True/False

1. If at $x = a$, $f''(a) < 0$, it is guaranteed that $(a, f(a))$ is a local max for $f(x)$.

2. If $(a, f(a))$ is a critical point, it must be the case that $f'(a) = 0$.

3. If f is continuous on $[a, b]$, $f(a) < 0$ and $f(b) > 0$, the Bisection Method is guaranteed to find a sequence of points which converge to an x-intercept of f.

4. Newton's method is always faster than the Bisection Method.

5. Rolle's Theorem is a specific instance of the Mean Value Theorem

6. If $F(t)$ is an antiderivative of $f(t)$, then for any constant C, $F(t) + C$ is an antiderivative of $f(t)$

7. Suppose $f(x) = rx + c$ is the updating function for a DTDS. Then the general solution is $x_t = r^t x_0 + c$ is the general solution for this DTDS.

8. Every continuous function is also differentiable at every point.

9. If $f''(a) = 0$ then $(a, f(a))$ is an inflection point.

10. The extreme value theorem states that if $f(x)$ is continuous and restricted to the closed interval $[a, b]$, then f attains a maximum and a minimum somewhere on $[a, b]$

The force in Newtons of a rocket's propulsion system is given as a function of the radius in meters of the rocket:

$$F(r) = 50r^2(5 - r)$$

for $0 \leq r \leq 10$ Find the radius that maximizes force, and the force at this radius; fully justify your answer.

A DTDS evolves according to:

$$x_{t+1} = \frac{x_t^2}{x_t^2 - 1}$$

a) Give the updating function for this DTDS

b) Find the non-negative equilibrium points

c) Determine analytically whether the non-negative equilibrium points are stable or unstable.

d) (Use graphing technology to graph the updating function. In the test we will provide you with this). Cobweb for three steps starting at $x_0 = 1/2$.

1. Find the derivative of $f(x) = arctan(\sqrt{1 - x^2})$

2. Find the derivative of $g(x) = sin(2^x ln(x))$

3. Let $ln(f(x)) - x^2 = (f(x))^2$. Find $f'(x)$ in terms of x and $f(x)$.

4. Find the derivative of

$$h(t) = \frac{arcsin(t)}{e^{t^3 - 2t^2 + t + 5}}$$

Graph the following functions. State the domain, determine the behavior of the functions near vertical asymptotes, determine horizontal asymptotes, determine intervals of increase and decrease, determine intervals of concavity, determine local mins and local maxs, determine points of inflection.

a)

$$f(x) = \frac{3x}{x^3 - 8}$$

b)

$$g(x) = x^2 ln|x|$$

Consider the function

$$f(x) = x^4 + x^3 + x^2 + x - 1$$

Use the intermediate value theorem to prove that there is at least one root for this function in the interval $[0, 1]$.

Use four iterations of Newton's method to approximate a root in $[0, 1]$.

Find the fourth degree Taylor polynomial of:

a) $arctan(x)$ with base $a = 1$

b) $\sqrt{x^2 + 1}$ with base $a = 0$

Compute the following indefinite integrals:

1. $$\int x^3 sin(x)dx$$

2. $$\int \frac{cos(\sqrt{x+1})}{\sqrt{x+1}}dx$$

3. $$\int \frac{(1/x+\sqrt{x})^2}{x^3}dx$$

4. $$\int e^x cos(x)dx$$

Calculate the following limits analytically. Do not simply plug in numbers and infer trends.

$$\lim_{x \longrightarrow \infty} xe^{1/x} - x$$

$$\lim_{x \longrightarrow 0^+} x^x$$

$$\lim_{x \longrightarrow 4^+} \frac{(x+4)|x-4|}{x-4}$$