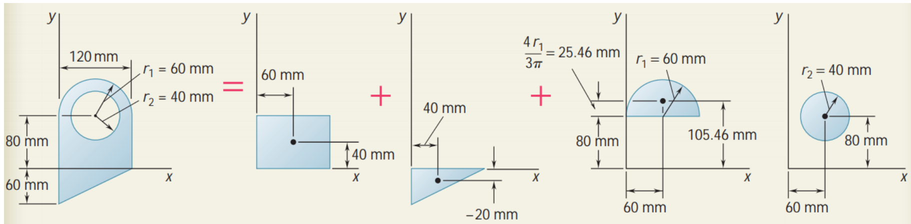
Note: If axis are not given, place them so that as many forces as possible lie on them. That way those forces don't affect the moment about that axis.

Centroids:

- If an object has two lines of symmetry that intersect at 90 degrees, they intersect at the axis of symmetry.

$$Centroid\ (do\ for\ x\ and\ y) = \bar{x} = \frac{1}{A_{total}}\int xdA = \frac{1}{A_{total}}\sum_i A_i\bar{x}_i \qquad x_{center\ of\ mass}m_{total} = \int xdm = \sum_i m_i x_i$$

Total $\bar{x}$*(A_{total})= $\bar{x}_1A_1+\bar{x}_2A_2$, meaning you can add up all the areas and all their centroids, while subtracting the negative areas.



Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

b. Location of Centroid. Substituting the values given in the table into the equations defining the centroid of a composite area, we obtain

$\bar{X}\Sigma A = \Sigma\bar{x}A$: $\quad \bar{X}(13.828 \times 10^3\ \text{mm}^2) = 757.7 \times 10^3\ \text{mm}^3$

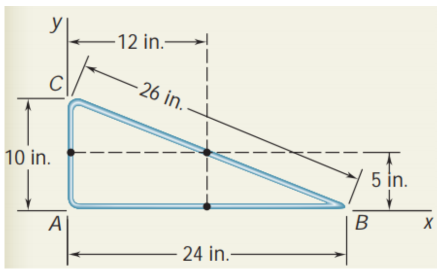
$\bar{X} = 54.8$ mm ◀

$\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\quad \bar{Y}(13.828 \times 10^3\ \text{mm}^2) = 506.2 \times 10^3\ \text{mm}^3$

$\bar{Y} = 36.6$ mm ◀

Steps:

1) Make table of values for L or A, $\bar{x}$, $\bar{y}$, $\bar{x}(L\ or\ A)$, $\bar{y}(L\ or\ A)$
2) Sum all the values.
3) Divide $\bar{x}A$ value by total area to find $\bar{x}$. (same thing for L).



Segment	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
AB	24	12	0	288	0
BC	26	12	5	312	130
CA	10	0	5	0	50
	$\Sigma L = 60$			$\Sigma\bar{x}L = 600$	$\Sigma\bar{y}L = 180$

Substituting the values obtained from the table into the equations defining the centroid of a composite line, we obtain

$\bar{X}\Sigma L = \Sigma\bar{x}L$: $\quad \bar{X}(60\ \text{in.}) = 600\ \text{in}^2 \qquad \bar{X} = 10$ in. ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L$: $\quad \bar{Y}(60\ \text{in.}) = 180\ \text{in}^2 \qquad \bar{Y} = 3$ in. ◀

Notes:

- A center of symmetry is an axis of symmetry that is just 0 so on the origin. This is when there are two perpendicular lines to an axis that divides a shape. Then the centroid is on that intersection.

First moment:

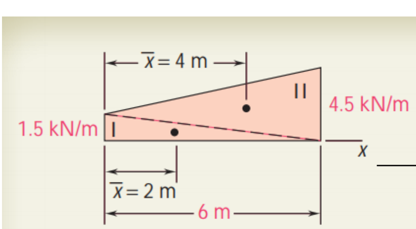
- $Q_y = \bar{x}A_{total} = \int xdA \qquad Q_x = \bar{y}A_{total} = \int ydA$

Distributed loads:

- A distributed load can be replaced by a single force of the same magnitude located at the centroid of the distributed load. This load is also equal to the magnitude of the area under the curve or the integral of that curve.
- If you have a form load made of various shapes, split it up and find each single force with relation to each centroid.

