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BROCK UNIVERSITY

Final Examination: April 2011
Course: Math 1P98
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Number of Students:1205
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Instructors: D. Miners, S. Yee, A.Dar

Name_____

Student Number_____

VERSION B

Write your name on the scantron sheet and shade in the corresponding boxes for your student number.

Write 'Version A' on the top right corner of the scantron sheet.

One piece of paper, containing hand writing and nothing glued, attached, printed or photocopied on this paper, and a Sharp EL 510-R calculator are the only aids permitted.

Use or possession of unauthorized materials will result in a grade of zero for this examination.

Also note that in order to pass this course you must obtain at least 30% on this examination.

All of the multiple choice answers are correct to 3 significant digits unless otherwise specified.

All questions are worth 1 mark each. Circle the answer on this paper and shade the appropriate box on the scantron.

Consider the function $f(x)=\sqrt{2x+1}$ for the next two questions.

1. The domain of f(x) is
a. $(-\infty,\infty)$
b. $[-\infty,2)$
c. $[-0.5,\infty)$
d. $[-2,\infty)$
e. $(-\infty,0.5]$

2. $f'(4)=$
a. 1/3
b. 2/3
c. 1/6
d. 1/8
e. none of the above

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Consider the function $g(x)=\dfrac{x-2}{x^2+3x-10}$ for the next 3 questions.

3. The function is discontinuous at
a. x = -5 only
b. x = -5 and x = 2
c. x = 2 only
d. x = -2 and x = 5
e. it is continuous everywhere

4. $\lim_{x\to 2} g(x)=$
a. 0
b. ∞
c. 1/7
d. 2/7
e. does not exist

5. $\lim_{x\to \infty} g(x)=$
a. 0
b. ∞
c. 2/3
d. 2/7
e. does not exist

If a rock is thrown upward on the planet Mars with a velocity of 10m/s, its height (in meters) after t seconds is given by $h = 10t - 1.86t^2$. Use this information for the next 3 questions.

6. The velocity of the rock after 2 seconds is
a. 2.56 m/s
b. 7.44 m/s
c. 3.72 m/s
d. 8.14 m/s
e. none of the above

7. The acceleration of the rock after 2 seconds is
a. -9.81 m/s^2
b. -3.72 m/s^2
c. 8.14 m/s^2
d. -2.64 m/s^2
e. none of the above

8. The average velocity between t = 1 second and t = 4 seconds is
a. 2.10 m/s
b. 3.22 m/s
c. 0.70 m/s
d. 4.21 m/s
e. none of the above

9. The profit P (in dollars) from selling x units of Calculus textbooks is given by
$P(x) = -0.05x^2 + 200x - 1000$
The marginal profit when x=1500 is
a. $50
b. $500
c. $150
d. $1500
e. none of above

10. The slope of the tangent to the curve y=e^x is parallel to the line y=2x at
a. x =e^2
b. x = ln2
c. x = 2
d. $x = \frac{\ln 2}{2}$
e. Not parallel at any point.

11. When $f(x) = x^3 - 2x + 1$, find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ and evaluate at x = 2
a. 25
b. 22
c. 18
d. 11
e. 10

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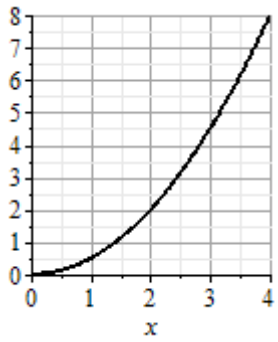
Consider the graph shown, illustrating the height of a plant(cm) after x weeks, for the next two questions.



12. The average growth rate between x = 2 and x = 4 is
a. 3 cm/week
b. 4.5 cm/week
c. 8 cm/week
d. 2 cm/week
e. none of the above

13. The instantaneous growth rate at x = 2 is approximately
a. 0.5 cm/week
b. 1 cm/week
c. 2 cm/week
d. 3 cm/week
e. 4 cm/week

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14. The equation of the tangent to the graph of $f(x) = \ln(x^2 - 3) + 4$, at (2,4) is:
a. $y = 3x - 7$
b. $y = 4x - 4$
c. $y = 2x - 8$
d. $y = 4x + 1$
e. none of the above

15. Using the function of the previous question, when $f(x) = 10, \quad x =$
a. 406.4
b. 148.4
c. 20.2
d. 35.6
e. none of the above

