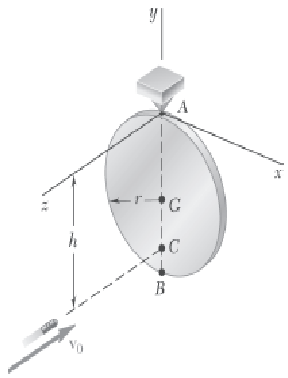
ASSIGNMENT# 4 SOLUTIONS



PROBLEM 17.89

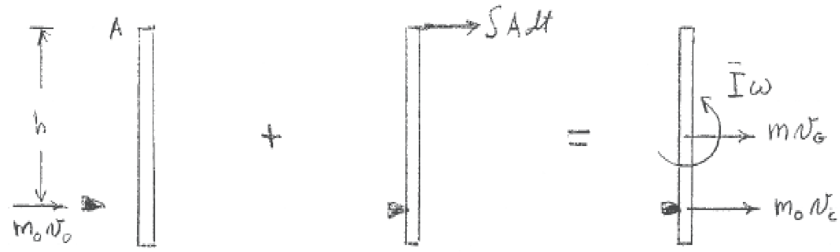
A 30-kg uniform circular plate of radius r is supported by a ball-and-socket joint at point A and is at rest in the vertical xy plane when a bullet with a mass of 15 g is fired with the velocity $\mathbf{v}_0 = -(210 \text{ m/s})\mathbf{k}$ and hits the plate at point C. Knowing that $r = 400$ mm and $h = 700$ mm, determine (*a*) the angular velocity of the plate immediately after the bullet becomes embedded, (*b*) the impulsive reaction at point A, assuming that the bullet becomes embedded in 1.1 ms.

SOLUTION

Moment of inertia: $\bar{I} = \frac{1}{4}mr^2 = \frac{1}{4}(30)(0.4)^2 = 1.2 \text{ kg}\cdot\text{m}^2$

Kinematics: $v_G = r\omega = 0.4\omega, \quad v_C = h\omega = 0.7\omega$

Kinetics:



Syst Momenta$_1$ + **Syst Ext Imp$_{1\to 2}$** = **Syst Momenta$_2$**

↻ moments about A: $m_0 v_0 h + 0 = m_0 v_C h + m v_G r + \bar{I}\omega$

$$(0.015)(210)(0.7) + 0 = (0.015)(0.7\omega)(0.7) + (30)(0.4\omega)(0.4) + 1.2\omega$$

(*a*) $2.205 = 6.00735\omega, \quad \omega = 0.36705$ $\boldsymbol{\omega} = (0.367 \text{ rad/s})\mathbf{i}$ ◀

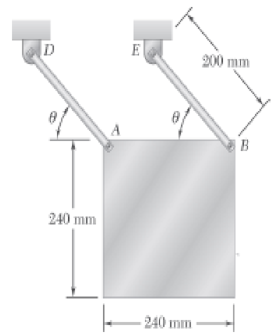
(*b*) z-components: $-m_0 v_0 - \int A\,dt = -m v_G - m_0 v_C$

$$\int A\,dt = -m_0 v_0 + m v_G + m_0 v_C$$

$$= -(0.015)(210) + (30)(0.4)(0.36705) + (0.015)(0.7)(0.36705)$$

$$= 1.25846 \text{ N}\cdot\text{s}$$

$$A = \frac{\int A\,dt}{\Delta t} = \frac{1.25846}{1.1\times 10^{-3}} = 1144 \text{ N}$$ $\mathbf{A} = -(1144 \text{ N})\mathbf{k}$ ◀



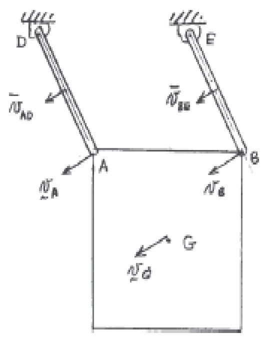
PROBLEM 17.36

A 5-kg uniform square plate is supported by two identical 1.5-kg uniform slender rods AD and BE and is released from rest in the position $\theta = 45°$. Knowing that the angular velocity of AD as it passes through the vertical position is 5.2 rad/s, determine the amount of energy dissipated in friction.

SOLUTION

Moment of inertia of one rod: $\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}(1.5)(0.200)^2 = 0.005 \text{ kg}\cdot\text{m}^2$

Kinematics:



Let ω be the angular velocity of rod AD. It is also the angular velocity of rod BE.

$$\bar{v}_{AD} = \bar{v}_{BE} = \frac{l}{2}\omega = 0.100\omega$$

$$v_A = v_B = v_G = l\omega = 0.200\omega$$

Note that the plate is in translation.

