

PROBLEM 7.38

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a) FBD Beam:

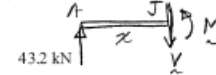
$$\sum M_A = (4.5 \text{ m})B - (3.6 \text{ m})(30 \text{ kN/m})(1.8 \text{ m}) - (1.8 \text{ m})(54 \text{ kN}) = 0$$

$$B = 64.8 \text{ kN} \uparrow$$

$$\uparrow \sum F_y = 0: A_y - 54 \text{ kN} - (30 \text{ kN/m})(1.8 \text{ m}) + 64.8 \text{ kN}$$

$$A_y = 43.2 \text{ kN} \uparrow$$

Along AC:



$$\uparrow \sum F_y = 0: 43.2 \text{ kN} - V = 0$$

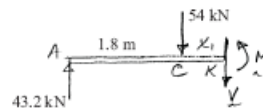
$$V = 43.2 \text{ kN}$$

$$\curvearrowleft \sum M_J = 0: M - x(43.2 \text{ kN}) = 0$$

$$M = (43.2 \text{ kN})x$$

$$M = 77.8 \text{ kN.m at } C \text{ (} x = 1.8 \text{ m)}$$

Along CD:



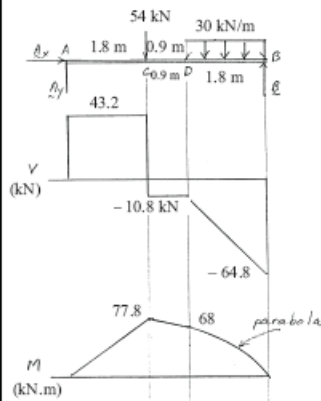
$$\uparrow \sum F_y = 0: 43.2 \text{ kN} - 54 \text{ kN} - V = 0$$

$$V = -10.8 \text{ kN}$$

$$\curvearrowleft \sum M_K = 0: M + x_1(54 \text{ kN}) - (1.8 \text{ m} + x_1)(43.2 \text{ kN}) = 0$$

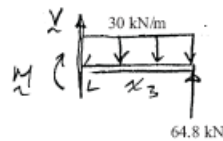
$$M = 77.8 \text{ kN.m} - (10.8 \text{ kN})x_1$$

$$M = 68 \text{ kN.m at } D$$



PROBLEM 7.38 CONTINUED

Along DB:



$$\sum F_y = 0: V - x_3(30 \text{ kN/m}) + 64.8 \text{ kN} = 0$$

$$V = -64.8 \text{ kN} + (30 \text{ kN/m})x_3$$

$$V = -10.8 \text{ kN at } D$$

$$\curvearrowleft \sum M_L = 0: M + \frac{x_3}{2}(30 \text{ kN/m})(x_3) - x_3(64.8 \text{ kN}) = 0$$

$$M = (64.8 \text{ kN})x_3 - (15 \text{ kN/m})x_3^2$$

$$M = 68 \text{ kN at } D \text{ (} x_3 = 1.8 \text{ m)}$$

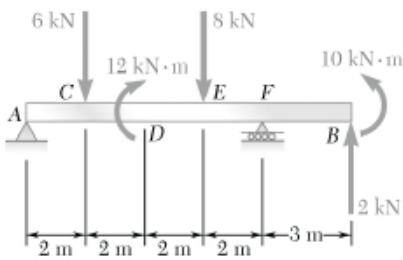
(b) From diagrams:

$$|V|_{\max} = 64.8 \text{ kN at } B \quad \blacktriangleleft$$

$$|M|_{\max} = 77.8 \text{ kN.m at } C \quad \blacktriangleleft$$

PROBLEM 7.71

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.