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16. Inflation fluctuates according to the function $\frac{10t}{t^2 + 9}$ %, after t months from the beginning of the year. This function has a horizontal tangent at
a. t = 3
b. t = 0
c. t = 2.71
d. t = 1.11
e. none of the above

17. The temperature on a cold spring day, varies according to $T(t) = t^2(3t-4)^3$ after t hours. What is the rate of change of temperature after 1 hour?
a. 2 ^{0}C/h
b. 3 ^{0}C/h
c. 5 ^{0}C/h
d. 7 ^{0}C/h
e. none of the above

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18. There is a critical point of $g(x) = 6xe^{3x}$ at
a. x = -1/3
b. x = 2
c. x = 1
d. x = -1
e. x =-3

19. The function $g(x)$ in the previous question has an absolute maximum on [-3, 1] of
a. 0.00222
b. 250.6
c. 15.31
d. 120.5
e. none of the above

.....

20. The function $h(x) = x^4$ has
a. no critical points
b. a relative maximum
c. a relative minimum
d. a saddle point
e. a critical point that is neither a relative maximum nor relative minimum

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21. The price a farmer receives for his crop varies according to the function $P(t) = 2t^3 - 15t^2 + 24t + 10$, dollars per kg, t weeks after it is ready for market. Use this information for the next two questions.
There is a critical point at
a. t = 4
b. t = -1
c. t = 2
d. t = 3
e. none of the above

22. There is a relative maximum price at
a. t = 1
b. t = 4
c. t = -1
d. t = 2
e. none of the above
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23. It has been determined that $g(p)$ has a maximum at $p = -47.6$. What can be said of the function's concavity at that point?

a. $g''(p) = 0$

b. There's no way to tell without first knowing what the specific function is.

c. The function is concave down

d.The function is concave up.

e. None of the above

24. Does f(x) = $(x + 2)^3 - 2$ have an inflection point? If so, where is it located?

a.Yes, at (-2, -2).

b. Yes, at (2, -2).

c. Yes, at (8, -2).

d.No.

e. None of the above

25. The 2nd derivative of a function at point P is 0, and the second derivative is positive for values to the right of P. What must the second derivative be to the left of P for P to be an inflection point?

a.The second derivative must also be positive.

b.The second derivative must be negative.

c.The second derivative must be zero.

d.The second derivative must be undefined.

e. None of the above

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The next 4 questions refer to the functions $f(x) = 2x^2 + 10$ and $g(x) = 4x + 16$

26. To find the point of intersection, solve the equation

a) $2x^2 - 4x + 26 = 0$

b) $2x^2 - 4x + 6 = 0$

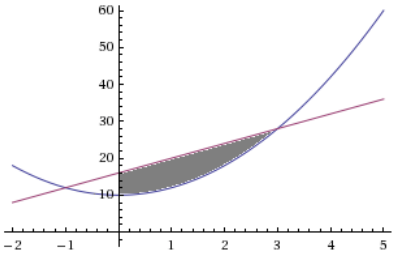
c) $2x^2 - 4x - 6 = 0$

d) $2x^2 + 4x - 6 = 0$

e) None of the above

27. The area bounded by f(x), g(x), x=0, x=3 is represented by the shaded area

a.



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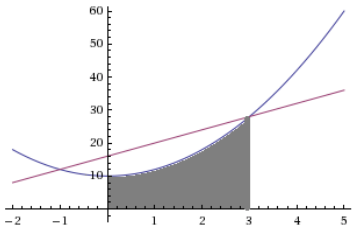
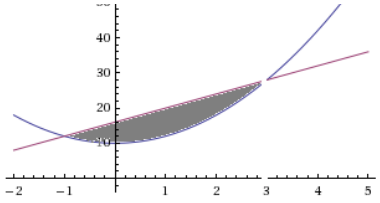
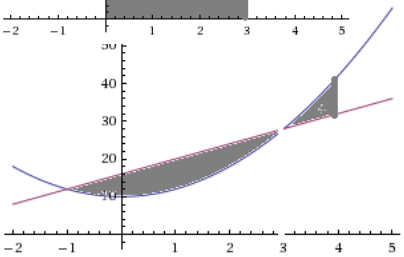


c.



d.



