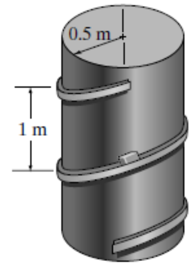
ASSIGNMENT# 3 SOLUTIONS

12–183. The box slides down the helical ramp which is defined by $r = 0.5$ m, $\theta = (0.5t^3)$ rad, and $z = (2 - 0.2t^2)$ m, where t is in seconds. Determine the magnitudes of the velocity and acceleration of the box at the instant $\theta = 2\pi$ rad.



Time Derivatives:

$$r = 0.5 \text{ m}$$

$$\dot{r} = \ddot{r} = 0$$

$$\dot{\theta} = \left(1.5t^2\right)\text{rad/s} \qquad \ddot{\theta} = 3(3t) \text{ rad/s}^2$$

$$z = 2 - 0.2t^2$$

$$\dot{z} = (-0.4t) \text{ m/s} \qquad \ddot{z} = -0.4 \text{ m/s}^2$$

When $\theta = 2\pi$ rad,

$$2\pi = 0.5t^3 \qquad t = 2.325 \text{ s}$$

Thus,

$$\dot{\theta}\big|_{t=2.325\text{ s}} = 1.5(2.325)^2 = 8.108 \text{ rad/s}$$

$$\ddot{\theta}\big|_{t=2.325\text{ s}} = 3(2.325) = 6.975 \text{ rad/s}^2$$

$$\dot{z}\big|_{t=2.325\text{ s}} = -0.4(2.325) = -0.92996 \text{ m/s}$$

$$\ddot{z}\big|_{t=2.325\text{ s}} = -0.4 \text{ m/s}^2$$

Velocity:

$$v_r = \dot{r} = 0$$

$$v_\theta = r\dot{\theta} = 0.5(8.108) = 4.05385 \text{ m/s}$$

$$v_z = \dot{z} = -0.92996 \text{ m/s}$$

Thus, the magnitude of the box's velocity is

$$v = \sqrt{v_r^2 + v_\theta^2 + v_z^2} = \sqrt{0^2 + 4.05385^2 + (-0.92996)^2} = 4.16 \text{ m/s} \qquad \textbf{Ans.}$$

Acceleration:

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 0.5(8.108)^2 = -32.867 \text{ m/s}^2$$

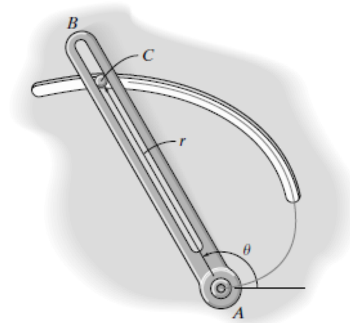
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5(6.975) + 2(0)(8.108)^2 = 3.487 \text{ m/s}^2$$

$$a_z = \ddot{z} = -0.4 \text{ m/s}^2$$

Thus, the magnitude of the box's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \sqrt{(-32.867)^2 + 3.487^2 + (-0.4)^2} = 33.1 \text{ m/s}^2 \qquad \textbf{Ans.}$$

12–186. The slotted arm AB drives pin C through the spiral groove described by the equation $r = a\theta$. If the angular velocity is constant at $\dot{\theta}$, determine the radial and transverse components of velocity and acceleration of the pin.



Time Derivatives: Since $\dot{\theta}$ is constant, then $\ddot{\theta} = 0$.

$$r = a\theta \qquad \dot{r} = a\dot{\theta} \qquad \ddot{r} = a\ddot{\theta} = 0$$

