CONCORDIA UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MULTIVARIABLE CALCULUS (MATH-264)

SAMPLE EXAM 1

Fall semester 2015-2016; instructor prof. A. Shnirelman

Solve as many problems as you can; each problem is 10% worth. No books and notes are permitted, only calculators if necessary.

Problem 1. Find the length of the curve $x=3t^2+1$, $y=-4t^3+2$, $z=3t^4-2$ ($-1\le t\le 1$).

Problem 2. A plane Π contains the points $A(1,2,-1)$, $B(-2,1,3)$, and $C(0,1,2)$. A line l is orthogonal to the plane Π and contains the point $D(2,1,0)$. Find the intersection point of the line l and the plane Π.

Problem 3. (a) Find the point of intersection between the circle $C:\ (x-1/2)^2+y^2=1$ and the positive part of y - axis.
(b) Find the angle between the circle and the y - axis at their point of intersection.

Problem 4. Find the area between the curve $x=2\cos t+1,\ y=3\sin t-1$ and the x - and y - axes in the first quadrent (**Hint:** find first the values of t such that $x(t)=0$ and $y(t)=0$.

Problem 5. Find the curvature of the parametric curve $x=t(1-t)$, $y=t(1/2-t)(1-t)$ at the point $(1/4,0)$ (what is the value of parameter t for this point?).

Problem 6. Find the equations of the tangent plane and the normal line to the surface $x^2+y^4+z^6=26$ at the point $P(3,2,1)$.

Problem 7. Let $f(x,y,z)=\sin\left(xe^{y/z}\right)$; let $x=u+v$, $y=uv$, $z=u/v$. Find $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial v}$.

Problem 8. Find the equation of an ellipse whose foci are $F_1(1,-2)$, $F_2(1,3)$, and which contains the origin $O(0,0)$.

Problem 9. Find all the critical points of the function $f(x,y)=x^4-3x^2-y^6+y^2$ and classify them (i.e. find which of them are points of local minimum, local maximum, or saddle points).

Problem 10. Using the Lagrange multipliers method, find the minimum of the function $f(x,y)=x^2-2y^2$ on the line $2x+3y=1$.

GOOD LUCK!