e. none of the above

29. What is the minimum number of separate integrals that are required to find the area bounded by $f(x) = 2x^2 + 10$ and $g(x) = 4x + 16, \quad x = -1$ and $x = 4$

a. 1
b. 2
c. 3
d. 4

30. The area bounded by $f(x) = 2x^2 + 10$ and $g(x) = 4x + 16, \quad x = 0$ and $x = 3$ is

a) 18
b) -18
c) 78
d) 54
e) None of the above

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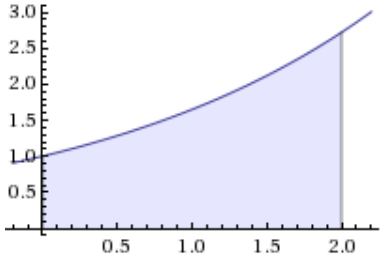
26. A flower bed in a mathematician's house is built in the shape of the area bounded by $e^{x/2}, x = 0, x = 2, y = 0$ This area of this flower bed is

a. 2(e-1)
b. 2e-1
c. e-1
d. 2e
e. (e-1)/2



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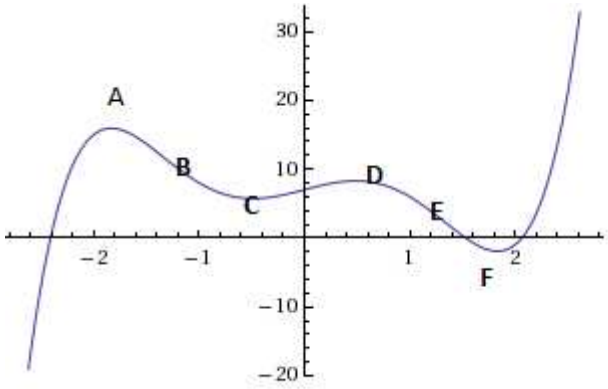
31. Consider the plot of the function f(x) for the next two questions.



A relative maximum occurs at

a) D only
b) A only
c) D and A
d) There is no relative max
e) None of the above

32. At x = -2

a. $f'(x) < 0, f''(x) < 0$

b. $f'(x) < 0, f''(x) > 0$

c. $f'(x) > 0, f''(x) > 0$

d. $f'(x) > 0, f''(x) < 0$

e. $f'(x) > 0, f < 0$

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Consider the function $f(x,y) = \ln(x^2 - y^2)$ for the next 2 questions.

33. The domain of the function includes all x and y with the restriction that

a. $x^2 < y^2$

b. $x > y$

c. $x^2 \geq y^2$

d. $x < y$

e. none of the above

34. The height of the function above the x-y plane when x = e and y = 0 is

a. 0

b. e

c. 2

d. 1

e. none of the above

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35. When $f(x,y) = e^{x^2 y^2}$, find $f_x(1,3) =$

a. $18e^9$

b. $9e^9$

c. $3e^9$

d. $6e^9$

e. none of the above

36. When $f_x = 6x - 2y$ and $f_y = 2y - 2x$, what is $d(0,0)$?

a. 4

b. 8

c. 0

d.16

e. none of the above

37. When $f_{xx} = -4,\ f_{yy} = -5,\ f_{xy} = -3$ at the critical point, what is the nature of the critical point?

a. relative maximum

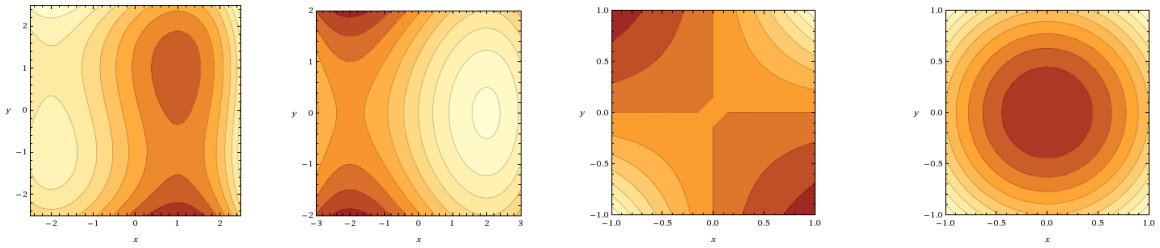
b. relative minimum

c. inflection point

d. saddle point

e. cannot be known unless the function is given

38. If a function had one relative maximum, one relative minimum and two saddle points, which of the following contour plots could illustrate this function?



a. b. c. d.