Velocity: Applying Eq. 12–25, we have

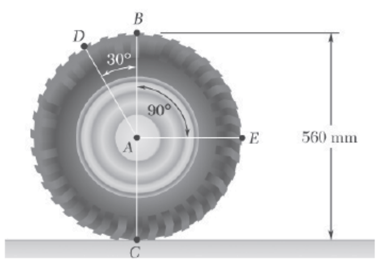
$$v_r = \dot{r} = a\dot{\theta} \qquad \textbf{Ans.}$$

$$v_\theta = r\dot{\theta} = a\theta\dot{\theta} \qquad \textbf{Ans.}$$

Acceleration: Applying Eq. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - a\theta\dot{\theta}^2 = -a\theta\dot{\theta}^2 \qquad \textbf{Ans.}$$

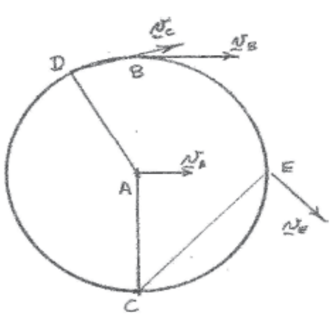
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(a\dot{\theta})(\dot{\theta}) = 2a\dot{\theta}^2 \qquad \textbf{Ans.}$$



PROBLEM 15.69

An automobile travels to the right at a constant speed of 80 km/h. If the diameter of the wheel is 560 mm, determine the velocities of points B, C, D, and E on the rim of the wheel.

SOLUTION



$$\mathbf{v}_A = 80 \text{ km/h} = 22.222 \text{ m/s} \downarrow \qquad \mathbf{v}_C = 0 \blacktriangleleft$$

$$d = 560 \text{ mm}, \qquad r = \frac{d}{2} = 280 \text{ mm} = 0.28 \text{ m}$$

$$\omega = \frac{v_A}{r} = \frac{22.222}{0.28} = 79.364 \text{ rad/s} \circlearrowright$$

$$v_{B/A} = v_{D/A} = v_{E/A} = r\omega$$

$$= (0.28)(79.364) = 22.222 \text{ m/s}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = [22.222 \text{ m/s} \longrightarrow] + [22.222 \text{ m/s} \longrightarrow]$$

$$\mathbf{v}_B = 44.4 \text{ m/s} \longrightarrow \blacktriangleleft$$

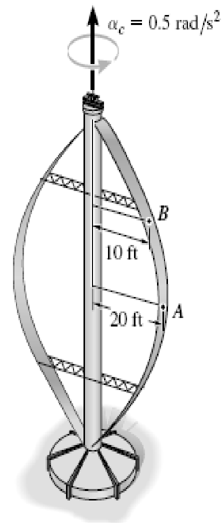
$$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A} = [22.222 \text{ m/s} \longrightarrow] + [22.222 \text{ m/s} \measuredangle 30°]$$

$$\mathbf{v}_D = 42.9 \text{ m/s} \measuredangle 15.0° \blacktriangleleft$$

$$\mathbf{v}_E = \mathbf{v}_A + \mathbf{v}_{E/A} = [22.222 \text{ m/s} \longrightarrow] + [22.222 \text{ m/s} \downarrow]$$

$$\mathbf{v}_E = 31.4 \text{ m/s} \searrow 45.0° \blacktriangleleft$$

16–19. The vertical-axis windmill consists of two blades that have a parabolic shape. If the blades are originally at rest and begin to turn with a constant angular acceleration of $\alpha_c = 0.5 \text{ rad/s}^2$, determine the magnitude of the velocity and acceleration of points A and B on the blade after the blade has rotated through two revolutions.



Angular Motion: The angular velocity of the blade after the blade has rotated $2(2\pi) = 4\pi$ rad can be obtained by applying Eq. 16–7.

