(1) Two different manufacturers make a product. Manufacturers M1 and M2 produce 55% and 45% of the product, respectively. It is known from the past experience that 1% and 2% of the products manufactured by each manufacturer, respectively, are defective.
a) If a finished product is randomly selected, what is the probability that it is defective?
b) If a finished product is randomly selected and is found to be defective, what is the probability that the manufacturer M1 has manufactured this product? In this case, which manufacturer was most likely used?

Solution:
(a)

Let,

D = be the event that the selected product is defective.
M1 = the event that the selected product is manufactured by manufacturer 1.
M2 = the event that the selected product is manufactured by manufacturer 2.

Using total probability:

$$P(D) = P(D \mid M1)P(M1) + P(D \mid M2)P(M2) = 0.01 * 0.55 + 0.02 * 0.45 = 0.0145$$

(b) Using Bayes' Theorem:

$$P(M1 \mid D) = \frac{P(D \mid M1) \times P(M1)}{P(D)} = \frac{0.01 \times 0.55}{0.0145} = \frac{11}{29} = 0.379$$

$$P(M2 \mid D) = 1 - P(M1 \mid D) = 1 - \frac{11}{29} = \frac{18}{29} = 0.621$$

(2) A car manufacturer produces one car every hour. It is found that 15% of the manufactured cars have a minor problem.
a) Find the probability that 2 or 3 of the produced cars in the first 12 hours have minor problems.
b) What is the probability that the first car with minor problem is produced in the 13th hour?
c) What is the probability that the second car with minor problem is produced in the 10th hour?

Solution:
(a)

Let, X = number of cars with minor problems.
The random variable X follows binomial distribution with $n = 12$ and $p=0.15$:

$$P(X = x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$P(2 \le X \le 3) = P(X = 2) + P(X = 3) = \binom{12}{2} 0.15^2 0.85^{10} + \binom{12}{3} 0.15^3 0.85^9$$

$$= \frac{12!}{10!\ 2!} 0.15^2 0.85^{10} + \frac{12!}{9!\ 3!} 0.15^3 0.85^9 = 0.4643$$

(b) Let, X = Hour of occurrence of first car with minor problem
Here, X will follow geometric distribution with p=0.15:

$$P(X = x) = f(x) = (1-p)^{x-1}p$$

$$P(X = 13) = 0.85^{12}0.15^{1} = 0.02133$$

(c) Let, X = Hour of occurrence of second car with minor problem
For this problem the variable X will follow negative binomial distribution with p=0.15 and r=2:

$$P(X = x) = f(x) = \begin{pmatrix} x-1 \\ r-1 \end{pmatrix}(1-p)^{x-r}p^{r}$$

$$P(X = 10) = \begin{pmatrix} 9 \\ 1 \end{pmatrix}0.15^{2}0.85^{8} = \frac{9!}{8!\ 1!}0.15^{2}0.85^{8} = 0.05518$$

(3) Measurements of scientific systems are always subject to variation, some more than the others. Suppose that the measurement error X of a certain physical quantity is decided by the density function, $f(x) = 0.5x - 1$ for $2 < x < 4$
a) Determine the cumulative distribution function of the measurement error.
b) For a particular measurement, it is not desirable to have a magnitude of error exceeding 3. What is the probability that this occurs?
c) Determine the mean of the measurement error.

Solution:
(a)

$$F(x) = \int_{2}^{x}(0.5x-1)\,dx = 0.5\frac{t^2}{2} - t\Big|_{2}^{x} = \frac{x^2}{4} - x + 1$$

Then,

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{x^2}{4} - x + 1, & 2 \le x < 4 \\ 1, & 4 \le x \end{cases}$$

(b)

$$P(X \le 3) = \int_{2}^{3}(0.5x-1)\,dx = \left(0.5\frac{x^2}{2} - x\right)\Big|_{2}^{3} = \left(\frac{9}{4} - 3\right) - \left(\frac{4}{4} - 2\right) = \frac{1}{4}$$

(c) Mean,

$$E(X)=\int_2^4 x(0.5x-1)\,dx = 0.5\frac{x^3}{3}-\frac{x^2}{2}\Bigg|_2^4 = \left(\frac{32}{3}-8\right)-\left(\frac{4}{3}-2\right)=\frac{10}{3}$$

(4) Based on past experience, 40% of all customers at Miller's Automotive Service Station pay for their purchases with a credit card. If a random sample of 200 customers is selected, what is the *approximate* probability that

a) At least 75 pay with a credit card?
b) Not more than 70 pay with a credit card?
c) Between 70 and 75 customers, inclusive, pay with a credit card?

Solutions:

Here,

Number of customers, n = 200
Probability of a customer paying by credit card, p = 0.4.

Check:
np = 80 ≥ 5 and n(1− p) = 120 ≥ 5

So standard normal distribution can be approximated.

Mean, μ = *np* = 80
Standard Deviation, σ = $\sqrt{np(1-p)}$ = 6.9282

Standard Normal Variable, $Z=\dfrac{X-\mu}{\sigma}$

(a) P (X ≥ 75) ≅ P (X ≥ 74.5) = P (Z ≥ − 0.7939) = 0.7864

(b) P (X ≤ 70) ≅ P (X ≤ 70.5) = P (Z ≤ − 1.3712) = 0.0852

(c) P (70 ≤ X ≤ 75) ≅ (69.5 ≤ X ≤ 75.5) = P (−1.5155 ≤ Z ≤ − 0.6495) = 0.1932