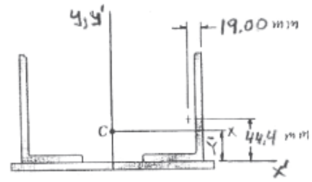
ASSIGNMENT# 2DEVOIR#2

SOLUTION



Angle:

$$A = 2420\text{ mm}^2$$
$$\bar{I}_x = 3.93 \times 10^6\text{ mm}^4$$
$$\bar{I}_y = 1.06 \times 10^6\text{ mm}^4$$

Plate:

$$A = (200\text{ mm})(10\text{ mm}) = 2000\text{ mm}^2$$
$$\bar{I}_x = \frac{1}{12}(200\text{ mm})(10\text{ mm})^3 = 0.01667 \times 10^6\text{ mm}^4$$
$$\bar{I}_y = \frac{1}{12}(10\text{ mm})(200\text{ mm})^3 = 6.6667 \times 10^6\text{ mm}^4$$

Centroid

$$\bar{X} = 0$$
$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

or

$$\bar{Y} = \frac{2(2420\text{ mm}^2)(44.4\text{ mm}) + 2000\text{ mm}^2(-5\text{ mm})}{\left[2(2420) + 2000\right]\text{mm}^2} = \frac{204896\text{ mm}^3}{6840\text{ mm}^2}$$
$$= 29.9556\text{ mm}$$

Now

$$\bar{I}_x = 2(I_x)_{\text{angle}} + (I_x)_{\text{plate}}$$
$$= 2\left[3.93 \times 10^6 + (2420)(44.4 - 29.9556)^2\right]\text{mm}^4$$
$$+\left[0.01667 \times 10^6 + (2000)(29.9556 + 5)^2\right]\text{mm}^4$$
$$= 2(4.4349 \times 10^6)\text{ mm}^4 + 2.4605 \times 10^6\text{ mm}^4$$
$$= 11.3303 \times 10^6\text{ mm}^4$$

or $\bar{I}_x = 11.33 \times 10^6\text{ mm}^4$ ◀

PROBLEM 9.55 CONTINUED

Also
$$\bar{I}_y = 2(I_y)_{\text{angle}} + (\bar{I}_y)_{\text{plate}}$$

Where
$$(\bar{I}_y)_{\text{angle}} = 1.06 \times 10^6 \text{ mm}^4 + (2420 \text{ mm}^2)(b - 19.0 \text{ mm})^2$$

$$= \left[1.06 \times 10^6 + (2420)(b^2 - 38b + 361)\right] \text{mm}^4$$

$$= \left[2420b^2 - 91960b + 1.93362 \times 10^6\right] \text{mm}^4$$

and
$$(\bar{I}_y)_{\text{plate}} = 6.6667 \times 10^6 \text{ mm}^4$$

Now
$$\bar{I}_y = 3(\bar{I}_x)$$

Then
$$2\left[2420b^2 - 91960b + 1.93362 \times 10^6\right] \text{mm}^4 + 6.6667 \times 10^6 \text{ mm}^4 = 3\left[(11.33 \times 10^6) \text{mm}^4\right]$$

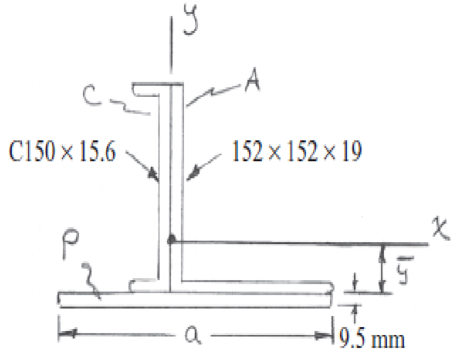
or
$$2420b^2 - 91960b + 1.93362 \times 10^6 - 13.662 \times 10^6 = 0$$

$$b^2 - 38.0b - 4846.5 = 0$$

$$b = 91.16 \text{ mm}$$

or $b = 91.2$ mm ◀

SOLUTION



(*a*) Using shape data from Fig. 9.13B

$$x_A = 44.9 \text{ mm} \qquad A_A = 5420 \text{ mm}^2$$

$$x_C = -12.5 \text{ mm} \qquad A_C = 1980 \text{ mm}^2$$

$$x_P = \left(150 - \frac{a}{2}\right) \text{mm} \qquad A_P = 19\text{a mm}^2$$

From the condition $\bar{x} = \dfrac{\Sigma \bar{x} A}{\Sigma A} = 0$ or $\Sigma \bar{x} A = 0$