39. $f(x,y) = 12x - x^3 - 4y^2$. Has a saddle point at

a. (0,-2)
b. (2,0)
c. (-2,0)
d. (0,2)
e. none of the above

40. Two cell phone plans are being marketed by the same company. When Plan A is sold for x dollars per month and Plan B is sold for y dollars per month the total profit is $P(x,y) = -5x^2 + 10xy - 20x - 7y^2 + 240y - 5300$.

This function has one critical point and it is a relative maximum.
What should be the price of Plan A in order to maximize profit?

a. \$95
b. \$64
c. \$74
d. \$53
e. none of the above

41. Evaluate $\int_{-2}^{-1} \frac{7}{5x}\,dx$

a. 1.4
b. -0.9704
c. -0.5136
d. -2.461
e. none of the above

42. Evaluate $\int_{1}^{4} 2 + \frac{3}{\sqrt{x}}\,dx$

a. -4
b. 0
c. 28
d. 12
e. none of the above

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The slope of the tangent to the function $f(x)$ is $f'(x) = 2e^{0.2x}$. The function passes through the point (0,4). Use this information for the next two questions.

43. The constant of integration is :
a. -4
b. 10
c. 14
d. -6
e. none of the above

44. $f(1)$ =
a. 16.2
b. 8.21
c. 18.2
d. 6.21
e. none of the above

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45. The rate at which an investment grows is $30\sqrt{t}$ dollars/year after t years. The initial investment was $1000. How much is the investment worth after 4 years?
a. $1080
b. $1015
c. $1160
d. $1120
e. none of the above

46. The rate at which an employee packages boxes of flowers for shipment is $P'(t) = 10t - 2t^2$ packages per hour, t hours after 7:00 am. How many boxes of flowers are packaged between 8:00 and 10:00 am? Round to the nearest box.
a. 63
b. 15
c. 23
d. 76
e. none of the above

47. A northern cottage has a flat rock area near the shore. Part of the of rock is black and part pink and crystalline. To determine the area of the black rock measurements are taken as below, at 1 meter intervals. Using the trapezoid rule find the area of this black rock.
a. 22.5 m^2
b. 11.3 m^2
c. 12.5 m^2
d. 8.9 m^2
e. none of the above

48. Consider the integral $\int_1^5 (\ln x)^2\,dx$. This can be approximated using the trapezoid rule with n = 8 as $\frac{1}{4}(a+b+c+d+e+f+g+h+i)$

What is *b* ?
a. 0.405
b. 1.02
c. 0.329
d. 0.811
e. none of the above

49. Find the slope of the tangent to the function defined implicitly as $x^2+2y^3=3xy$ at (1,1).
a. -2/3
b. 1/2
c. 1/3
d. 3/4
e. none of the above

50. The lid of a jar is very difficult to open. Hot water is poured on the lid to make it expand. What is the rate of change of the area of the lid if the radius is 2.5 cm and it expands at 0.02 cm/s?
a. 19.63 cm^2/s
b. 0.314 cm^2/s
c. 0.126 cm^2/s
d. 0.514 cm^2/s
e. none of the above

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A participant in a weight loss programme loses weight at a rate of $R'(t)=-5t(3-t^2)^2$ kg/month. Initially this person weighed 90 kg. Use this information for the next 4 questions.

51. The integral can be expressed as $\int Wf(u)\,du$, where W is a constant. What is W?
a. 1.25
b. 5/3
*c. 2.5
d. 5
e. 0.6

52. After integrating you obtain $Lg(u)+C$, where L is a constant and C is the constant of integration. L =
a. 5/12
b. 1/5
c. 5/9
d. 5/4
e.5/6

53. C =
a. 90 kg
b. 79.2 kg
c. 83 kg
d. 67.5 kg
e. none of the above

54. The weight after 2 months is
a. 66.7 kg
b. 83 kg
c. 64.6 kg
d. 90.4 kg
e. 85.2 kg

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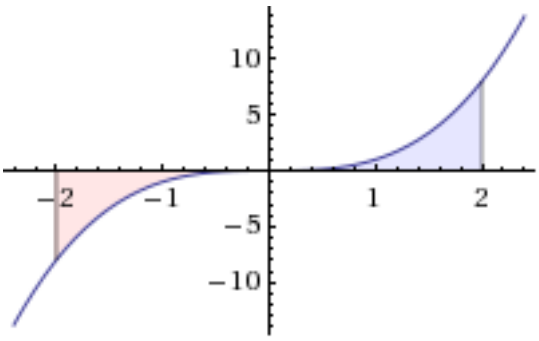
55. When integrating by substitution to find $\int (12x-3)e^{2x^2-x+7}\,dx$, an integral is formed of the form $\int Ae^u\,du$. What is A?

