CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

Probability and Statistics in Engineering
(ENGR 371 Winter 2009)

Final

- Read carefully all questions
- Write all the steps you need to find the solution
- Please do not write in red (colour used for correction)
- Everything which is not readable will not be corrected

Problem 1 (8 points)

a) Three dice were rolled. Given that no two faces were the same, what is the probability that there was a "1"?
b) What is the probability of throwing an 11 at least three times in five throws of a pair of fair dice?

Problem 2 (8 points)

a) If the unemployment rate in Canada is 5%, what is the probability that a random sample of 10,000 will contain between 450 and 550 unemployed?
b) An oil well is to be drilled in a certain location where the soil is either rock (probability 0.53), clay (probability 0.21), or sand. If it is a rock, a geological test gives a positive result with 35% accuracy, if it is clay, this test gives a positive result with 45% accuracy, and if it is sand, the test gives a positive result with 75% accuracy. Given that the test is positive, what is the probability that the soil is rock?

Problem 3 (4 points)

Given the joint density function

$$f(x,y) = \begin{cases} \dfrac{x(1+3y^2)}{4} & 0 < x < 2, 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the conditional probability P(1/4 < X < 1/2 | Y=1/3).

Problem 4 (4 points)

The lengths of time, in minutes, that 10 patients waited in a doctor's office before receiving treatment were recorded as follows: 5, 11, 9, 5, 10, 15, 6, 10, 5, and 10. Assuming that the waiting time is normal distributed with variance $\sigma^2 = 10$.

a) Find the 98% confidence interval for the mean of a patient's waiting time.
b) How large a sample is required if we want to be 98% confident that our estimate of the mean differs from the real mean by less than 1 minute.

Problem 5 (4 points)

Suppose that we wish to test the hypothesis

$$H_0 : \mu = 68 \text{ kilograms}$$
$$H_1 : \mu > 68 \text{ kilograms}$$

For the weight of male students at a certain college using an $\alpha = 0.05$ level of significance when it is known that $\sigma = 5$.

Find the sample size required if the power of our test is to be 0.95 when the true mean is 69 kilograms.

Problem 6 (8 points)

A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 Volts, and the manufacturer wishes to test

$$H_o : \mu = 5\text{Volts}$$
$$H_1 : \mu \neq 5\text{Volts}$$

using n=8 units.

a) The acceptance region is $4.85 \leq \bar{x} \leq 5.15$. Find the value of α.
b) Find the power of the test for detecting a true mean output voltage of 5.1 Volts.
c) Find the boundary of the critical region if the type I error probability is α=0.01.
d) Find the probability of a type II error if the true mean output is 5.05 volts and α=0.01 and n=16.

Problem 7 (4 points)

If the standard deviation of a hole diameter exceeds 0.01 mm, there is an unacceptable high probability that the rivet will not fit. Suppose that n=15 and s=0.008mm.

a) Is there a strong evidence to indicate that the standard deviation of the hole diameter exceeds 0.01mm? Use α=0.01. State the necessary assumptions about the underling distribution of the data.
b) Find the P-value of the test.