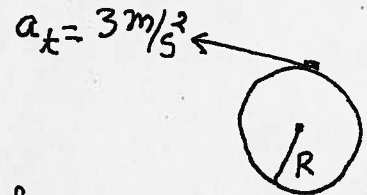
Solutions

Assignment #7

Physics 204

Q.1 A racing car starts from rest and accelerates uniformly along the track with an acceleration of ~~3.0 m/s~~^{2}. If the track is circular with the radius of 244 m, at what speed is the centripetal acceleration equal to the tangential acceleration? How long does it take to achieve the speed?
(27.1 m/s, 9.0 s)



Solution: Tangential acceleration,

$$a_t = 3\ m/s^2$$

If v is the required speed for

$$a_r = a_t$$

$$\frac{v^2}{R} = \alpha \quad , \quad \alpha \rightarrow \text{angular acceleration}$$

$$\therefore v = \sqrt{R\alpha} = \sqrt{244 \times 3} = \boxed{27.1\ m/s.}$$

– $v_0 = 0$, If t is the time taken to attain this speed,

$$\therefore v = v_0 + at =$$

$$\text{or} \quad 27.1 = 0 + 3t \qquad \therefore t = \frac{27.1}{3} = \boxed{9.0\,S.}$$

Q.2 The turbine of a jet engine has a moment of inertia of 500 kg.m^2. It is to be accelerated uniformly from rest to an angular velocity of 300 rad/s in 25 seconds. Find the angular acceleration, the unbalanced torque required, the angle turned through during the acceleration, and the kinetic energy of the turbine at its final angular velocity. (12rad/s^2, 6000 N.m, 3750 rad, 2.25x10^7 J)

Solution: Moment of inertia of the turbine, $I = 500\ kg.m^2$

Angular velocity, $\omega = 300\ rad/s$

time to attain $\omega = 25\ s.$

$$\therefore \text{using,} \quad \omega = \omega_0 + \alpha t$$

$$300 = 0 + \alpha(25)$$

$$\therefore \alpha = \frac{300}{25} = \boxed{12\ rad/s^2}$$

– Torque required, $\tau = I\alpha = (500)(12) = \boxed{6000\ N.m}$

The angle turned through is given by,

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

$$\therefore (300)^2 - 0 = 2 \times (12)\,\theta$$

$$\therefore \theta = \frac{(300)^2}{24} = \boxed{3750 \text{ rad}}$$

- Kinetic energy of rotation, $T_R = \frac{1}{2} I \omega^2$

$$= \frac{1}{2}(500)(300)^2 = \boxed{2.25 \times 10^7 \text{ J}}$$

Q.3 A flywheel has a moment of inertia of 5 kg.m^2, what angular speed does the wheel attain if 100,000 joules of work are done in producing rotational kinetic energy? What torque is required to bring this flywheel to rest in 25 seconds? (200 rad/s, 40 N.m)

Solution: Moment of inertia of the fly wheel, $I = 5$ kg. m^2

If ω is the angular velocity,

Kinetic Energy of rotation, $T_R = \frac{1}{2} I \omega^2 = 100{,}000$

$$\therefore \omega = \sqrt{\frac{2 \times 100{,}00}{5}} = \boxed{200 \text{ rad/s}}$$

Angular deceleration α is given by

$$0 - 200 = 25 \times \alpha$$

$$\text{or } \alpha = -8 \text{ rad/s}^2$$

$\therefore$ Required torque $\rightarrow T = I\alpha = 5 \times 8 = 40$ N.M.