SOLUTION

(a)

FBD Beam:

$$\left(\sum M_A = 0: (8 \text{ m})F + (11 \text{ m})(2 \text{ kN}) + 10 \text{ kN}\cdot\text{m} - (6 \text{ m})(8 \text{ kN}) \right.$$

$$\left. - 12 \text{ kN}\cdot\text{m} - (2 \text{ m})(6 \text{ kN}) = 0 \quad F = 5 \text{ kN} \uparrow \right.$$

$$\uparrow \sum F_y = 0: A_y - 6 \text{ kN} - 8 \text{ kN} + 5 \text{ kN} + 2 \text{ kN} = 0$$

$$A_y = 7 \text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, E, F, G

Moment Diag:

M is piecewise linear with a discontinuity equal to the couple at D .

$$M_C = (7 \text{ kN})(2 \text{ m}) = 14 \text{ kN}\cdot\text{m}$$

$$M_{D^-} = 14 \text{ kN}\cdot\text{m} + (1 \text{ kN})(2 \text{ m}) = 16 \text{ kN}\cdot\text{m}$$

$$M_{D^+} = 16 \text{ kN}\cdot\text{m} + 12 \text{ kN}\cdot\text{m} = 28 \text{ kN}\cdot\text{m}$$

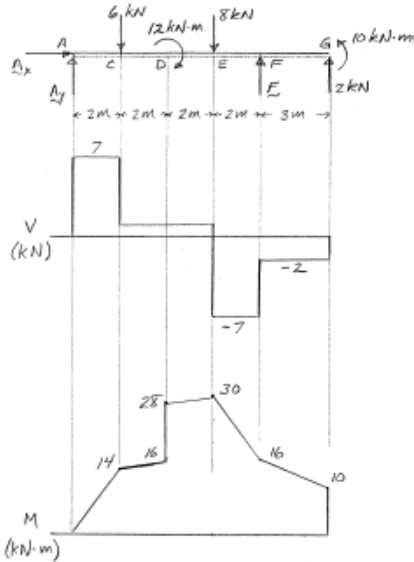
$$M_E = 28 \text{ kN}\cdot\text{m} + (1 \text{ kN})(2 \text{ m}) = 30 \text{ kN}\cdot\text{m}$$

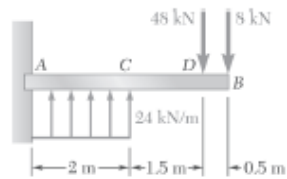
$$M_F = 30 \text{ kN}\cdot\text{m} - (7 \text{ kN})(2 \text{ m}) = 16 \text{ kN}\cdot\text{m}$$

$$M_G = 16 \text{ kN}\cdot\text{m} - (2 \text{ kN})(3 \text{ m}) = 10 \text{ kN}\cdot\text{m}$$

$$(b) \quad |V|_{\max} = 7.00 \text{ kN} \blacktriangleleft$$

$$|M|_{\max} = 30.0 \text{ kN}\cdot\text{m} \blacktriangleleft$$





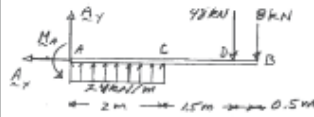
PROBLEM 7.38

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a) FBD Beam:

$$\leftarrow \Sigma F_x = 0: \quad A_x = 0$$

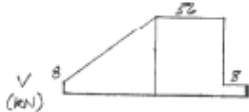


$$\uparrow \Sigma F_y = 0: \quad A_y + (2 \text{ m})(24 \text{ kN/m}) - 48 \text{ kN} - 8 \text{ kN} = 0$$

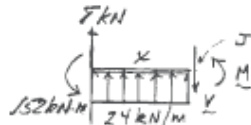
$$A_y = 8 \text{ kN} \uparrow$$

$$\left(\Sigma M_A = 0: \quad M_A + (1 \text{ m})(2 \text{ m})(24 \text{ kN/m}) - (3.5 \text{ m})(48 \text{ kN}) \right.$$

$$\left. - (2 \text{ m})(8 \text{ kN}) = 0, \quad M_A = 152 \text{ kN}\cdot\text{m} \right)$$



Along AC:

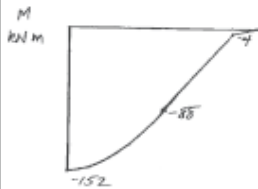


$$\uparrow \Sigma F_y = 0: \quad 8 \text{ kN} + x(24 \text{ kN/m}) - V = 0$$

$$V = 8 \text{ kN} + (24 \text{ kN/m})x$$

$$\left(\Sigma M_J = 0: \quad M + 152 \text{ kN}\cdot\text{m} - x(8 \text{ kN}) - \frac{x}{2}(24 \text{ kN/m})x = 0 \right.$$

$$\left. M = (12 \text{ kN/m})x^2 + (8 \text{ kN})x - 152 \text{ kN}\cdot\text{m} \right)$$



Along DB:



$$\uparrow \Sigma F_y = 0: \quad V - 8 \text{ kN} = 0$$

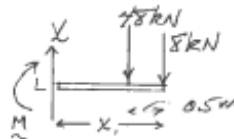
$$V = 8 \text{ kN}$$

$$\left(\Sigma M_K = 0: \quad M + x_1(8 \text{ kN}) = 0, \quad M = -(8 \text{ kN})x_1 \right)$$

continued

PROBLEM 7.38 CONTINUED

Along CD:



$$\uparrow \Sigma F_y = 0: \quad V - 48 \text{ kN} - 8 \text{ kN} = 0, \quad V = 56 \text{ kN}$$

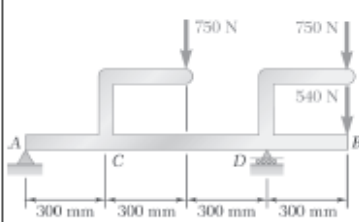
$$\left(\Sigma M_L = 0: \quad M + (x_1 - 0.5 \text{ m})(48 \text{ kN}) + x_1(8 \text{ kN}) = 0 \right.$$

$$\left. M = 24 \text{ kN}\cdot\text{m} - (56 \text{ kN})x_1 \right)$$

(b)

$$|V|_{\max} = 56.0 \text{ kN along CD} \blacktriangleleft$$

$$|M|_{\max} = 152.0 \text{ kN}\cdot\text{m at A} \blacktriangleleft$$

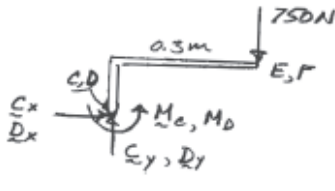


PROBLEM 7.46

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

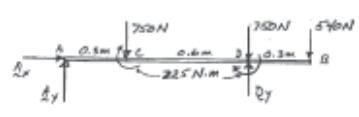
SOLUTION

FBD CE or DF:

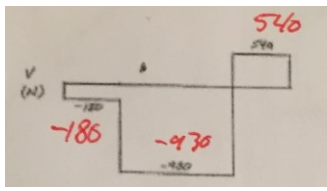


$$\begin{aligned} \rightarrow \Sigma F_x = 0: \quad C_x, D_x = 0 \\ \uparrow \Sigma F_y = 0: \quad C_y - 750 \text{ N} = 0, \quad C_y = 750 \text{ N} \\ D_y = 750 \text{ N} \\ \curvearrowright \Sigma M_C = 0: \quad M_C - (0.3 \text{ m})(750 \text{ N}) = 0 \\ M_C = 225 \text{ N}\cdot\text{m} = M_D \end{aligned}$$

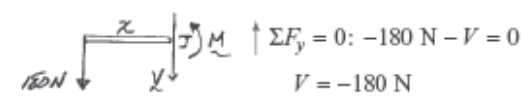
Beam AB:



$$\begin{aligned} \curvearrowright \Sigma M_A = 0: \quad (0.9 \text{ m})D_y - 2(225 \text{ N}\cdot\text{m}) - (0.3 \text{ m})(750 \text{ N}) \\ - (0.9 \text{ m})(750 \text{ N}) - (1.2 \text{ m})(540 \text{ N}) = 0 \\ D_y = 2220 \text{ N} \\ \uparrow \Sigma F_y = 0: \quad A_y - 2(750 \text{ N}) - 540 \text{ N} + 2220 \text{ N} = 0 \\ A_y = -180 \text{ N} \quad A_y = 180 \text{ N} \downarrow \end{aligned}$$