Distributed loads on submerged bodies:

- $W = bp = bgh = brgh$
  - Where: p is the pressure at depth h, g is the specific weight of the liquid, r is the specific density of the liquid, b is the width of the submerged surface.
  - The line of action of the resulting force is perpendicular to the surface of the submerged plane.
- Use this W as a distributed load.



Component	A, kN	$\bar{x}$, m	$\bar{x}A$, kN · m
Triangle I	4.5	2	9
Triangle II	13.5	4	54
	$\Sigma A = 18.0$		$\Sigma\bar{x}A = 63$

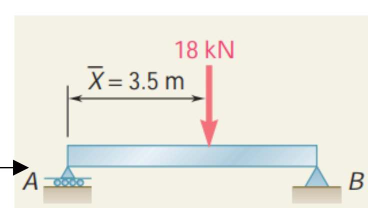
Thus, $\bar{X}\Sigma A = \Sigma\bar{x}A$: $\quad \bar{X}(18\ \text{kN}) = 63\ \text{kN} \cdot \text{m} \qquad \bar{X} = 3.5$ m

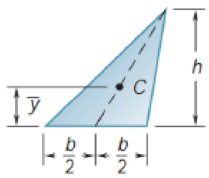
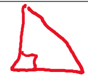
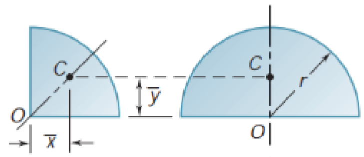
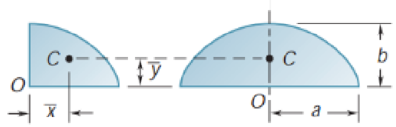
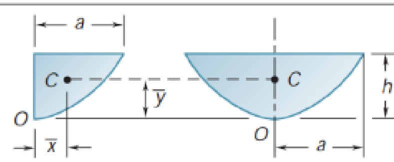
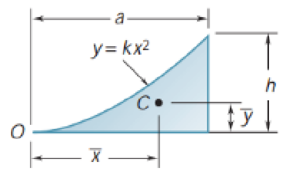
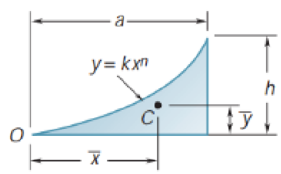
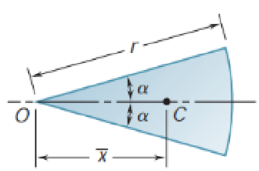
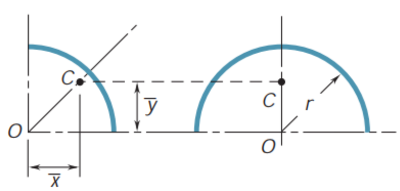
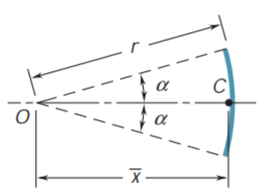
The equivalent concentrated load is

W = 18 kNw ◀

and its line of action is located at a distance

$\bar{X}$ = 3.5 m to the right of A ◀



Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area		 $\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r\sin\alpha}{\alpha}$	0	$2\alpha r$

Internal Forces in members:

- If you cut a member, you will find the internal forces:
  - F which is the axial force and is directed axially.
  - V which is the sheering force and is perpendicular to the member.
  - M which is the bending moment at that point.
- Note that on one side of the cut, the values are positive and on the other side they are negative.

To solve these, use the regular three equations with the added axial force, sheer force and bending moment.