Q.4 A force of 50 newton stretches a spring 75 cm. What is the force constant of the spring? How much force is required to stretch the spring an additional 15 cm? What is the potential energy of the spring when it is stretched 90 cm? (66.7 N/m, 60 N, 27

Solution: 50 N force stretches the spring by 0.75 m

$$\therefore \text{Spring Constant} \rightarrow k = \frac{F}{S} = \frac{50}{0.75} = \boxed{66.7 \text{ N/m}}$$

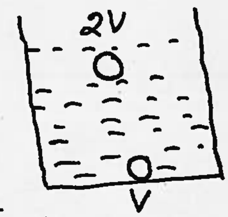
- Force required to stretch additional 0.15 m is

$$F = kS = (66.7)(0.75 + 0.15) = \boxed{60 \text{ N}}$$

- Potential energy $\rightarrow E_P = \frac{1}{2} k S^2 = \frac{1}{2}(66.7)(0.9)^2$

$$= \boxed{27 \text{ J}}$$

Q.5 An air bubble released at the bottom of a pond expands to twice its original volume by the time it reaches the surface. How deep is the pond if atmospheric pressure is 10^5 N/m^2) (10.2 m)



Solution: If P is the pressure on bubble at the bottom and P′ (atmospheric pressure) on the surface, using gas equation.

$$PV = P'V' \quad (1)$$

$$P \times (V) = 10^5 \times (2V)$$

$$\therefore P = 2\times10^5 \text{ N/m}^2$$

$P' = 10^5$ N/m^2
V = volume of bubble at the bottom
2V = volume on the surface

Also $P - P' = \rho g h$ (2)

ρ(water) = 1000 kg/m^3

ρ = density of water
h = depth of the pond

∴ From eq. (2)

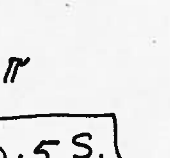
$$(2\times10^5) - (10^5) = (1000)(9.8)(h)$$

$$\therefore h = \frac{10^5}{10^3\times9.8} = 10.2\,m$$

Q.6 A body describing simple harmonic motion has a maximum acceleration of 8π m/s^2 and a maximum speed of 2 m/s. Find the time period and the amplitude. (0.50 s, 0.16 m)

Solution: In a simple harmonic motion a particle attains maximum velocity at the center of oscillation and attains max. acceleration at the end of amplitude.

$v_{max} = A\omega$



$$\therefore v_{max} = A\omega = 2 \quad (1) \qquad \therefore A = \frac{2}{\omega}$$

$$a_{max} = \omega^2 A = 8\pi \quad (2)$$

$$\text{or } \omega^2\left(\frac{2}{\omega}\right) = 8\pi \qquad \therefore \omega = 4\pi$$

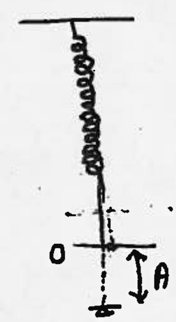
Time period → $T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \boxed{0.5\,s.}$

and $A = 2/\omega = \frac{2}{4\pi} = \boxed{0.16\,m}$

Q.7 A 5-kg mass hangs in equilibrium from a spring of spring constant 80 N/m. How far must the spring be stretched to give the mass an acceleration of $4 m/s^2$ upward when released? What is the period of the motion? Find the speed of the mass 0.1 s after it passes the equilibrium position. (0.25 m, 1.57 s, 0.92 m/s)

Solution: Constant of spring, $k = 80\ N/m$.

To have the required acceleration, the particle must be at the end of amplitude of S.H.M.



$\therefore\ a = 4 = A\omega^2$ (1)

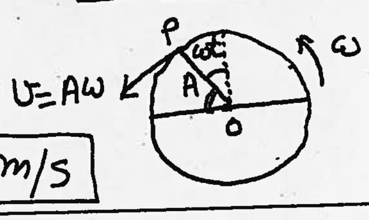
where $\omega^2 = \frac{k}{m} = \frac{80}{5} = 16$

$\therefore\ \omega = 4\ rad/s.$

From (1), Amplitude $\rightarrow A = \frac{4}{\omega^2} = \frac{1}{4} = \boxed{0.25 m}$.

Time period, $T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \boxed{1.57\ s.}$

x - Component of v.



$v_x = A\omega \cos \omega t$

$= (\frac{1}{4})(4) \cos\{(4)(0.1)\} = \boxed{0.92\ m/s}$

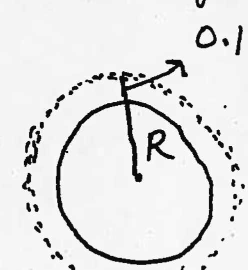
Q.8 A rocket is 637 km (0.1 earth radius) above the surface of the earth. What is g at this height? Find the weight of 5-kg mass at this point.