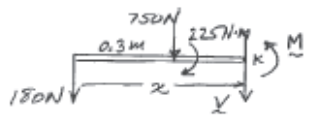
Along AC:



$$\uparrow \Sigma F_y = 0: \quad -180 \text{ N} - V = 0 \quad V = -180 \text{ N}$$

$$\curvearrowright \Sigma M_J = 0: \quad M + x(180 \text{ N}) = 0 \quad M = (180 \text{ N})x$$

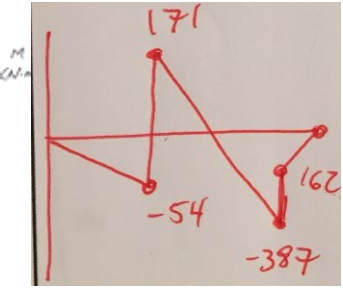
Along CD:



$$\uparrow \Sigma F_y = 0: \quad -180 \text{ N} - 750 \text{ N} - V = 0, \quad V = -930 \text{ N}$$

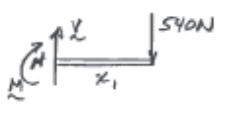
$$\curvearrowright \Sigma M_K = 0: \quad M - 225 \text{ N}\cdot\text{m} + (x - 0.3 \text{ m})(750 \text{ N}) + x(180 \text{ N}) = 0$$

$$M = 450 \text{ N}\cdot\text{m} - (930 \text{ N})x$$



PROBLEM 7.46 CONTINUED

Along DB:



$$\uparrow \Sigma F_y = 0: \quad V - 540 \text{ N} = 0$$

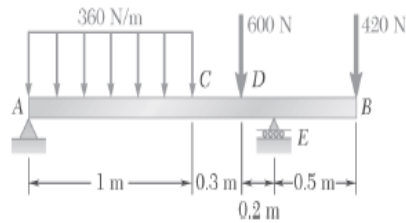
$$V = 540 \text{ N}$$

$$\curvearrowright \Sigma M_N = 0: \quad M + x_1(540 \text{ N}) = 0 \quad M = -(540 \text{ N})x_1$$

Note: The discontinuities in *M*, at *C* and *D*, equal 225 N·m, *M_C* and *M_D*

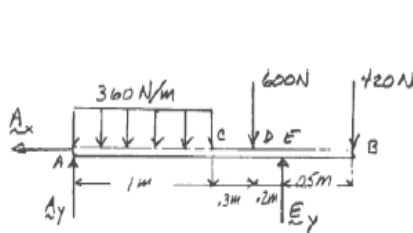
From the diagrams $|V|_{\max} = 930 \text{ N}$ along *CD* ◀
 $|M|_{\max} = 387 \text{ N}\cdot\text{m}$ at *D* ◀

PROBLEM 7.64



For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

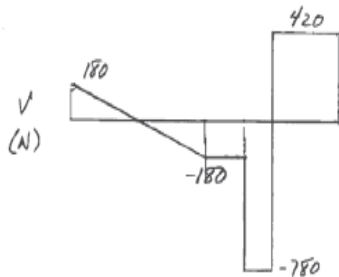
SOLUTION



$$\begin{aligned} \sum M_A = 0: & \quad (1.5 \text{ m})E_y - \left(\frac{1}{2} \text{ m}\right)(360 \text{ N/m})(1 \text{ m}) - (1.3 \text{ m})(600 \text{ N}) \\ & \quad - (2 \text{ m})(420 \text{ N}) = 0 \quad E_y = 1200 \text{ N} \uparrow \end{aligned}$$

$$\uparrow \sum F_y = 0: \quad A_y + 1200 \text{ N} - (360 \text{ N/m})(1 \text{ m}) - 600 \text{ N} - 420 \text{ N} = 0$$

$$(a) \quad A_y = 180.0 \text{ N} \uparrow$$

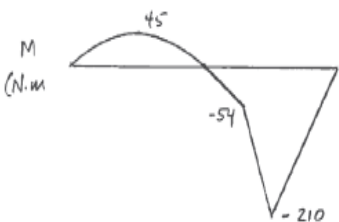


Shear Diag:

V jumps to $A_y = 180 \text{ N}$ at A , then decreases linearly

$$\left(\frac{dV}{dx} = -w = -360 \text{ N/m}\right) \text{ to } 180 \text{ N} - (360 \text{ N/m})(1 \text{ m}) = -180 \text{ N} \text{ at } C.$$

From C , V is piecewise constant ($w = 0$) with jumps of -600 N at D , $+1200 \text{ N}$ at E , -420 N at B .



Moment Diag:

M starts at zero at A with slope $\frac{dM}{dx} = V = 180 \text{ N/m}$, decreasing to zero

at $x = 0.5 \text{ m}$. There $M = \frac{1}{2}(180 \text{ N})(0.5 \text{ m}) = 45 \text{ N}\cdot\text{m}$. M is zero again

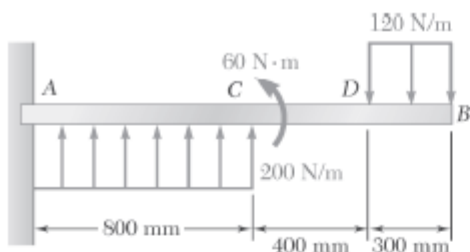
at C , decreasing to $-(180 \text{ N})(0.3 \text{ m}) = -54 \text{ N}\cdot\text{m}$ at D . M then decreases

by $(780 \text{ N})(0.2 \text{ m}) = 156 \text{ N}\cdot\text{m}$ to $-210 \text{ N}\cdot\text{m}$ at E , and increases by

$(420 \text{ N})(0.5 \text{ m}) = 210 \text{ N}\cdot\text{m}$ to zero at B .

$$(b) \quad \text{From the diagrams,} \quad |V|_{\max} = 780 \text{ N along } EB \blacktriangleleft$$

$$|M|_{\max} = 210 \text{ N}\cdot\text{m at } E \blacktriangleleft$$

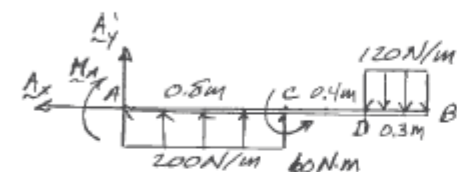


PROBLEM 7.70

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(a)



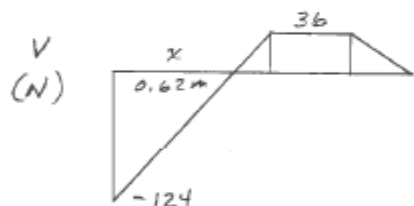
$$\uparrow \Sigma F_y = 0: \quad A_y + (200 \text{ N/m})(0.8 \text{ m}) - (120 \text{ N/m})(0.3 \text{ m}) = 0$$

$$A_y = -124 \text{ N} \quad A_y = 124 \text{ N} \downarrow$$

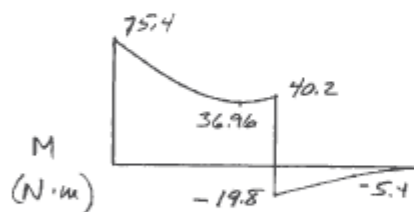
$$\curvearrowleft \Sigma M_A = 0: \quad M_A - 60 \text{ N}\cdot\text{m} - (0.4 \text{ m})(200 \text{ N/m})(0.8 \text{ m})$$

$$+ (1.35 \text{ m})(120 \text{ N/m})(0.3 \text{ m}) = 0 \quad M_A = 75.4 \text{ N}\cdot\text{m}$$

Shear Diag:



V drops to 124 N at A . $\frac{dV}{dx} = 200 \text{ N/m}$ from A to C and V increases by $(200 \text{ N/m})(0.8 \text{ m}) = 160 \text{ N}$ to $+36 \text{ N}$ at C . It remains at 36 N to D , then decreases linearly to zero at B . Note $V = 0$ where $\frac{x}{0.8 \text{ m}} = \frac{124 \text{ N}}{160 \text{ N}}$, or $x = 0.62 \text{ m}$.