Kinetic energy: $T = \frac{1}{2}(1.5)\bar{v}_{AD}^2 + \frac{1}{2}(0.005)\omega^2 + \frac{1}{2}(1.5)\bar{v}_{BE}^2 + \frac{1}{2}(0.005)\omega^2 + \frac{1}{2}(5)v_G^2$

$$= \frac{1}{2}\left\{2\left[(1.5)(0.100)^2 + (0.005)\right] + (5)(0.200)^2\right\}\omega^2 = 0.120\omega^2$$

Potential energy: Datum is a level line through points D and E.

$$V = -(1.5)(9.81)(0.100\sin\theta) - (1.5)(9.81)(0.100\sin\theta) - (5)(9.81)(0.200\sin\theta + 0.120)$$

$$= -12.753\sin\theta - 5.886$$

Position 1. $\theta = 45°$, $\omega = 0$

$$T_1 = 0$$

$$V_1 = -12.753\sin 45° - 5.886 = -14.9037 \text{ J}$$

$$E_1 = T_1 + V_1 = -14.9037 \text{ J}$$

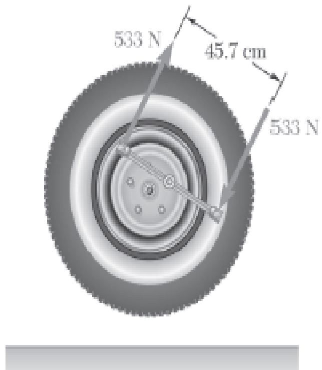
Position 2. $\theta = 90°$, $\omega = 5.2$ rad/s

$$T_2 = (0.120)(5.2)^2 = 3.2448 \text{ J}$$

$$V_2 = -12.753\sin 90° - 5.886 = -18.639 \text{ J}$$

$$E_2 = T_2 + V_2 = -15.3942 \text{ J}$$

Energy lost: $E_1 - E_2 = 0.490 \text{ J}$ ◀



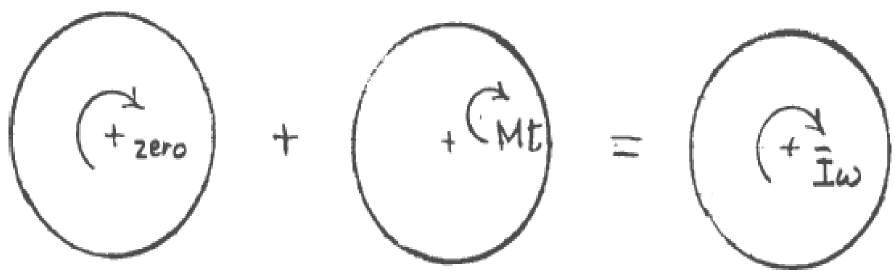
PROBLEM 17.45

A bolt located 5 cm from the center of an automobile wheel is tightened by applying the couple shown for 0.10 s. Assuming that the wheel is free to rotate and is initially at rest, determine the resulting angular velocity of the wheel. The wheel weighs 19 kg and has a radius of gyration of 27.4 cm.

SOLUTION

Moment of inertia: $\bar{I} = m\bar{k}^2 = (19\text{ kg})(.274\text{ kg})^2 = 1.4264\text{ N}\cdot\text{m}\cdot\text{s}^2$

Applied couple: $M = (533\text{ N})(.457\text{ m}) = 243.58\text{ N}\cdot\text{m}$



Syst Momenta$_1$ + **Syst Ext Imp$_{1\to2}$** = **Syst Momenta$_2$**

Moments about axle: $0 + Mt = \bar{I}\omega$

$$0 + (243.58\text{ N}\cdot\text{m})(0.10\text{ s}) = (1.4264\text{ N}\cdot\text{m}\cdot\text{s}^2)\omega$$

$\omega = 17\text{ rad/s}$ ◀

PROBLEM 19.4

A 9 kg block is initially held so that the vertical spring attached as shown is undeformed. Knowing that the block is suddenly released from rest, determine (*a*) the amplitude and frequency of the resulting motion, (*b*) the maximum velocity and maximum acceleration of the block.