a. 4x-1
b. 4
c. 0.25
d. 3
e. 12x -3

56. Consider the integral $\int_{-2}^{2} f(x)dx$ represented by the graph. If $\int_{0}^{2} f(x)dx = -\int_{-2}^{0} f(x)dx$, = 4
Then the average value of the function f(x) is



a. 4
b. 2
c. 8
d. 0
e. none of the above

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57. At a resort the number of people on the chairs at the beach increases exponentially after breakfast. At 8:00 there are 4 people. Two hours later there are 50 people. What is k, the growth constant?
a.12.5
b. 1.26
c. -1.263
d. 0.00541
e. none of the above

58. Using the information from the previous question, how many people were there at 9:00 am? (Rounded to the nearest person.)
a. 25
b. 14
c. 18
d. 32
e. none of the above

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A garden maze consists of 3 rectangles inside each other. The rectangles are made of fencing covered with climbing flowers, with archways to go between the rectangles. The width of each of the paths between the fences are 1 m. There is no fence on the north side of the innermost rectangle as in the diagram. Use this information for the next 3 questions.

59. Write an expression for the total length of fence.
a. L = 5x + 6y – 20
b. L = 6x + 6y -42
c. L = 8x + 8y – 21
d. L = 2(x + y -3.5)
e. L = 11x -35

60. The area if the maze is to be 80 m^2. In order to find the dimensions for minimum fence length, the derivative of the fence length can be expressed as:
a. $L' = 4 - \frac{200}{x}$
b. $L' = 8 - 64x^{-2}$
c. $L' = 5 - \frac{480}{x^2}$
d. $L' = 500x^2 + x$
e. $L' = 6 - 300x^{-2}$

61. The value of y for minimum fence length is
a. 8.16 m
b. 8.94 m
c. 9.62 m
d. 10.10 m
e. none of the above

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62. Express the following statement as a differential equation. The rate at which a person spends cash , $M(t)$, is proportional to the square of the amount of cash in the person's wallet.

a. $\frac{dM}{dt} = kM^2$

b. $\frac{dM}{dt} = k^2 M$

c. $\frac{dM}{dt} = k^2 - M$

d. $\frac{dM}{dt} = (k - M)^2$

e. $\frac{dM}{dt} = \frac{k}{M^2}$

63. The function $y = e^{2x} + e^{-2x}$ is a solution to which differential equation

a. $y' - y = e^{2x}$

b. $2y - y' = 2e^{-2x}$

c. $y'' - 4y = 0$

d. $y'' - y' - y = 0$

e. none of the above

64. Solve $y' = \frac{3x^2}{2y}$, $y(0) = 3$, to find $y(1)$. In this case consider y is positive.

a. 0.54

b.2.31

c. 4

d. 3.16

e. 2

65. When solving a differential equation , an answer is obtained in the form of $\ln(2y - 3) = 5x^2 + C$. This can be simplified, changing the C, to

a. $y = 1.5 + Ce^{5x^2}$

b. $y = \ln(5x^2 + C) + 3/2$

c. $y = 1.5e^{5x^2 + C}$

d. $y = 0.5\ln(3 + e^{5x^2 + C})$

e. none of the above

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Some frozen fish are put under running water to defrost before cooking. The rate of change of temperature is proportional to the difference between the fish and the tap water. The tap water is at 10^0C and initially the fish were at -11^0C.

After 15 minutes the fish are at -6^0C. Use this information for the next 3 questions.

66. The differential equation expressing the rate of change of temperature $T(t)$ at time t minutes is

a. $T'(t) = k(t-10)$

b. $T'(t) = k(T-11)$

c. $T'(t) = -11kT$

d. $T'(t) = k(T-10)$

e. none of the above

67. Solve the differential equation to find k.

a. -0.00812

b. 0.00781

c. 0.01813

d. -0.0452

e. none of the above

68. The temperature of the fish after 25 minutes is

a. $-1.44\ ^0$C

b. $-2.36\ ^0$C

c $4.61\ ^0$C

d. $-3.35\ ^0$C

e. none of the above

Have you shaded in the boxes for your student number? This is the only way the computer can read and enter your mark.

Have you written your name on the front of this exam paper and on the top of the scantron?

Have you written a large A or B in the top right corner of the scantron?

If so then you are all done, have a great summer !