($8.1 m/s^2$, 40.5 N)

Solution: If g_s is the value of g on the surface of the earth,

$g_s = 9.8 = -\frac{GM_E}{R^2}$ (1) where M_E = Mass of earth
R = Radius of earth

If h is the height of the rocket from the center of the earth, then the gravity at height h is,



$g_h = -\frac{GM_E}{(1.1)^2 R^2}$ (2)

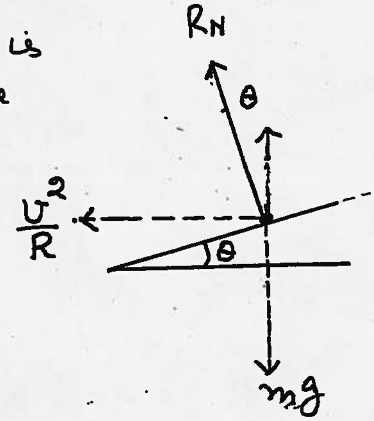
From (1) and (2) $\frac{g_h}{9.8} = \frac{R^2}{(1.1)^2 R^2}$

$\therefore\ g_h = \frac{9.8}{(1.1)^2} = \boxed{8.1\ m/s^2}$

- Weight of 5-kg mass is, $W_h = (8.1)(5) = 40.5\ N.$

Q.9 A curve of 200 m radius is banked at 15 °. At what speed must an automobile go around this curve if no frictional forces are to be used to keep the automobile on its circular path?
(22.9 m/s)

Solution: The Centripetal acceleration is due to Component of R_N towards the Center of the path.



$$\therefore \quad R_N \cos\theta = mg \quad (1)$$

$$R_N \sin\theta = \frac{mv^2}{R} \quad (2)$$

$$\therefore \quad \tan\theta = \frac{mv^2/R}{mg} = \frac{v^2}{gR}$$

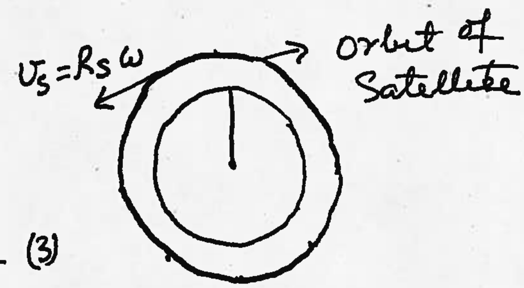
Given $\theta = 15^\circ$, $\therefore \quad v^2 = gR\tan\theta$

$$= (9.8)(200)\tan 15^\circ = 525.2$$

$$\therefore \quad v = \sqrt{525.2} = 22.9\,m/s.$$

Q10 Find the radius of the orbit of a satellite which revolves about the earth in one sidereal day (86164 s). What is the orbital velocity?
($4.2\text{x}10^4$ km, 3060 m/s)

Solution: g- on earth $\quad g = -\frac{GM_E}{R_E^2} \quad (1)$



g- on Satellite $\quad g_s = -\frac{GM_E}{R_S^2} \quad (2)$

From (1) and (2) $\quad g_s = g\left(\frac{R_E}{R_S}\right)^2 = 9.8\,\frac{R_E^2}{R_S^2} \quad (3)$

Radius of the earth $\rightarrow R_E = 6.37\times10^6$ m.