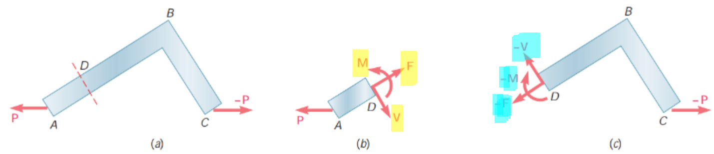
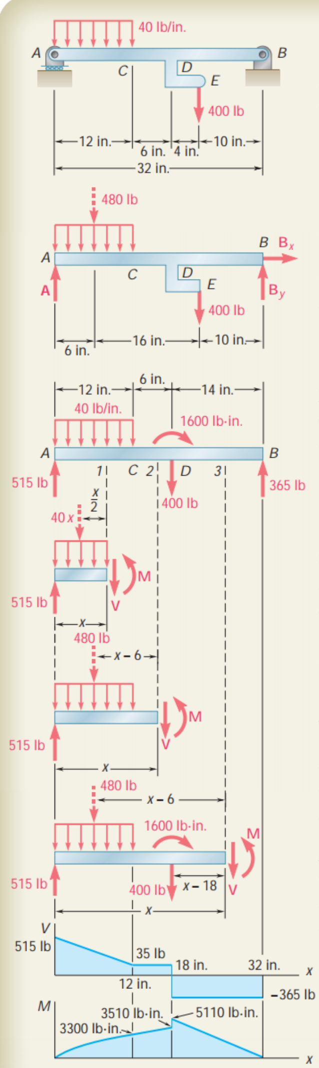


Fig. 7.4



SAMPLE PROBLEM 7.3

Draw the shear and bending-moment diagrams for the beam *AB*. The distributed load of 40 lb/in. extends over 12 in. of the beam, from *A* to *C*, and the 400-lb load is applied at *E*.

SOLUTION

Free-Body: Entire Beam. The reactions are determined by considering the entire beam as a free body.

$+\circlearrowleft \Sigma M_A = 0$: $B_y(32 \text{ in.}) - (480 \text{ lb})(6 \text{ in.}) - (400 \text{ lb})(22 \text{ in.}) = 0$
$B_y = +365 \text{ lb}$ $\quad \mathbf{B}_y = 365 \text{ lb}\uparrow$

$+\circlearrowleft \Sigma M_B = 0$: $(480 \text{ lb})(26 \text{ in.}) + (400 \text{ lb})(10 \text{ in.}) - A(32 \text{ in.}) = 0$
$A = +515 \text{ lb}$ $\quad \mathbf{A} = 515 \text{ lb}\uparrow$

$\xrightarrow{+} \Sigma F_x = 0$: $B_x = 0$ $\quad \mathbf{B}_x = 0$

The 400-lb load is now replaced by an equivalent force-couple system acting on the beam at point *D*.

Shear and Bending Moment. *From A to C.* We determine the internal forces at a distance *x* from point *A* by considering the portion of the beam to the left of section *1*. That part of the distributed load acting on the free body is replaced by its resultant, and we write

$+\uparrow \Sigma F_y = 0$: $515 - 40x - V = 0$ $\quad V = 515 - 40x$
$+\circlearrowleft \Sigma M_1 = 0$: $-515x + 40x(\frac{1}{2}x) + M = 0$ $\quad M = 515x - 20x^2$

Since the free-body diagram shown can be used for all values of *x* smaller than 12 in., the expressions obtained for *V* and *M* are valid throughout the region $0 < x < 12$ in.

From C to D. Considering the portion of the beam to the left of section *2* and again replacing the distributed load by its resultant, we obtain

$+\uparrow \Sigma F_y = 0$: $515 - 480 - V = 0 V = 35 \text{ lb}$
$+\circlearrowleft \Sigma M_2 = 0$: $-515x + 480(x - 6) + M = 0$ $\quad M = (2880 + 35x) \text{ lb} \cdot \text{in.}$

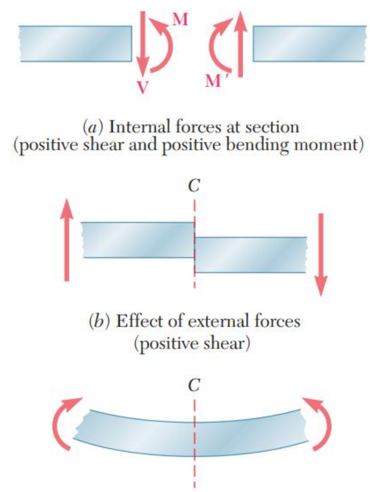
These expressions are valid in the region 12 in. $< x <$ 18 in.