$$(44.9)(5420) - (12.5)(1980) + \left[\left(150 - \frac{a}{2}\right)\right](19\text{a}) = 0$$

or $a^2 - 300 - 23\,011 = 0$ $\qquad a = 363$ mm

or $a = 363$ mm ◀

and $A_P = (19)(363) = 6897 \text{ mm}^2$

(*b*) Locate centroid

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{(44.9)(5420) + \left(\frac{150}{2}\right)(1980) - (9.5)(6897)}{5420 + 1980 + 6897}$$

$$= 22.8 \text{ mm}$$

PROBLEM 9.56 CONTINUED

Now $\bar{I}_x = (I_x)_A + (I_x)_C + (I_x)_P$

$$= \left[11.6 \times 10^6 + (5420)(76 - 22.8)^2\right]$$

$$+ \left[6.21 \times 10^6 + (1980)(75 - 22.8)^2\right]$$

$$+ \left[\frac{1}{12}(363)(19)^3 + (6897)(9.5 + 22.8)^2\right]$$

$$= \left(26.94 \times 10^6 + 11.61 \times 10^6 + 7.40 \times 10^6\right) \text{mm}^4$$

$$= 45.95 \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 46 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_y = (I_y)_A + (I_y)_C + (I_y)_P$

$$= \left[11.6 \times 10^6 + (5420)(44.9)^2\right] + \left[0.347 \times 10^6 + (1980)(12.5)^2\right]$$

$$+ \left[\frac{1}{12}(19)(363)^3 + (6897)\left(150 - \frac{363}{2}\right)^2\right]$$

$$= (22.53 + 0.66 + 82.58)10^6$$

$$= 105.77 \times 10^6 \text{ mm}^4$$

or $\bar{I}_y = 105.8 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.53 CONTINUED

$$= \left[\left(32.6 \times 10^6 + 1.07234 \times 10^6\right) + \left(0.517 \times 10^6 + 2.8767 \times 10^6\right) + \left(0.517 \times 10^6 + 0.151675 \times 10^6\right)\right] \text{mm}^4$$

$$\bar{I}_x = 37.7 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

And
$$\bar{I}_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3$$

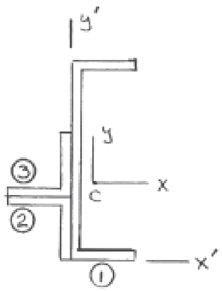
$$= \left[1.14 \times 10^6 \text{ mm}^4 + \left(3780 \text{ mm}^2\right)\left(15.3 \text{ mm} - 3.1794 \text{ mm}\right)^2\right] + 2\left[0.517 \times 10^6 \text{ mm}^4 + \left(932 \text{ mm}^4\right)\left(3.1794 \text{ mm} + 21.4 \text{ mm}\right)^2\right]$$

$$= \left[\left(1.14 \times 10^6 + 0.55532 \times 10^6\right) + 2\left(0.517 \times 10^6 + 0.56306 \times 10^6\right)\right] \text{mm}^4$$

$$\bar{I}_y = 3.86 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

SOLUTION

Locate centroid:



$\bar{x}_1 = 15.3$ mm $\quad \bar{y}_1 = 127$ mm $\quad A_1 = 3780\text{ mm}^2$

$\bar{x}_2 = -21.4$ mm $\quad \bar{y}_2 = 76\text{ mm} - 21.4\text{ mm} = 54.6$ mm $\quad A_2 = 932\text{ mm}^2$

$\bar{x}_3 = -21.4$ mm $\quad \bar{y}_3 = 76\text{ mm} + 21.4\text{ mm} = 97.4$ mm $\quad A_3 = 932\text{ mm}^2$

Then

$$\bar{x} = \frac{\Sigma \bar{x}_i A_i}{\Sigma A_i}$$

$$= \frac{(15.3\text{ mm})(3780\text{ mm}^2) + 2\left[(-21.4\text{ mm})(932\text{ mm}^2)\right]}{3780\text{ mm}^2 + 2(932\text{ mm}^2)}$$

$$= 3.1794\text{ mm}$$

And

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$$

$$= \frac{(127\text{ mm})(3780\text{ mm}^2) + (76\text{ mm})(2 \times 932\text{ mm}^2)}{3780\text{ mm}^2 + 2(932\text{ mm}^2)}$$