The dynamical equation of the Satellite is

$$\frac{m_s v_s^2}{R_s} = m_s g_s \quad \text{or} \quad v_s^2 = R_s g_s$$

from (3)

$$\text{or} \quad R_s^2\omega^2 = R_s g_s = R_s\left(9.8\times\frac{R_E^2}{R_s^2}\right) \quad (4)$$

$$\omega = \left(\frac{2\pi}{86164}\right) = 7.29\times10^{-15}\ rad/s$$

From (4) $$R_s^3 = \frac{9.8\times R_E^2}{(7.29\times10^{-5})^2} = \frac{9.8\times(6.37\times10^6)^2}{(7.29\times10^{-5})^2}$$

$$\text{or} \quad R_s = 4.2\times10^7\,m = \boxed{4.2\times10^4\,km}$$

orbital velocity $\quad v = R_s\,\omega = (4.2\times10^7)(7.29\times10^{-5}) = \boxed{3062\,m/s}$

Q11 A rocket engine ejects 30 kg/s of hot gases at a speed of 2000 m/s. Find the thrust and the vertical acceleration available at launch for a missile of 4000 kg tota'

(60,000 N, 5.2 m/s²)

Solution: Rate of ejection of hot gases $\rightarrow -\frac{dm}{dt} = 30$ kg/s

Speed of exhaust gases = 2000 m/s.

Resulting thrust of the rocket $= 30 \times 2000$

$= 60{,}000$ N.

∴ vertical acceleration a of the rocket is → $a = \frac{\text{Thrust} - \text{weight}}{\text{Mass}}$

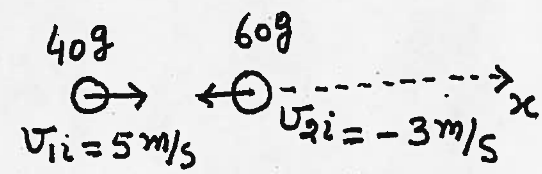
$= \frac{60{,}000 - 4000 \times 9.8}{4000}$

$= \boxed{5.2 \text{ m/s}^2}$

Q12 A 40-g ball traveling east with a speed of 5.0 m/s has a "head-on" collision with a 60-g ball traveling 3.0 m/s west. If the collision is elastic, find the velocities after the collision.

($v_{1f} = -4.6$ m/s, $v_{2f} = 3.4$ m/s)

Solution:



Since the Collision is elastic

$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$

$5 - (-3) = -v_{1f} + v_{2f}$ (1)

v_{1f} → velo. of 40g-ball after Collision

v_{2f} → velo. of 60g-ball after Collision

Using the Conservation of mom.

Mom. before Coll. = Mom. after Coll.

$(.040 \times 5) - .060 \times .3) = .040\, v_{1f} + .060\, v_{2f}$

or $20 = 40 v_{1f} + 60 v_{2f}$

or $2 v_{1f} + 3 v_{2f} = 1$ (2)

From (1) $v_{1f} = v_{2f} - 8$, Therefore from (2)

$2 v_{2f} - 16 + 3 v_{2f} = 1$ or $5 v_{2f} = 17$

∴ $v_{2f} = \boxed{3.4 \text{ m/s}}$

From (1) $v_{1f} = 3.4 - 8 = -4.6$ m/s

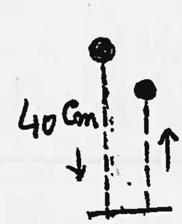
∴ 40-g ball will rebound in −x-direction and 60g ball will go in the +ve x-direction.

Q.13 If a ball is dropped from a height of 40 cm, and if the coefficient of restitution is 0.80, find the height attained after the first bounce.

(25.6 cm)

Solution: Coefficient of restitution,

$$e = 0.8$$



velocity of the ball when it hits the ground is,

$$v^2 - 0 = 2 \times (0.4) \times 9.8$$

$$\therefore v = \sqrt{0.8 \times 9.8} = 2.8 \text{ m/s}$$

v is the velocity of the ball when it hits the ground.

velocity v' of rebound after the ball hits ground is given by Newton's rule of Collision

$$(v_{1i} - v_{2i}) = -e(v_{1f} - v_{2f}) \quad (1)$$

or (Relative velo. before Collision) = -e (Relative velocity after Collision)

Therefore $v' = -ev = -0.8 \times 2.8 = -2.24 \text{ m/s}$ (+ve y direction)