Moment Diag:

M jumps to 75.4 at A where $\frac{dM}{dx} = -124 \text{ N}$. The slope increases to zero

at $x = 0.62 \text{ m}$, where $M = 75.4 \text{ N}\cdot\text{m} - \frac{1}{2}(124 \text{ N})(0.62 \text{ m})$

$= 36.96 \text{ N}\cdot\text{m}$. The slope then increases as M increases by

$\frac{1}{2}(36 \text{ N})(0.18 \text{ m}) = 3.24 \text{ N}\cdot\text{m}$ to $40.2 \text{ N}\cdot\text{m}$ at C , where it drops

$60 \text{ N}\cdot\text{m}$ to $-19.8 \text{ N}\cdot\text{m}$. M increases linearly by

$(36 \text{ N})(0.4 \text{ m}) = 14.4 \text{ N}\cdot\text{m}$ to $-5.4 \text{ N}\cdot\text{m}$, and finally M increases

quadratically by $\frac{1}{2}(36 \text{ N})(0.3 \text{ m}) = 5.4 \text{ N}\cdot\text{m}$ to zero at B where $\frac{dM}{dx}$ is also zero.

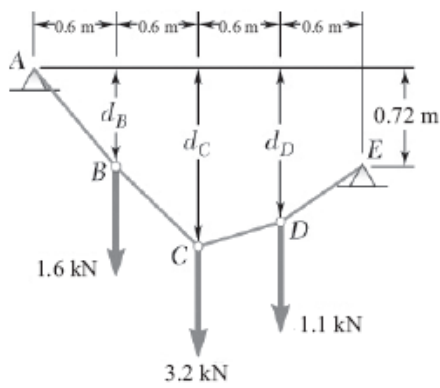
From the diagrams,

$$|V|_{\max} = 124.0 \text{ N at } A \blacktriangleleft$$

(b)

$$|M|_{\max} = 75.4 \text{ N}\cdot\text{m at } A \blacktriangleleft$$

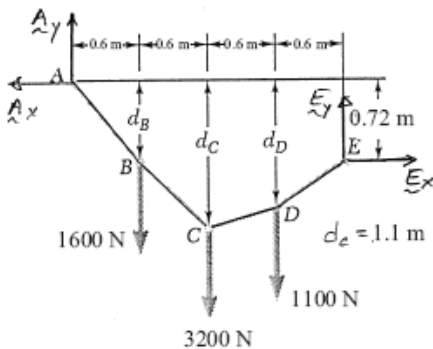
PROBLEM 7.92



Cable $ABCDE$ supports three loads as shown. Knowing that $d_C = 1.1$ m, determine (a) the reaction at E , (b) the distances d_B and d_D .

SOLUTION

FBD Cable:



$$\begin{aligned} (a) \quad \left(\sum M_A = 0: (0.72 \text{ m}) E_x + (2.4 \text{ m}) E_y - (0.6 \text{ m})(1600) \right. \\ \left. - (1.2 \text{ m})(3200 \text{ N}) - (1.8 \text{ m})(1100 \text{ N}) = 0 \right. \\ \left. 0.3 E_x + E_y = 2825 \text{ N} \right. \end{aligned} \quad (1)$$

$$\left(\sum M_C = 0: -(0.38 \text{ m}) E_x + (1.2 \text{ m}) E_y - (0.6 \text{ m})(1100 \text{ N}) = 0 \right.$$

$$\left. -0.3167 E_x + E_y = +550 \text{ N} \right. \quad (2)$$

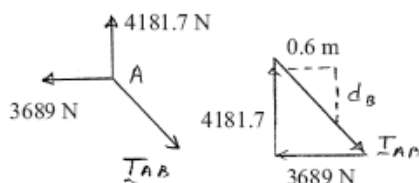
Solving (1) and (2) $E_x = 3689 \text{ N}$ $E_y = 1718.3 \text{ N}$