$$= 110.157\text{ mm}$$

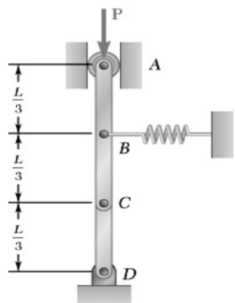
Now

$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= \left[32.6 \times 10^6\text{ mm}^4 + (3780\text{ mm}^2)(127\text{ mm} - 110.157\text{ mm})^2\right]$$
$$+ \left[0.517 \times 10^6\text{ mm}^4 + (932\text{ mm}^2)(110.157\text{ mm} - 54.6\text{ mm})^2\right]$$
$$+ \left[0.517 \times 10^6\text{ mm}^4 + (932\text{ mm}^2)(110.157\text{ mm} - 97.4\text{ mm})^2\right]$$

continue

Potential energy – work of a force / Énergie potentielle-travail d'une force :



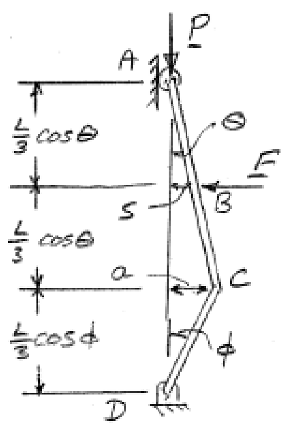
PROBLEM 10.93

Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

For small values of ϕ and θ:

$$\phi = 2\theta$$

SOLUTION



First note $\quad a = \dfrac{2L}{3}\sin\theta = \dfrac{L}{3}\sin\phi$

and $\quad s = \dfrac{L}{3}\sin\theta$

For small values of ϕ and θ: $\quad \phi = 2\theta$

$$V = P\left(\frac{2L}{3}\cos\theta + \frac{L}{3}\cos\phi\right) + \frac{1}{2}ks^2$$

$$= \frac{PL}{3}\left(2\cos\theta + \cos 2\theta\right) + \frac{1}{2}k\left(\frac{L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}\left(-2\sin\theta - 2\sin 2\theta\right) + \frac{kL^2}{9}\sin\theta\cos\theta$$

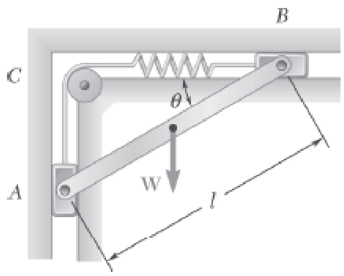
$$= -\frac{2PL}{3}\left(\sin\theta + \sin 2\theta\right) + \frac{kL^2}{18}\sin 2\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{2PL}{3}\left(\cos\theta + 2\cos 2\theta\right) + \frac{kL^2}{9}\cos 2\theta$$

When $\quad \theta = 0$: $\quad \dfrac{d^2V}{d\theta^2} = -2PL + \dfrac{kL^2}{9}$

For stability: $\quad \dfrac{d^2V}{d\theta^2} > 0$: $\quad -2PL + \dfrac{kL^2}{9} > 0$

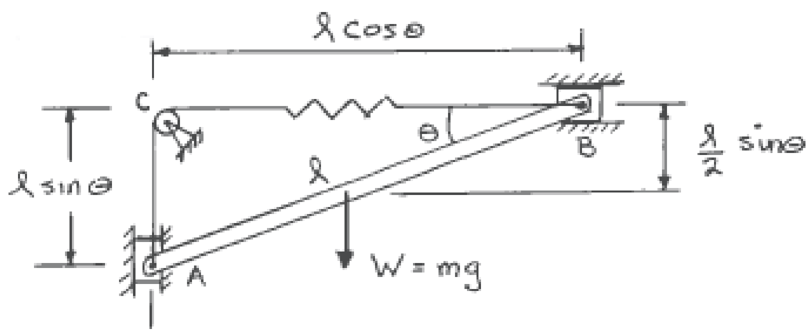
$$0 \le P < \frac{1}{18}kL \blacktriangleleft$$



PROBLEM 10.112

A slender rod AB of mass m is attached to two blocks A and B that can move freely in the guides shown. Knowing that the spring is unstretched when AB is horizontal, determine three values of θ corresponding to equilibrium when $m = 125$ kg, $l = 320$ mm, and $k = 15$ kN/m. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Elongation of Spring:

$$s = l\sin\theta + l\cos\theta - l$$
$$= l(\sin\theta + \cos\theta - 1)$$

Potential Energy:

$$V = \frac{1}{2}ks^2 - W\frac{l}{2}\sin\theta$$
$$= \frac{1}{2}kl^2(\sin\theta + \cos\theta - 1)^2 - mg\frac{l}{2}\sin\theta$$
$$\frac{dV}{d\theta} = kl^2(\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{1}{2}mgl\cos\theta$$