From D to B. Using the portion of the beam to the left of section 3, we obtain for the region 18 in. $< x <$ 32 in.

$+\uparrow \Sigma F_y = 0$: $515 - 480 - 400 - V = 0$ $\quad V = -365 \text{ lb}$
$+\circlearrowleft \Sigma M_3 = 0$: $-515x + 480(x - 6) - 1600 + 400(x - 18) + M = 0$
$M = (11{,}680 - 365x) \text{ lb} \cdot \text{in.}$

Shear and Bending-Moment Diagrams. The shear and bending-moment diagrams for the entire beam can now be plotted. We note that the couple of moment 1600 lb · in. applied at point *D* introduces a discontinuity into the bending-moment diagram.

Positive or Negative Sheer and Bending



(*a*) Internal forces at section (positive shear and positive bending moment)

(*b*) Effect of external forces (positive shear)

(*c*) Effect of external forces (positive bending moment)

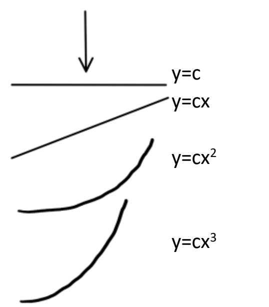
Steps for making the diagram

1) Find reaction forces
2) Draw the blank diagrams with discontinuities anywhere where there is a force on the FBD.
3) Sheer:
   Start from the left add or subtract forces on the plot as you walk along the beam. In the example here, start at 515 lb because of the reaction at A, then subtract 480lb linearly slope until x=12in. Then the force remains constant until x=18in. Then subtract 400lb and keep that value until x=32in. Then reaction at B adds 365lb.

Note: if its in equilibrium and you don't end at 0lb for bending moment, there is a mistake somewhere.

The bending moment diagram can be found by integrating the sheer force diagram or by calculating the area of each shape.

Order of the lines:



Load	P	Constant	Linear
Shear	Constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic

Positive area under shear gives positive slope of moment. Negative area gives negative slope.

$$\frac{dV}{dx} = -w \quad \textbf{(7.1)}$$

$$\frac{dM}{dx} = V \quad \textbf{(7.3)}$$

$$V_D - V_C = -(\text{area under load curve between } C \text{ and } D) \quad \textbf{(7.2}'\textbf{)}$$

$$M_D - M_C = (\text{area under shear curve between } C \text{ and } D) \quad \textbf{(7.4}'\textbf{)}$$

Taking these relations into account, you can use the following procedure to draw the shear and bending-moment diagrams for a beam.

1. Draw a free-body diagram of the entire beam, and use it to determine the reactions at the beam supports.

2. Draw the shear diagram. This can be done as in the preceding section by cutting the beam at various points and considering the free-body diagram of one of the two resulting portions of the beam [Sample Prob. 7.3]. You can, however, consider one of the following alternative procedures.

a. The shear *V* at any point of the beam is the sum of the reactions and loads to the left of that point; an upward force is counted as positive, and a downward force is counted as negative.

b. For a beam carrying a distributed load, you can start from a point where you know V and use Eq. (7.2′) repeatedly to find V at all other points of interest.

3. Draw the bending-moment diagram, using the following procedure.

a. Compute the area under each portion of the shear curve, assigning a positive sign to areas above the x axis and a negative sign to areas below the x axis.

b. Apply Eq. (7.4′) repeatedly [Sample Probs. 7.4 and 7.5], starting from the left end of the beam, where $M = 0$ (except if a couple is applied at that end, or if the beam is a cantilever beam with a fixed left end).

c. Where a couple is applied to the beam, be careful to show a discontinuity in the bending-moment diagram by *increasing* the value of M at that point by an amount equal to the magnitude of the couple if the couple is *clockwise,* or *decreasing* the value of M by that amount if the couple is *counterclockwise* [Sample Prob. 7.7].

