Systems and Simulation SYSC 3600 C

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Scope

- This course provides a unified view of dynamical engineering systems:
  - Electrical
  - Mechanical
  - Electro-mechanical
  - Thermal
  - Hydraulic
  - … etc

Learning Objectives

This course is focused on a class of systems known as "Linear Time Invariant (LTI) systems. By the end of the term, attendants should understand that:

1) Different physical engineering systems can be designed and analyzed using similar modelling and simulation techniques.
2) Systems can be analyzed in "time-domain" or "frequency domain", and we can go back and forth between the two domains
3) Most of the time engineers rely on frequency domain analysis
4) There are different methods to describe, analyze and simulate LTI systems

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Introduction and Definitions

Points covered in this section :

- What do we mean by a "system"?
- Classifications of systems
- Definition of: Linear Time Invariant (LTI) system
- Order of systems
- Unified mathematical modelling of LTI dynamical systems

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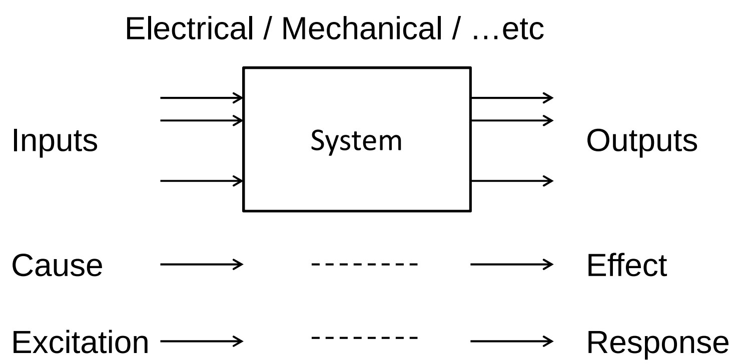
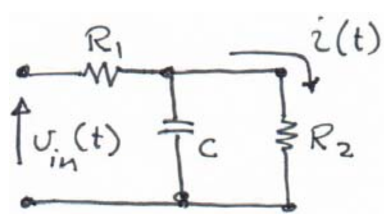
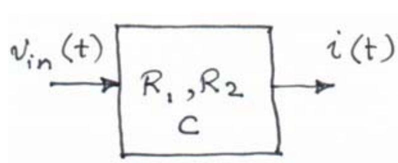
What is a System?

A system is any physical arrangement with input(s) and output(s)



Simple Electrical Example

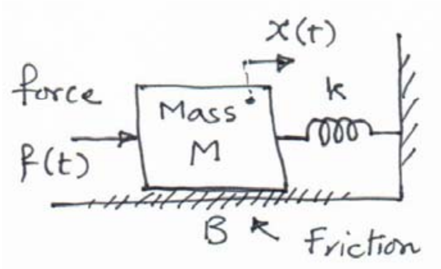


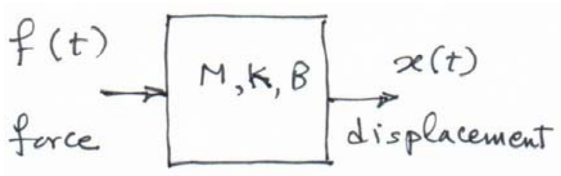


The system is a black box. It is subjected to external forces called "inputs". The system responds with "outputs"

Simple Mechanical Example

Mechanical Example



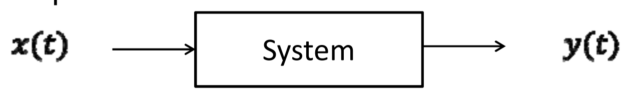


Remarks

- Inputs are "independent" variables
- Outputs are "dependent" variables
- Systems are classified as:
  - Linear or Non-linear
  - Time-Invariant or Time-Variant
  - Stable or unstable
  - Causal or not causal
  - …etc
- We are only interested in Linear Time Invariant stable causal systems

Linear Time Invariant (LTI) Systems

- A **LINEAR** system is one that obeys the super-position principles.



$$if\ x_1(t) \text{ produces } y_1(t)$$
$$and\ x_2(t) \text{ produces } y_2(t)$$
$$and\ a_1 x_1(t) + a_2 x_2(t) \text{ produces } a_1 y_1(t) + a_2 y_2(t)$$
$$Then\ the\ system\ is\ linear$$
$$a_1 \text{ and } a_2\ are\ arbitrary\ constants$$

- A **TIME-INVARIANT** system is one that responds to the same input with exactly the same output regardless of "when" the input is applied

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Stable and Causal Systems

- A **stable** system is one that produces a **bounded** output when the input is **bounded**
- When the input of a stable system is removed, the output will stop or gradually dies away.
- If the system output continues to exist at constant level after removing the input, the system is called "**Marginally Stable**" [example: an oscillator]
- A **Non-Causal** system is one that responds to an input before the input is applied. The opposite of that is a causal system.

