Solution.

CONCORDIA UNIVERSITY
Department of Economics

ECON 221 SECTIONS A, B and C
STATISTICAL METHODS I
Fall 2017 – MIDTERM EXAM
October 21, 6:15pm – 8:15 pm

Name:
Section: **I.D:**

INSTRUCTIONS

- ☐ This is a two-hour exam.
- ☐ This paper is graded out of 90 marks.
- ☐ The examination comprises nine (9) problems. You should attempt **ALL** questions.
- ☐ All answers should be written in the spaces provided. You may use the back pages for rough work.
- ☐ You may not tear pages from the examination paper package, it must be returned intact at the end of the examination.
- ☐ Statistical tables are provided.
- ☐ You are allowed to use a non-programmable calculator.
- ☐ Round to two decimal places where necessary.
- ☐ Notes or formula crib-sheets are **NOT** allowed. You may use either pen or pencil to provide your answers.
- ☐ Write your name clearly at the top of the first page.

	FOR EXAMINERS' USE ONLY									
Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**T**
Marks	13	8	10	8	9	15	8	9	10	90

1. **(13 marks)** Consider the following seven observations for the two random variables X and Y.

X	Y
120	7
100	3
50	2
80	9
140	22
56	10
115	9

a. **(2 marks)** Calculate the sample mean of X.

$$\bar{X} = \frac{120+100+50+80+140+56+115}{7} = \frac{661}{7} = 94.43$$

b. **(2 marks)** Calculate the sample mean of Y.

$$\bar{Y} = \frac{7+3+2+9+22+10+9}{7} = \frac{62}{7} = 8.86$$

c. **(3 marks)** Calculate the sample variance of X.

$$s^2 = \frac{653.82 + 31.09 + 1974.03 + 208.23 + 2076.63 + 1476.87 + 423.13}{6}$$

$$\Rightarrow s^2 = 1140.619 \qquad \sqrt{s^2} = 33.77$$

d. **(3 marks)** Calculate the covariance between X and Y.

$$Cov(x, y) = s_{xy} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = 130.0714$$

e. **(3 marks)** Show if X and Y are independent?

~~Cov~~ Since the $Cov(X,Y) = S_{XY} \neq 0$

$\Rightarrow X, Y$ are not independent.

2. **(8 marks)** The coaching staff for a national soccer team has to select 25 players to participate in the upcoming World Cup in 2018. They have a list of 30 talented players from which to choose.

a. **(4 marks)** Calculate how many different combinations of 25 players are possible.

$n = 30 \qquad x = 25$

$C^{n=30}_{x=25} = \frac{30!}{5!\,25!} = 142,506$

b. **(4 marks)** Of the 30 available players, 20 are white. Calculate the probability that the final selected team would have 17 white players.

$C^{20}_{17} = \frac{20!}{3!\,17!} = 1140$

$C^{10}_{8} = \frac{10!}{2!\,8!} = 45$

P(of having 17 white players) $= \frac{(1140)(45)}{142506} = a\,35$

3. **(10 marks)** A manager has 16 employees who could be assigned to a project-monitoring task. Three of the employees are women and thirteen are men. Four of the men are brothers. The manager is to make the assignment at random so that each of the 16 employees is equally likely to be chosen. Let A be the event "chosen employee is aman" and B be the event "chosen employee is one of the brothers."

a. **(3 marks)** Calculate the probability of A.

$A = \{ \text{is a man} \}$

$N(A) = 13 \qquad P(A) = \frac{13}{16} = 0.8125$

b. **(3 marks)** Calculate the probability of B

$B = \{ \text{The employee is one of the four brothers} \}$

$N(B) = 4$

$\Rightarrow P(B) = \frac{4}{16} = 0.25$

c. **(2 marks)** Calculate the probability of A intersect B.

In this particular case $\quad P(A \cap B) = P(B) = 0.25$

d. **(2 marks)** Calculate the probability of A union B.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Recall that $P(A \cap B) = P(B)$

$\Rightarrow P(A \cup B) = P(A) = 0.8125$

4. **(8 marks)** Assume that the number of network errors experienced in a day on a local area network (LAN) is distributed as a Poisson random variable. The mean number of network errors experienced in a day is 1.8.

a. **(4 marks)** Calculate the probability that, in any given day, less than two network errors will occur.