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

$$\omega^2 = 0^2 + 2(0.5)(4\pi - 0)$$

$$\omega = 3.545 \text{ rad/s}$$

Motion of A and B: The magnitude of the velocity of point A and B on the blade can be determined using Eq. 16–8.

$$v_A = \omega r_A = 3.545(20) = 70.9 \text{ ft/s} \qquad \textbf{Ans.}$$

$$\omega_B = \omega r_B = 3.545(10) = 35.4 \text{ ft/s} \qquad \textbf{Ans.}$$

The tangential and normal components of the acceleration of point A and B can be determined using Eqs. 16–11 and 16–12 respectively.

$$(a_t)_A = \alpha r_A = 0.5(20) = 10.0 \text{ ft/s}^2$$

$$(a_n)_A = \omega^2 r_A = \left(3.545^2\right)(20) = 251.33 \text{ ft/s}^2$$

$$(a_t)_B = \alpha r_B = 0.5(10) = 5.00 \text{ ft/s}^2$$

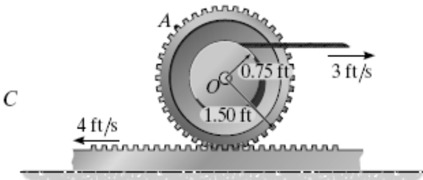
$$(a_n)_B = \omega^2 r_B = \left(3.545^2\right)(10) = 125.66 \text{ ft/s}^2$$

The magnitude of the acceleration of points A and B are

$$(a)_A = \sqrt{(a_t)_A^2 + (a_n)_A^2} = \sqrt{10.0^2 + 251.33^2} = 252 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$(a)_B = \; = \sqrt{(a_t)_B^2 + (a_n)_B^2} = \sqrt{5.00^2 + 125.66^2} = 126 \text{ ft/s}^2 \qquad \textbf{Ans.}$$

16–59. Determine the angular velocity of the gear and the velocity of its center O at the instant shown.

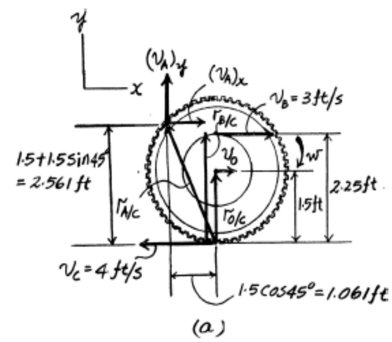


General Plane Motion: Applying the relative velocity equation to points B and C and referring to the kinematic diagram of the gear shown in Fig. a,

$$\mathbf{v}_B = \mathbf{v}_C + \omega \times \mathbf{r}_{B/C}$$

$$3\mathbf{i} = -4\mathbf{i} + \left(-\omega\mathbf{k}\right) \times \left(2.25\mathbf{j}\right)$$

$$3\mathbf{i} = \left(2.25\omega - 4\right)\mathbf{i}$$



Equating the **i** components yields

$$3 = 2.25\omega - 4 \qquad \textbf{(1)}$$

$$\omega = 3.111 \text{ rad/s} \qquad \textbf{Ans. (2)}$$

For points O and C,

$$\mathbf{v}_O = \mathbf{v}_C + \omega \times \mathbf{r}_{O/C}$$

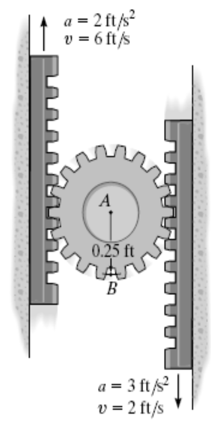
$$= -4\mathbf{i} + \left(-3.111\mathbf{k}\right) \times \left(1.5\mathbf{j}\right)$$

$$= [0.6667\mathbf{i}] \text{ ft/s}$$

Thus,

$$v_O = 0.667 \text{ ft/s} \rightarrow \qquad \textbf{Ans.}$$

16–127. At a given instant, the gear racks have the velocities and accelerations shown. Determine the acceleration of points A and B.