$\therefore$ Height h attained by the ball;

$$2 \times 9.8 \times h = (2.24)^2$$

$$\text{or } h = \frac{(2.24)^2}{2 \times 9.8} = 0.256 \text{ m} = \boxed{25.6 \text{ Cm}}$$

Q.14 A 2-g bullet is fired from a 2.5-kg rifle with a velocity of 350 m/s north. Find the momentum of the bullet, and the recoil velocity of the rifle, assuming that no other bodies are involved> (0.70 kg-m/s north, 0.28 m/s south)

Solution: Momentum of bullet $= mv = (0.002)(350) = 0.7 \frac{\text{kg.m}}{\text{s}}$ North

If v' is the velocity of recoil of gun, then

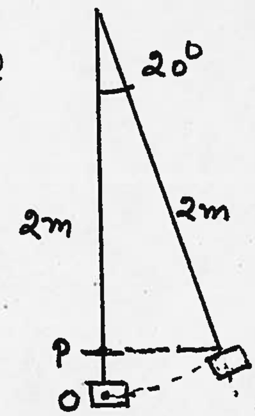
From principle of momentum i.e

(Mom. of bullet and rifle before) = (Mom. of bullet and rifle after)

$$\therefore \quad 0 = (0.002)(350) + 2.5 \times v'$$

$$\text{or } v' = -\frac{.7}{2.5} = \boxed{-0.28} \text{ i.e Southward}$$

Q 15 A 5-g bullet is fired horizontally into 2-kg block of wood suspended from a cord 2 m long. The block swings because of the impact, deflecting the cord to a position 20° from the vertical. Find the initial speed of the bullet. (615 m/s)



Solution: Let v be the initial speed of the bullet, and v' be the speed of block and bullet after impact.

Using Conservation of momentum

$$0.005 \times v = v' \times (2.0 + 0.005)$$

$$\text{or} \quad v' = \frac{0.005v}{2.005} \quad (1)$$

Kinetic energy of block and bullet after impact

$$T_f = \frac{1}{2}(2.005)v'^2 \quad (2)$$

At 20°, this kinetic energy is changed into potential energy.

$$E_p = (2.005)(2 - 2\cos 20°)g \quad (2)$$

$$OP = 2(1 - \cos 20°)$$

$$\therefore \quad T_f = E_p$$

$$\frac{1}{2}(2.005)v'^2 = (2.005)(2 - 2\times\cos 20°)\,9.8$$

$$\therefore v' = \sqrt{4 \times 0.06 \times 9.8} = 1.53 \text{ m/s}.$$

From (1) $\therefore$ $$v = v'\frac{2.005}{0.005} = \frac{1.53 \times 2.005}{0.005} = \boxed{614 \text{ m/s}}$$

Q16 A 2-g bullet is fired into a 2398-kg block of wood suspended from a long cord. The bullet is embedded in the block, and the two start off together with a speed of 0.700 m/s. Find the velocity of the bullet before collision. (840 m/s).

Solution:

If v is the velocity of bullet before collision, principle of momentum gives

$$0.002 \times v = (2398 + 0.002) \times 0.7$$

$$\therefore v = \frac{24 \times 0.7}{0.002} = 840 \text{ m/s}$$

Q.17 An 80-kg man is lifted by an elevator through a distance of 50 m in 30 seconds. What is the increase in his potential energy? What power is expanded in raising him?

(39,200 J, 1307 W)

Solution: Increase in potential energy $= mgh$

$= 80 \times 9.8 \times 50$

$= 39{,}200 \text{ J}$

$$\text{Power} = \frac{39{,}200}{30} = 1307 \text{ W}$$

Q18 A proton of mass $1.66\text{x}10^{-27}$ kg collides head on with a helium atom at rest. The helium atom has a mass of $6.64\text{x}10^{-27}$ kg, and it recoils with a speed of $8\text{x}10^{5}$ m/s. If the collision was elastic, find the initial and final speeds of the proton and the fraction of its initial energy transferred to the helium atom. ($2\text{x}10^{6}$ m/s, $-1.2\text{x}10^{6}$ m/s, 64 per cent)

Solution: Since the collision is elastic,

v_{1i} → Proton $\qquad v_{2i} = 0$ He

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$\therefore v_{1i} - 0 = -v_{1f} + 8 \times 10^{5} \quad (1)$$

After Collision $v_{2f} = 8 \times 10^{5}$ m/s.