Let X be the number of errors in a given day. $\Rightarrow X \sim$ a Poisson distribution

Recall that $E(X) = \lambda \Rightarrow \lambda = 1.8$

$P(X<2) = P(X=0) + P(X=1)$

$= \frac{e^{-1.8}(1.8)^0}{0!} + \frac{e^{-1.8}(1.8)^1}{1!} = 0.1653 + 0.2975$

$\Rightarrow \boxed{P(X<2) = 0.4628}$

b. **(4 marks)** Assume that the mean time between two network errors in a given day is 2.3 hours. Calculate the probability that three hours would pass before the next network error occurs.

The time between two errors ~~should~~ could be modelled as an exponential distribution with

$\lambda = 2.3$

$\Rightarrow P(T>3) = 1 - P(T \leq 3) = 1 - (1 - e^{-\lambda t})$

$= e^{-(2.3)(3)} = 0.001$

5. **(9 marks)** The number of computers sold per day at a computer store is defined by the probability distribution below.

X	0	1	2	3	4	5	6
P(x)	0.020	0.090	0.200	0.005	0.050	0.300	0.335

a. **(3 marks)** Calculate the probability that the store sells more than three computers in a given day.

See the following page

b. **(3 marks)** Calculate the probability that the store sells between two and four computers inclusive in a given day.

See the following page

c. **(3 marks)** Calculate the expected number of computers to be sold in a given day.

See the following page.

5~~6~~. (9 marks) The number of computers sold per day at a computer store is defined by the probability distribution below.

x	0	1	2	3	4	5	6
P(x)	0.02	0.09	0.2	0.005	0.05	0.3	0.335

a. (3 marks) Calculate the probability that the store will sell greater than three computers in a given day.

$P(X>3) = P(X=4) + P(X=5) + P(X=6)$

$= 0.05 + 0.3 + 0.335$

$\Rightarrow P(X>3) = 0.685$

b. (3 marks) Calculate the probability that the store will sell between two and four computers in a given day.

$P(2 \le X \le 4) = P(X=4) + P(X=3) + P(X=2).$

$= 0.05 + 0.005 + 0.2 = 0.225$

c. (3 marks) What is the expected number of computers to be sold in a given day?

$E(X) = \sum x\, P(x) = 0(0.02) + 1(0.09) + 2(0.2) + 3(0.005)$
$+ 4(0.05) + 5(0.3) + 6(0.335)$

$= 0 + 0.09 + 0.4 + 0.015 + 0.2 + 1.5 + 2.01$

$\Rightarrow E(x) = 4.215$

6 ~~4~~. (15 marks) Consider the following discrete joint probability distribution:

		Y					
		100	200	300	400	500	P(x)
X	1	0.01	0.2	0.07	0.02	0.03	0.33
	2	0.06	0.03	0.06	0.15	0.04	0.34
	3	0.15	0.04	0.04	0.01	0.09	0.33
P(y)	$= f_y(y)$	0.22	0.27	0.17	0.18	0.16	

3 Marks

a. Calculate the marginal probability densities of X and Y.

3 Marks

b. Calculate the expected value of X.

$$E(x) = \sum x\,P(x) = 1(0.33) + 2(0.34) + 3(0.33)$$

$$= 0.33 + 0.68 + 0.99$$

$$\Rightarrow \quad E(x) = 2$$

3 Marks

c. Calculate the P(X=2, Y=200).

$$P(X=2, Y=200) = 0.03$$

$$P(X|Y=400) = \frac{P(\ldots)}{P(Y=400)}$$

	$Y=400$
$X=1$	$\frac{0.02}{0.18} = 0.111$
$X=2$	$\frac{0.15}{0.18} = 0.833$
$X=3$	$\frac{0.01}{0.18} = 0.056$

3 Marks

d. Calculate the P(X=3| Y=400).

$$\Rightarrow P(X=3|Y=400) = \frac{P(X=3, Y=400)}{P(Y=400)} = \frac{0.01}{0.18} = 0.056$$

3 Marks

e. Calculate the E(X|Y=400).

$$E(X|Y=400) = \sum X\, P(X|Y=400)$$
$$= 1(0.111) + 2(0.833) + 3(0.056)$$
$$= 1.9437$$

~~f.~~ Calculate the Var(X|Y=400). (Cancelled) (Use to practice.