Velocity Analysis: The angular velocity of the gear can be obtained by using the method of instantaneous center of zero velocity. From similar triangles,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{v_C}{r_{C/IC}}$$

$$\frac{6}{r_{D/IC}} = \frac{2}{r_{C/IC}} \qquad \textbf{[1]}$$

Where

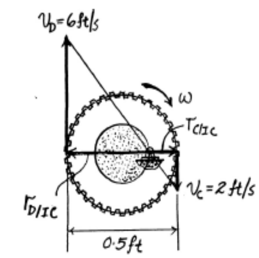
$$r_{D/IC} + r_{C/IC} = 0.5 \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields

$$r_{D/IC} = 0.375\text{ ft} \qquad r_{C/IC} = 0.125\text{ ft}$$

Thus,

$$\omega = \frac{v_D}{r_{D/IC}} = \frac{6}{0.375} = 16.0\text{ rad/s}$$



Acceleration Equation: The angular acceleration of the gear can be obtained by analyzing the angular motion point C and D. Applying Eq. 16–18 with $\mathbf{r}_{D/C} = \{-0.5\mathbf{i}\}$ ft, we have

$$\mathbf{a}_D = \mathbf{a}_C + \alpha \times \mathbf{r}_{D/C} - \omega^2\,\mathbf{r}_{D/C}$$

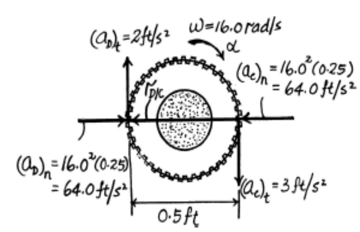
$$64.0\mathbf{i} + 2\mathbf{j} = -64.0\mathbf{i} - 3\mathbf{j} + (-\alpha\mathbf{k}) \times (-0.5\mathbf{i}) - 16.0^2(-0.5\mathbf{i})$$

$$64.0\mathbf{i} + 2\mathbf{j} = 64.0\mathbf{i} + (0.5\alpha - 3)\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components, we have

$$64.0 = 64.0\ (\textbf{\textit{Check!}})$$

$$2 = 0.5\,\alpha - 3 \qquad \alpha = 10.0\text{ rad/s}^2$$

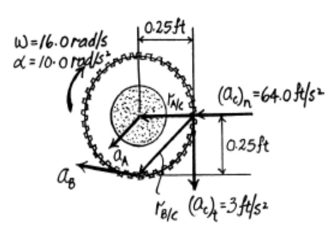


The acceleration of point A can be obtained by analyzing the angular motion point A and C. Applying Eq. 16–18 with $\mathbf{r}_{A/C} = \{-0.25\mathbf{i}\}$ ft, we have

$$\begin{aligned}\mathbf{a}_A &= \mathbf{a}_C + \alpha \times \mathbf{r}_{A/C} - \omega^2\,\mathbf{r}_{A/C}\\ &= -64.0\mathbf{i} - 3\mathbf{j} + (-10.0\mathbf{k}) \times (-0.25\mathbf{i}) - 16.0^2(-0.25\mathbf{i})\\ &= \{0.500\mathbf{j}\}\text{ ft/s}^2\end{aligned}$$

Thus,

$$a_A = 0.500\text{ ft/s}^2 \downarrow \qquad \textbf{Ans.}$$



The acceleration of point B can be obtained by analyzing the angular motion point B and C. Applying Eq. 16–18 with $\mathbf{r}_{B/C} = \{-0.25\mathbf{i} - 0.25\mathbf{j}\}$ ft, we have

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_C + \alpha \times \mathbf{r}_{B/C} - \omega^2\mathbf{r}_{B/C}\\ &= -64.0\mathbf{i} - 3\mathbf{j} + (-10.0\mathbf{k}) \times (-0.25\mathbf{i} - 0.25\mathbf{j}) - 16.0^2(-0.25\mathbf{i} - 0.25\mathbf{j})\\ &= \{-2.50\mathbf{i} + 63.5\mathbf{j}\}\text{ ft/s}^2\end{aligned}$$

The magnitude and direction of the acceleration of point B are given by

$$a_C = \sqrt{(-2.50)^2 + 63.5^2} = 63.5\text{ ft/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{63.5}{2.50} = 87.7^\circ \qquad \textbf{Ans.}$$