$$(a) \quad \mathbf{E} = 4069.6 \text{ N} \angle 24.98^\circ \blacktriangleleft$$

$$(b) \text{ cable: } \rightarrow \sum F_x = 0: -A_x + E_x = 0 \quad A_x = E_x = 3689 \text{ N}$$

$$\uparrow \sum F_y = 0: A_y - 1600 \text{ N} - 3200 \text{ N} - 1100 \text{ N} + 1718.3 \text{ N} = 0$$

Point A:

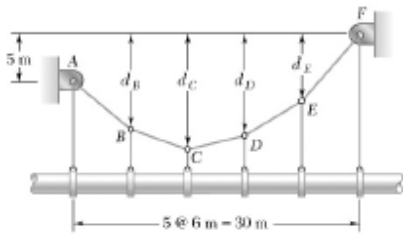


$$A_y = 4181.7 \text{ N}$$

$$\frac{d_B}{0.6 \text{ m}} = \frac{4181.7 \text{ N}}{3689 \text{ N}} \quad d_B = 0.680 \text{ m} \blacktriangleleft$$

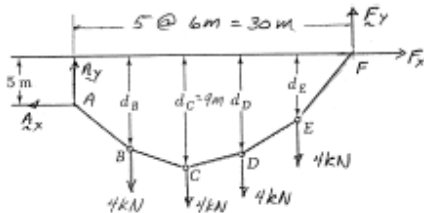
PROBLEM 7.95

Solve Prob. 7.94 assuming that $d_C = 9$ m.



SOLUTION

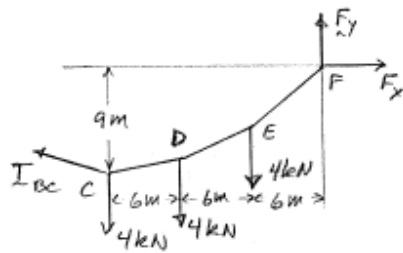
FBD Cable:



Note: 4 kN loads at A and F act directly on supports, not on cable.

$$\begin{aligned} \left(\sum M_A = 0: (30 \text{ m})F_y - (5 \text{ m})F_x \right. \\ \left. - (6 \text{ m})[1(4 \text{ kN}) + 2(4 \text{ kN}) + 3(4 \text{ kN}) + 4(4 \text{ kN})] = 0 \right. \\ \left. F_x - 6F_y = -48 \text{ kN} \right. \end{aligned} \quad (1)$$

FBD CDEF:



$$\begin{aligned} \left(\sum M_C = 0: (18 \text{ m})F_y - (9 \text{ m})F_x - (12 \text{ m})(4 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0 \right. \\ \left. F_x - 2F_y = -8 \text{ kN} \right. \end{aligned} \quad (2)$$

Solving (1) and (2)

$$F_x = 12 \text{ kN} \rightarrow \quad F_y = 10 \text{ kN} \uparrow$$

$$T_{EF} = \sqrt{(10 \text{ kN})^2 + (12 \text{ kN})^2} = 15.62 \text{ kN}$$

Since slope $EF >$ slope AB this is T_{\max}

$$(a) \quad T_{\max} = 15.62 \text{ kN} \blacktriangleleft$$

Also could note from FBD cable

$$\uparrow \sum F_y = 0: A_y + F_y - 4(4 \text{ kN}) = 0$$

$$A_y = 16 \text{ kN} - 12 \text{ kN} = 4 \text{ kN}$$

$$\text{Thus } A_y < F_y \quad \text{and} \quad T_{AB} < T_{EF}$$

FBD DEF:

$$(b) \quad \left(\sum M_D = 0: (12 \text{ m})(10 \text{ kN}) - d_D(12 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0 \right.$$

$$d_D = 8.00 \text{ m} \blacktriangleleft$$