Equilibrium: $\quad \frac{dV}{d\theta} = 0: \quad (\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{mg}{2kl}\cos\theta = 0$

or $\quad \cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0$

Now with $\quad W = mg = (125\text{ kg})(9.81\text{ m/s}^2) = 1226.25\text{ N}$

$l = 320$ mm, and $k = 15$ kN/m,

$$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{1226.25\text{ N}}{2(15000\text{ N/m})(0.32\text{ m})}\right] = 0$$

or $\quad \cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - 0.12773\right] = 0$

PROBLEM 10.112 CONTINUED

By inspection, one solution is $\cos\theta = 0$ or $\theta = 90.0^\circ$

Solving numerically: $\theta = 0.38338 \text{ rad} = 9.6883^\circ$ and $\theta = 0.59053 \text{ rad} = 33.8351^\circ$

Stability

$$\frac{d^2V}{d\theta^2} = kl^2\left[(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta - 1)(-\sin\theta - \cos\theta)\right] + \frac{1}{2}mgl\sin\theta$$

$$= kl^2\left[\left(1 + \frac{mg}{2kl}\right)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

$$= (15000 \text{ N/m})(0.32 \text{ m})^2\left[\left(1 + \frac{(1226.25 \text{ N})}{2(15000 \text{ N/m})(0.32 \text{ m})}\right)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

$$= (1536 \text{ N}\cdot\text{m})[1.12773\sin\theta + \cos\theta - 2\sin 2\theta]$$

Thus

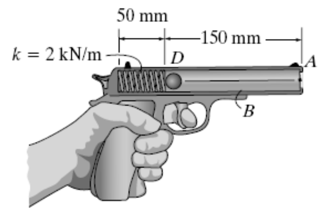
At $\theta = 90^\circ$: $\frac{d^2V}{d\theta^2} = 1732.2 > 0$ $\therefore\ \theta = 90.0^\circ$, Stable ◀

At $\theta = 9.6883^\circ$: $\frac{d^2V}{d\theta^2} = 786.4 > 0$ $\therefore\ \theta = 9.69^\circ$, Stable ◀

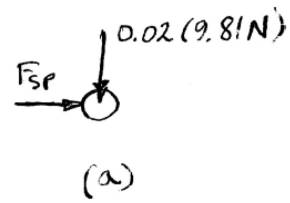
At $\theta = 33.8351^\circ$: $\frac{d^2V}{d\theta^2} = -600.6 < 0$ $\therefore\ \theta = 33.8^\circ$, Unstable ◀

Work of a force – kinetic energy: / Travail d'une force-l'énergie cinétique :

*__14–8.__ The spring in the toy gun has an unstretched length of 100 mm. It is compressed and locked in the position shown. When the trigger is pulled, the spring unstretches 12.5 mm, and the 20-g ball moves along the barrel. Determine the speed of the ball when it leaves the gun. Neglect friction.



Principle of Work and Energy: Referring to the free-body diagram of the ball bearing shown in Fig. *a*, notice that $\mathbf{F}_{sp}$ does positive work. The spring has an initial and final compression of $s_1 = 0.1 - 0.05 = 0.05$ m and $s_2 = 0.1 - (0.05 + 0.0125) = 0.0375$ m.



$$T_1 + \Sigma U_{1-2} = T_2$$

$$0 + \left[\frac{1}{2}ks_1^{\,2} - \frac{1}{2}ks_2^{\,2}\right] = \frac{1}{2}mv_A^{\,2}$$

$$0 + \left[\frac{1}{2}(2000)(0.05)^2 - \frac{1}{2}(2000)(0.0375^2)\right] = \frac{1}{2}(0.02)v_A^{\,2}$$

$$v_A = 10.5 \text{ m/s}$$ **Ans.**

14.8) Le ressort dans le pistolet de jouet a une longueur non étirée de 100 mm. Il est compressé et verrouillé dans la position indiquée. Lorsque le déclencheur est tiré, le ressort ne s'étend sur 12,5 mm, et la balle 20 g se déplace sur le long du cylindre. Déterminez la vitesse de la balle quand il quitte le pistolet. Frottement est négligé.