4. Determine the location and magnitude of $|M|_{max}$. The maximum absolute value of the bending moment occurs at one of the points where $dM/dx = 0$ [according to Eq. (7.3), that is at a point where V is equal to zero or changes sign]. You should

a. Determine from the shear diagram the value of $|M|$ where V changes sign; this will occur under a concentrated load [Sample Prob. 7.4].

b. Determine the points where $V = 0$ and the corresponding values of $|M|$; this will occur under a distributed load. To find the distance x between point C where the distributed load starts and point D where the shear is zero, use Eq. (7.2′). For V_C, use the known value of the shear at point C; for V_D, use zero and express the area under the load curve as a function of x [Sample Prob. 7.5].

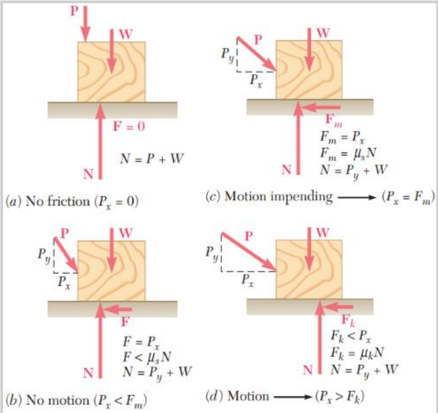
5. You can improve the quality of your drawings by keeping in mind that, at any given point according to Eqs. (7.1) and (7.3), the slope of the V curve is equal to $-w$ and the slope of the M curve is equal to V.

6. Finally, for beams supporting a distributed load expressed as a function $w(x)$, remember that you can obtain the shear V by integrating the function $-w(x)$, and you can obtain the bending moment M by integrating $V(x)$ [Eqs. (7.2) and (7.4)].

Static Friction: $F_{max}=\mu_s N$

Kinetic Friction: $F_k=\mu_k N$

- μ_s is the coefficient of static friction
- μ_k is the coefficient of kinetic friction



Angle of static friction

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

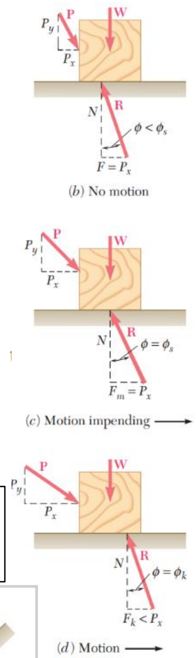
$$\tan\phi_s = \mu_s$$

Angle of kinetic friction

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

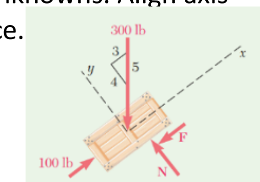
$$\tan\phi_k = \mu_k$$

Note: Force R has components F_s and N



Steps:

1. Draw a free body diagram and solve for unknowns. Align axis with surface.



2. Determine if the body is static, about to move or moving so that you can find your friction force.
3. Draw all cases. Is one block going to slide? Two? Tipping?
   a. Tipping is M_{net} about the tipping point.
   b. F_{nety} usually is only made up of the objects weight and normal forces. Use that to find N which will help you find F_s (if impending motion).

Note: Force R has components F_s and N

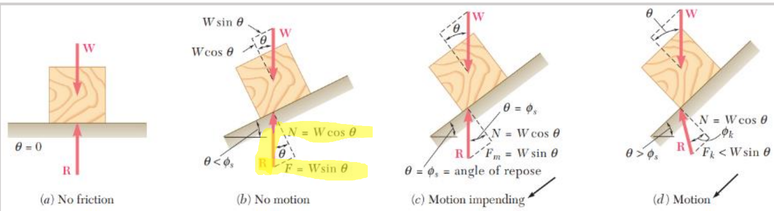
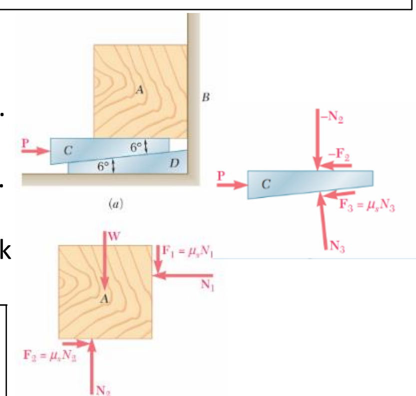


Fig. 8.4 (a) Block on horizontal board, friction force is zero; (b) board's angle of inclination is less than angle of static friction, no motion; (c) board's angle of inclination equals angle of friction, motion is impending; (d) angle of inclination is greater than angle of friction, forces are unbalanced and motion occurs.