SOLUTION

Simple Harmonic Motion:

$$\delta_s = \frac{W}{k} = \frac{88.29 \text{ N}}{15761 \text{ N/m}} = 0056 \text{ m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{15761}{9}} = 41.85 \text{ rad/s} = 2\pi f$$

(*a*) Amplitude $= \delta_s = x_m = .0056 \text{ m}$

$x_m = .0056 \text{ m}$ ◀

$$f = \frac{41.85 \text{ rad/s}}{2\pi} = 6.66$$

$f = 6.66 \text{ Hz}$ ◀

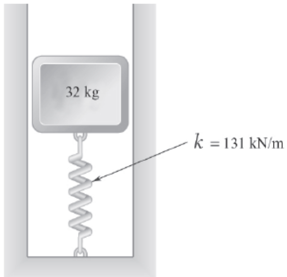
(*b*) $v_m = \omega_n x_m = (41.85 \text{ rad/s})(.0056 \text{ m}) = .234 \text{ m/s}$

$v_m = .234 \text{ m/s}$ ◀

$$a_m = \omega_n^2 x_m = (41.85 \text{ rad/s})^2 (.0056 \text{ m}) = 9.8 \text{ m/s}^2$$

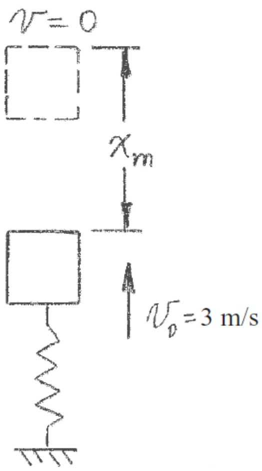
$a_m = 9.8 \text{ m/s}^2$ ◀

PROBLEM 19.5



A 32-kg block is attached to a spring and can move without friction in a slot as shown. The block is in its equilibrium position when it is struck by a hammer which imparts to the block an initial velocity of 3 m/s. Determine (*a*) the period and frequency of the resulting motion, (*b*) the amplitude of the motion and the maximum acceleration of the block.

SOLUTION



$$k = 131 \times 10^3 \text{ N/m}$$

Simple Harmonic Motion:

(*a*) $$x = x_m \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{131 \times 10^3 \text{ N/m}}{32 \text{ kg}}}$$

$$= 63.98 \text{ rad/s}$$

$$\tau_n = \frac{2\pi}{\omega_n} = .0982 \text{ s}$$

$\tau_n = .0982 \text{ s}$ ◀

$$f_n = \frac{1}{\tau_n} = 10.183 \text{ Hz}$$

$f_n = 10.183 \text{ Hz}$ ◀

(*b*) At $t = 0$: $x_0 = 0,\ \dot{x}_0 = v_0 = 3 \text{ m/s}$

$$x_0 = 0 = x_m \sin(\omega_n(0) + \phi) \Rightarrow \phi = 0$$

$$\dot{x}_0 = v_0 = x_m \omega_n \cos(\omega_n(0) + \phi) = x_m \omega_n$$

Substituting $3 \text{ m/s} = x_m 63.98 \text{ rad/s}$

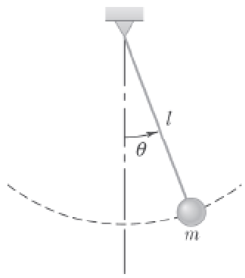
or $x_m = .047 \text{ m}$

$x_m = .047 \text{ m}$ ◀

$$a_m = x_m \omega_n^2 = (.047 \text{ m})(63.98 \text{ rad/s})^2$$

$$= 192.4 \text{ m/s}^2$$

$a_m = 192.4 \text{ m/s}^2$ ◀



PROBLEM 19.16

The bob of a simple pendulum of length l = 1.2 m is moving with a velocity of 180 mm/s to the right at time t = 0 when $\theta = 0$. Assuming simple harmonic motion, determine at t = 1.5 s (*a*) the angle θ, (*b*) the magnitudes of the velocity and acceleration of the bob.

SOLUTION

$$\theta = \theta_m \sin \omega_n t, \quad \dot{\theta} = \theta_m \omega_n \cos \omega_n t, \quad \omega_n = \sqrt{\frac{g}{l}}$$

$$\omega_n = \sqrt{\frac{9.81}{1.2}} = 2.8592 \text{ rad/s}, \qquad t = 0: \quad \dot{\theta} = \frac{0.18}{1.2} = \theta_m \omega_n$$

$$\therefore \theta_m = 0.052462 \text{ radians}$$

At $\quad t = 1.5 \text{ s}, \ \theta = 0.052462 \sin (2.8592)(1.5)$

(*a*) $$\theta = -0.047826 \text{ radians} = -2.74° \blacktriangleleft$$

(*b*) $$|v| = 1.2(0.052462)(2.8592)\cos(2.8592)(1.5)$$

$$|v| = 74.0 \text{ mm/s} \blacktriangleleft$$

$$|a| = 1.2(0.052462)(2.8592)^2 \sin(2.8592)(1.5)$$

$$|a| = 469 \text{ mm/s}^2 \blacktriangleleft$$