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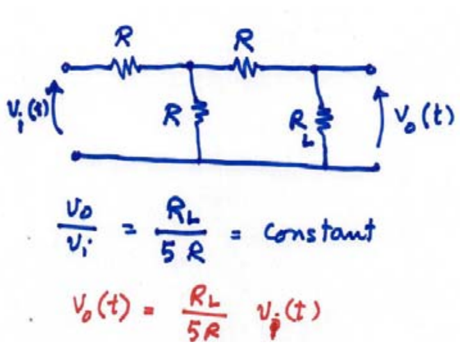
System Order

- The dynamics of an LTI system can be analyzed from an **energy** flow point of view.
- The input signal represents energy being injected into the system. Some of the energy will be stored within the system and the rest will be dissipated.
- If the system has no capacity to store any energy and all injected energy is instantly dissipated, we call that system **Order 0**. (not very interesting case)
- A system of **Order N** is a system that has N independent energy storage elements.

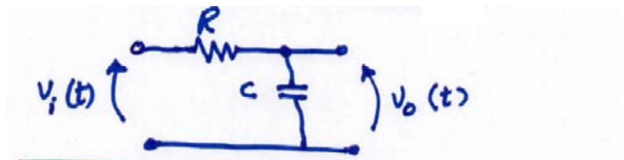
Example Order “0” System

Order 0 electric system:



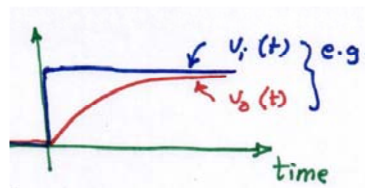
- No energy storage element
- The output is a scaled version of the input
- Any change at the input is instantly reflected at the output
- This is a zero order system

Example of First Order Electrical System



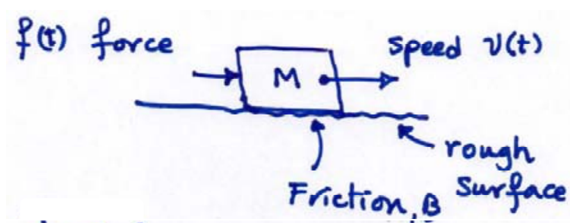
$$\frac{dv_o(t)}{dt} + \frac{1}{RC} v_o(t) = \frac{1}{RC} v_i(t) \qquad (1)$$

- One element capable of storing energy (Capacitor)
- The input and output are related by a first order differential equation (DE)



example

Example of First Order Mechanical System



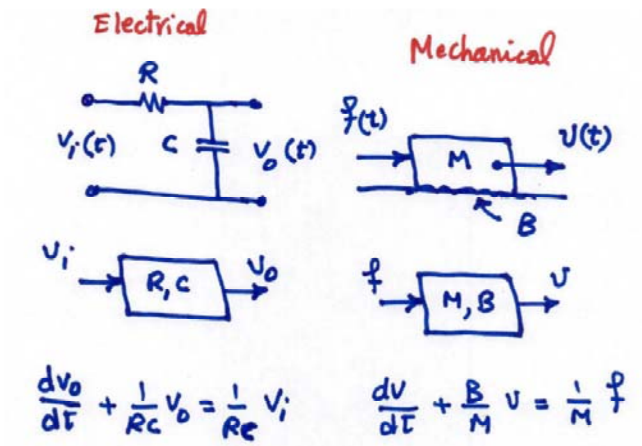
input force = Inertia force + Friction

$$f(t) = M \frac{dv(t)}{dt} + B\, v(t)$$

$$\frac{dv(t)}{dt} + \frac{B}{M} v(t) = \frac{1}{M} f(t) \qquad (2)$$

- One element capable of storing energy (Mass, M)
- Compare Eq. (2) to Eq. (1) on previous slide