Recall that $Var(X|Y=400) = E(X^2|Y=400) + E[(X|Y=400$

$$E(X^2|Y=400) = \sum x^2 P(X|Y=400)$$
$$= 1(0.111) + 4(0.833) + 9(0.056)$$
$$= 3.947$$

$$\Rightarrow Var(X|Y=400) = 3.947 - (1.9437)^2$$
$$= 0.169$$

7 ~~8~~. (8 **marks**) There are two types of traders, high ability trader and low ability trader. The probability that a trader is of high ability is 30%. Furthermore, the probability that a high-ability trader would beat the market is 70% and the probability that a low-ability trader would beat the market is 25%.

(4 marks)

a. A newly hired trader has beaten the market, what is the probability that the trader is a low-ability trader. Let B be the event that a trader did beat the market.

let H be a trader with high ability

L " " " " Low "

$\Rightarrow P(H) = 30\%$ $\quad P(L) = 70\%$

$P(B|H) = 70\% \Rightarrow P(\sim B|H) = 30\%$

$P(B|L) = 25\% \Rightarrow P(\sim B|L) = 75\%$

Recall that $P(B|L) = \dfrac{P(B \cap L)}{P(L)}$ and $P(L|B) = \dfrac{P(B \cap L)}{P(B)}$

	H	L	
B	(0.7)(0.3) = 0.21	(0.25)(0.7) = 0.175	P(B) = 0.385
~B	0.09	0.525	P(~B) = 0.615
	P(H) = 0.3	P(L) = 0.7	

$\Rightarrow P(B|L)P(L) = P(L|B)P(B)$

$\Rightarrow P(L|B) = \dfrac{P(B|L)P(L)}{P(B)}$

$= \dfrac{(0.25)(0.7)}{0.385} = 0.45$

(4 marks)

b. A newly hired trader has failed to beat the market, what is the probability that the trader is a high-ability trader.

Recall, $P(\sim B|H) = \dfrac{P(\sim B \cap H)}{P(H)}$; $P(H|\sim B) = \dfrac{P(H \cap \sim B)}{P(\sim B)}$

$\Rightarrow P(\sim B|H)P(H) = P(H|\sim B)P(\sim B)$

$\Rightarrow P(H|\sim B) = \dfrac{P(\sim B|H)P(H)}{P(\sim B)} = \dfrac{P(\sim B \cap H)}{P(\sim B)}$

$= \dfrac{0.09}{0.615}$

$\Rightarrow P(H|\sim B) = 0.146$

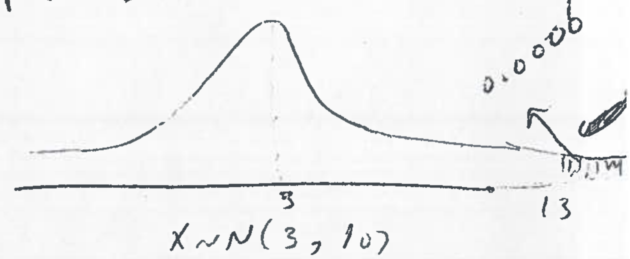
8. (9 marks) Stock returns, denoted by X, are normally distributed with a mean of 3 percent and a variance of 10 percent.

a. **(3 marks)** Calculate P (X > 13). Demonstrate your answer graphically.

$P(X > 13) = P\left(\frac{X-3}{\sqrt{10}} > \frac{13-3}{\sqrt{10}}\right) = P(Z > 3.16)$

$= 1 - P(Z \leq 3.16)$

$= 1 - 0.9994 = 0.0006 \approx 0$

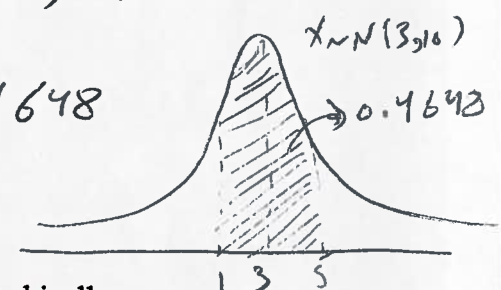


b. **(3 marks)** Calculate P (1 ≤ X ≤ 5). Demonstrate your answer graphically.