From Conservation of momentum, we get

$$(1.66 \times 10^{-27})\, v_{1i} + 0 = (1.66 \times 10^{-27})\, v_{1f} + (6.64 \times 10^{-27})(8 \times 10^{5})$$

Dividing by 1.66×10^{-27}, $\quad v_{1i} = v_{1f} + 32 \times 10^{5} \quad (2)$

Adding (1) and (2), $\quad v_{1i} = \boxed{2 \times 10^{6} \text{ m/s}}$

$\therefore$ From (2) $\quad v_{1f} = 2 \times 10^{6} - 3.2 \times 10^{6} = \boxed{-1.2 \times 10^{6} \text{ m/s}}$

K.E of proton before Collision $\quad T_i = \frac{1}{2}(1.66 \times 10^{-27})(2 \times 10^{6})^2 = 3.32 \times 10^{-15}$ J

K.E of proton After Collision $\quad T_f = \frac{1}{2}(1.66 \times 10^{-27})(-1.2 \times 10^{6})^2$

$= 1.20 \times 10^{-15}$ J.

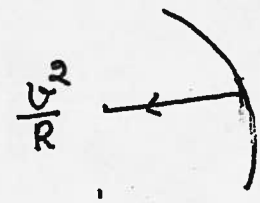
$$\text{Energy fraction transferred to He} = \frac{3.32 \times 10^{-15} - 1.2 \times 10^{-15}}{3.32 \times 10^{-15}}$$

$= 0.64$ or 64%

19 A pendulum bob hangs from the roof of a moving van. If the van is traveling 25 m/s around a curve of 150 m radius, find the angle which the cord makes with the vertical when the bob is at rest relative to the van. (23°)

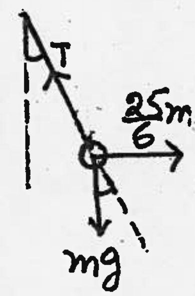
Solution:

Centrifugal force on the bob



$$ma = \frac{mv^2}{R} = \frac{m(25)^2}{150} = \frac{25}{6}m$$

Therefore, the bob of pendulum is in equilibrium under the effect of three forces, namely



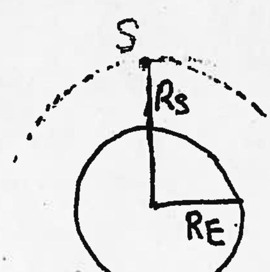
1. Tension in the string
2. weight of the bob
3. Centrifugal force at right angles to weigh

∴ The resultant of weight and Centrifugal force must be equal and opposite to tension to maintain equilibrium

$$\therefore \tan\theta = \frac{25/6\ m}{mg} = \frac{25}{6\times 9.8} = 0.425$$

$$\therefore \theta = \boxed{23^\circ}$$

20 . Find the velocity required for a space station to remain in circular orbit 600 km above the earth's surface and therefore some 7000 km from the center (7600 m/s) of the earth.



$R_S = 7000$ km
$R_E = 6400$ km

Solution: If v is the velocity of Space station where the gravity is g', then

$$\frac{mv^2}{(7000)\times 10^3} = mg' \quad (1)$$

$$\text{But } g' = g\left(\frac{6400}{7000}\right)^2 = 0.84g$$

$$\therefore \text{From (1)} \quad v^2 = 0.84g \times 7\times 10^6 = 0.84\times 9.8\times 7\times 10^6$$

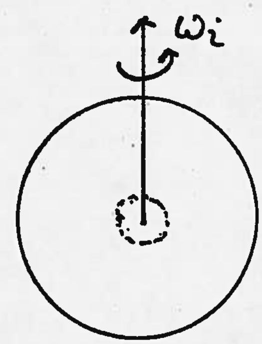
$$v = \sqrt{57.62\times 10^6} = \boxed{7.59\times 10^3\ m/s}$$

21 A star rotates with a period of 30 days about an axis through its center. After the star undergoes a supernova explosion, the stellar core, which has a radius of 1.0×10^4 km, collapses into a neutron star of radius 4.0 km. detemine the period of rotation of the neutron star. (0.23 s)

Solution:

Radius of steller Core $= 1.0\times10^4$ km.