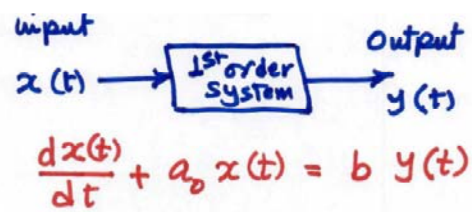
Compare the two previous Examples



Same DE with different symbol names

Important Principle

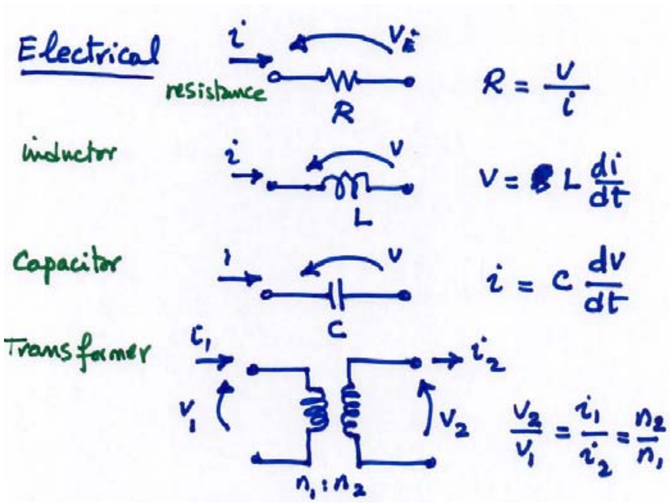
- Different **Physical Models** can produce the same **Mathematical Model**



"x", "y", "a" and "b" have different physical meaning but the mathematical model is the same

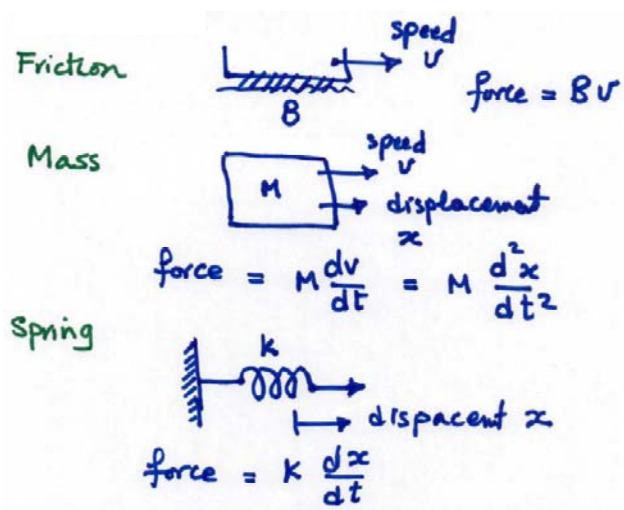
- This principles applies to all orders of systems and can be extended to a wide range of engineering models

Electrical Elements Used in This Course



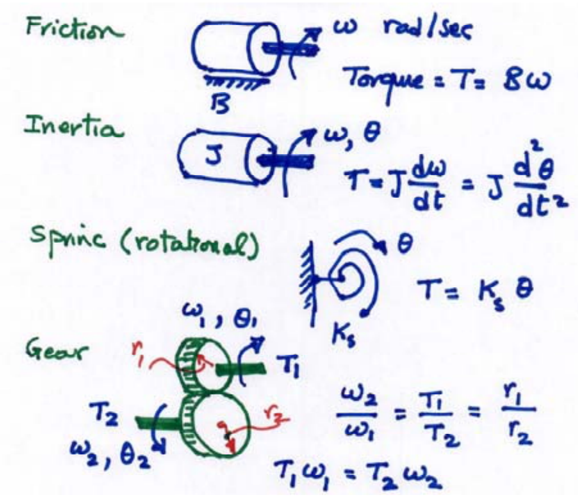
Resistors dissipate energy. Inductors and Capacitors store energy and Transformers are energy neutral

Mechanical Elements



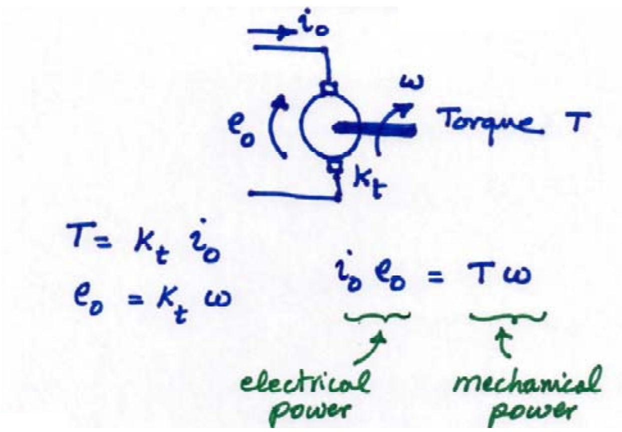
Friction dissipates energy. Mass and Spring store energy

Rotational Mechanical Elements



Friction dissipates energy. Inertia and Spring store energy.
Gears are energy neutral

Electro-Mechanical Elements



This is a simplified model for either a Motor (electric "in" mechanical "out") or a Generator (mechanical "in" electric "out").

Ideal Motors and Generators and energy neutral

Roadmap for the Rest of the Course