$P(1 \leq X \leq 5) = P(-0.63 \leq Z \leq 0.63) = P(Z \leq 0.63) - [1 - P(Z \leq 0.63)]$

$= 2P(Z \leq 0.63) - 1 = 2(0.7324) - 1$

$\Rightarrow P(1 \leq X \leq 5) = 0.4648$



c. **(3 marks)** Calculate P (-2 ≤ X ≤ 2). Demonstrate your answer graphically.

$P(-2 \leq X \leq 2) = P(-1.58 \leq Z \leq -0.32)$ P(

$= P(Z \leq -0.32) - P(Z \leq -1.58)$

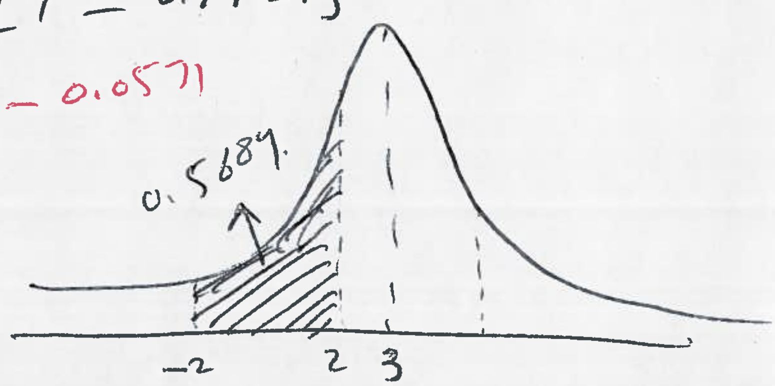
$= P(Z \leq -0.32) - [1 - P(Z \leq 1.58)]$

$= [1 - P(Z \leq 0.32)] - [1 - P(Z \leq 1.58)]$

$= [1 - 0.6255] - [1 - 0.9429]$

$= \cancel{0.5684}\ 0.3745 - 0.0571$

$= 0.3174.$



b. **(4 marks)** If a newly hired trader has failed to beat the market, calculate the probability that the trader is a high-ability trader.

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9. **(10 marks)** A hamburger stand sells hamburgers for \$1.45 each. Daily sales have a normal distribution with a mean of 530 and a standard deviation of 69.

a. **(2 marks)** Calculate the mean of daily total revenue from the sales of hamburgers.

Revenue = Price (Sales) = 1.45 Sales   Recall Sales $\sim N(530, \sigma = 69)$

$\Rightarrow E(\text{Revenue}) = E(1.45\,\text{Sales}) = 1.45\,E(\text{Sales}) = 1.45(530) = 768.5$

b. **(2 marks)** Calculate the standard deviation of daily total revenue.

$\text{Var}(\text{Revenue}) = \text{Var}(1.45\,\text{Sales}) = (1.45)^2\,\text{Var}(\text{Sales})$

$= 2.1(69)^2 = 10010$

c. **(2 marks)** What is the distribution of daily total revenue?

$\because$ Daily Sales are normally distributed $\Rightarrow$

& since Revenue is a linear function of daily Sales.

$\Rightarrow$ Revenue $\sim N(768.5, 10010)$

d. **(2 marks)** What is the probability that daily revenue would exceed \$900?

$$P(\text{Revenue} > 900) = P\left(Z > \frac{900 - 768.5}{\sqrt{10010}}\right)$$

$$= P(Z > 1.31) = 1 - P(Z \le 1.31)$$

$$= 1 - 0.9049$$

$$= 0.0951$$

e. **(2 marks)** What is the maximum daily revenue that would be achieved with 95 percent probability?

$$P(\text{Revenue} \le x^*) = 0.95$$

$$= P\left(Z \le \frac{x^* - 768.5}{\sqrt{10010}}\right) = 0.95$$

Recall that $P(Z \le 1.64) = 0.95$

$$\Rightarrow \frac{x^* - 768.5}{\sqrt{10010}} = 1.64 \Rightarrow x^* = 932.589$$