Note:

- If all the forces are perpendicular to the surface it won't have any friction forces.
- If F_s has not reached its maximum static value, then $F_s \neq \mu_s N$. Only use this equation if movement is impending. In this case, solve for F_s by F_{net} components.
- Friction is always opposing motion.
- Normal forces cause by other blocks have are in opposite direction on each block
- Always start with the block with the fewest unknowns.



Moment of inertia:

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$

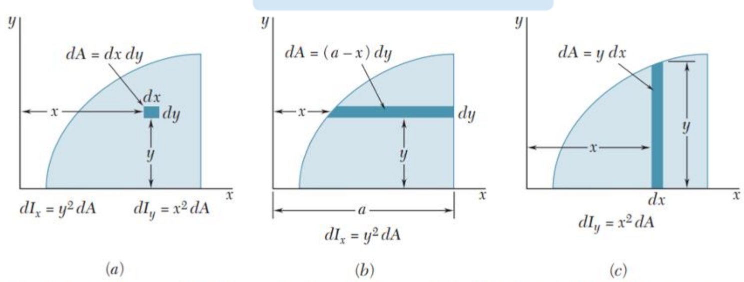


Fig. 9.3 (a) Rectangular moments of inertia dI_x and dI_y of an area dA; (b) calculating I_x with a horizontal strip; (c) calculating I_y with a vertical strip.

Polar moment of inertia:

$r^2=x^2+y^2$

$$J_O = I_x + I_y \quad | \quad J_O = \int r^2 dA = \int (x^2+y^2)dA = \int y^2 dA + \int x^2 dA$$

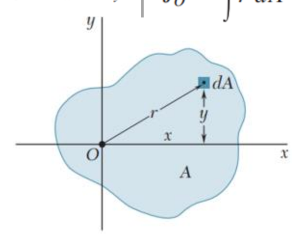


Fig. 9.6 Distance r used to evaluate the polar moment of inertia of area A.

Parallel axis theorem:

$$I = \bar{I} + Ad^2$$

Where:

->d is the distance between the two axis

->$\bar{I}$ is the moment of inertia with respect to the centroid.

Radius of gyration:

k_x is the distance to the x axis. And:

$$k_x = \sqrt{\frac{I_x}{A}} \quad | \quad k^2 = \bar{k}^2 + d^2 \quad | \quad J_O = \bar{J}_C + Ad^2 \text{ or } k_O^2 = \bar{k}_C^2 + d^2$$

$$k_O^2 = k_x^2 + k_y^2$$

$$I_y = k_y^2 A \qquad k_y = \sqrt{\frac{I_y}{A}}$$

$$J_O = k_O^2 A \qquad k_O = \sqrt{\frac{J_O}{A}}$$

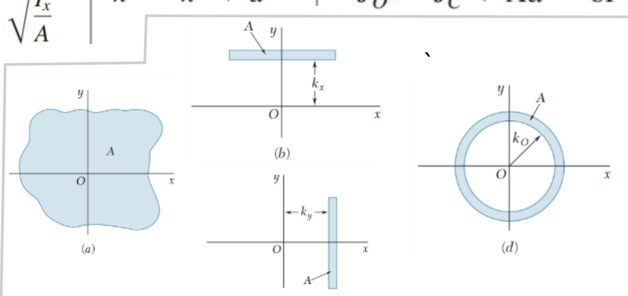


Fig. 9.7 (a) Area A with given moment of inertia I_x; (b) compressing the area to a horizontal strip with radius of gyration k_x; (c) compressing the area to a vertical strip with radius of gyration k_y; (d) compressing the area to a circular ring with polar radius of gyration k_O.