Radius of neutron star $= 4.0$ km.



Since there is no external torque acting on the star, we use the principle of Conservation of angular momentum.

$$\therefore \quad I_i \omega_i = I_f \omega_f \quad (1)$$

I_i, I_f are the moments of inertia of star before and after explosion and ω_i, ω_f are the angular speeds. Let T_i and T_f be the time periods of revolution before and after explosion. Eq. (1) Can be written as:

$$I_i\left(\frac{2\pi}{T_i}\right) = I_f\cdot\left(\frac{2\pi}{T_f}\right) \quad (2)$$

But $I_1 = \frac{2}{5} m R_i^2$ about its diameter

From 2 $\therefore \quad T_f = \dfrac{I_f T_i}{I_i} = \dfrac{\left(\frac{2}{5} m R_f^2\right).\ T_i}{\left(\frac{2}{5} m R_i^2\right)} = \dfrac{R_f^2\, T_i}{R_i^2} \quad (3)$

Putting $R_f = 4.0$ km, $R_i = 1.0\times10^4$ km, $T_i = 30$ d. in eq. (3) we get

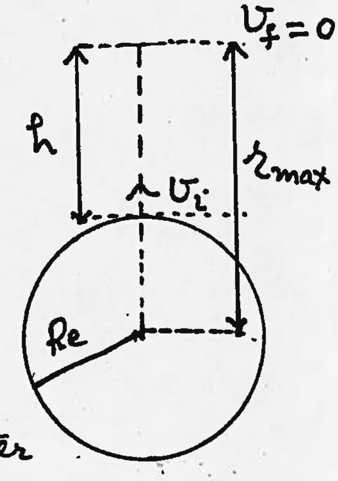
Time Period of neutron star $\rightarrow T_f = \left(\dfrac{4.0}{1\times10^4}\right)^2 (30\times24\times60\times60)$ s.

$$= 0.41 \text{ s.}$$

22 Calculate the escape speed from the Earth for a 5000-kg spacecraft , and determine the kinetic energy it must have at the Earth's surface in order to move infinitely far away from the Earth. (1.12×10^4 m/s, 3.14×10

<u>Solution:</u> Let v_i be the velocity of projection upward. we Can use energy Consideration to find the minimum value of v_i needed to allow the object to go to infinity. Since the total energy is Conserved in a gravitational field, we have:

$$\frac{1}{2} m v_i^2 - \frac{G M_e m}{R_e} = - \frac{G M_e m}{r_{max}} \quad (1)$$



where $M_e \rightarrow$ mass of the Earth

$R_e \rightarrow$ Radius of the Earth

$m \rightarrow$ mass of body projected.

$r_{max} \rightarrow$ maximum height from the Center of the Earth.

From (1) $$v_i^2 = 2 G M_e \left(\frac{1}{R_E} - \frac{1}{r_{max}} \right) \quad (2)$$

If h is the height above the Earth's surface where $v_f = 0$, $h = r_{max} - R_E$ (See figure)

To find the escape velocity we find the limiting value of v_i when $r_{max} \rightarrow \infty$ in eq. (2)

$$\therefore v_{esc} = \sqrt{\frac{2 G M_e}{R_e}} = \sqrt{\frac{2(6.67\times10^{-11})(5.98\times10^{24})}{6.37\times10^{6}}}$$

Escape velocity $\rightarrow v_{esc} = 1.12\times10^4$ m/s

Kinetic Energy of spacecraft $\rightarrow K = \frac{1}{2} m v_{esc}^2$

$$K = \frac{1}{2}(5\times10^3)(1.12\times10^4)^2 = 3.14\times10^{11} \text{ J}$$