Math 264 Midterm
Concordia University

Instructor: J. Macdonald — Time: 75 minutes — October 21, 2014

All solutions must include a carefully written explanation.

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1. [10 points] For the curve defined by the vector function

$$r(t) =< t^2 - 2t + 1, e^{2t-4}, \sqrt{t^2 + 5} >,$$

find an equation for the tangent line at the point $(1, 1, 3)$.

2. [10 points] Find the area enclosed by the curve $r = \sin(3\theta)$. Hint: the curve has three loops.

3. [10 points] Let $P = (1, 0, -2)$, $Q = (3, 1, 1)$, and $R = (2, -1, 0)$.

(a) Find an equation, in the form $ax + by + cz = d$, of the plane containing P, Q, and R.

(b) Find the area of the triangle formed by P, Q, and R.

4. [10 points] Find the Maclaurin series (i.e. Taylor series centred at $a = 0$) of

$$f(x) = \int_0^x t^2 e^{t^2}\, dt$$

and find its radius of convergence.

5. [10 points] Show that the curve defined by the parametric equations

$$\begin{aligned} x &= e^t \\ y &= \sqrt{e^{2t} - 1} \end{aligned}$$

where $t \geq 0$, lies on a hyperbola.