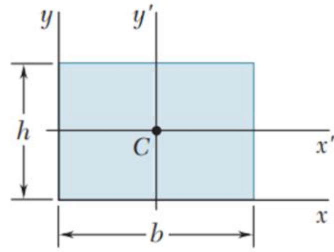
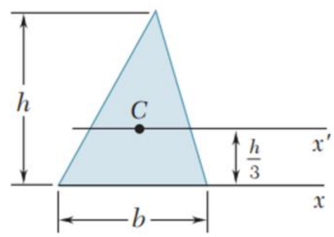
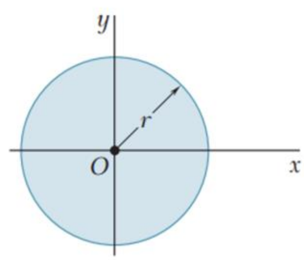
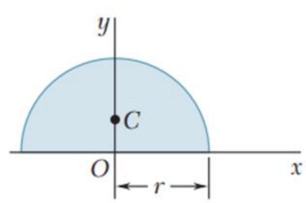
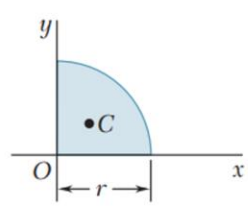
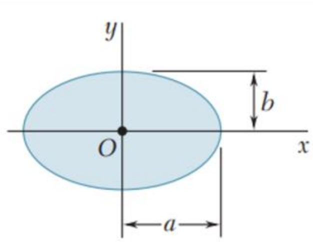
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Fig. 9.12 Moments of inertia of common geometric shapes.

APPENDIX:

Table 1.1 SI Prefixes

Multiplication Factor	Prefix†	Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto‡	h
10 = 10^{1}	deka‡	da
0.1 = 10^{-1}	deci‡	d
0.01 = 10^{-2}	centi‡	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

Table 1.2 Principal SI Units Used in Mechanics

Quantity	Unit	Symbol	Formula
Acceleration	Meter per second squared	. . .	m/s^2
Angle	Radian	rad	†
Angular acceleration	Radian per second squared	. . .	rad/s^2
Angular velocity	Radian per second	. . .	rad/s
Area	Square meter	. . .	m^2
Density	Kilogram per cubic meter	. . .	kg/m^3
Energy	Joule	J	N·m
Force	Newton	N	$kg·m/s^2$
Frequency	Hertz	Hz	s^{-1}
Impulse	Newton-second	. . .	kg·m/s
Length	Meter	m	‡
Mass	Kilogram	kg	‡
Moment of a force	Newton-meter	. . .	N·m
Power	Watt	W	J/s
Pressure	Pascal	Pa	N/m^2
Stress	Pascal	Pa	N/m^2
Time	Second	s	‡
Velocity	Meter per second	. . .	m/s
Volume			
Solids	Cubic meter	. . .	m^3
Liquids	Liter	L	10^{-3} m^3
Work	Joule	J	N·m

Table 1.3 U.S. Customary Units and Their SI Equivalents

Quantity	U.S. Customary Unit	SI Equivalent
Acceleration	ft/s^2	0.3048 m/s^2
	$in./s^2$	0.0254 m/s^2
Area	ft^2	0.0929 m^2
	in^2	645.2 mm^2
Energy	ft·lb	1.356 J
Force	kip	4.448 kN
	lb	4.448 N
	oz	0.2780 N
Impulse	lb·s	4.448 N·s
Length	ft	0.3048 m
	in.	25.40 mm
	mi	1.609 km
Mass	oz mass	28.35 g
	lb mass	0.4536 kg
	slug	14.59 kg
	ton	907.2 kg
Moment of a force	lb·ft	1.356 N·m
	lb·in.	0.1130 N·m
Moment of inertia		
Of an area	in^4	0.4162×10^6 mm^4
Of a mass	$lb \cdot ft \cdot s^2$	1.356 $kg \cdot m^2$
Momentum	lb·s	4.448 kg·m/s
Power	ft·lb/s	1.356 W
	hp	745.7 W
Pressure or stress	lb/ft^2	47.88 Pa
	lb/in^2 (psi)	6.895 kPa
Velocity	ft/s	0.3048 m/s
	in./s	0.0254 m/s
	mi/h (mph)	0.4470 m/s
	mi/h (mph)	1.609 km/h
Volume	ft^3	0.02832 m^3
	in^3	16.39 cm^3
Liquids	gal	3.785 L
	qt	0.9464 L
Work	ft·lb	1.356 J