Solutions Test 1 MATH2007 B Fall 2017

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1. $f(x) = 7e^x - 3x^2 + 1$

$\int f(x)\,dx = \int (7e^x - 3x^2 + 1)\,dx = 7e^x - x^3 + x + C$ gives all antiderivatives

Say $C = 0, 1$ and 2. Then we get
$$\boxed{\begin{array}{l} 7e^x - x^3 + x \\ 7e^x - x^3 + x + 1 \\ 7e^x - x^3 + x + 2 \end{array}}$$
are 3 antiderivatives

many other possibilities

2. $$\int_0^2 (5x^2 - 4x + 2)\,dx = \left(\frac{5x^3}{3} - 2x^2 + 2x\right)\Bigg|_0^2$$
$$= \frac{5(8)}{3} - 8 + 4 = \frac{40}{3} - 4 = \frac{40 - 12}{3} = \boxed{\frac{28}{3}}$$

3. $\int e^x \cos x\,dx$

$u = \cos x,\ dv = e^x\,dx$

$\frac{du}{dx} = -\sin x,\ v = e^x$

$uv - \int v\,du = e^x \cos x - \int e^x(-\sin x\,dx) = e^x \cos x + \int e^x \sin x\,dx$

$\therefore \boxed{\int e^x \cos x\,dx = e^x \cos x + \int e^x \sin x\,dx}$

$\int e^x \sin x\,dx$

$u = \sin x,\ dv = e^x\,dx$

$\frac{du}{dx} = \cos x,\ v = e^x$

$uv - \int v\,du = e^x \sin x - \int e^x \cos x\,dx$

$\boxed{\therefore \int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx}$

Hence, we have $\int e^x \cos x\,dx = e^x \cos x + \left(e^x \sin x - \int e^x \cos x\,dx\right)$

$\Rightarrow 2\int e^x \cos x\,dx = e^x \cos x + e^x \sin x$

$\Rightarrow \boxed{\int e^x \cos x\,dx = \frac{e^x \cos x}{2} + \frac{e^x \sin x}{2} + C}$

4. $\int \sin x \cos^3 x\,dx$

$u = \cos x,\ du = -\sin x\,dx$

$\Rightarrow \int \sin x\, u^3 \frac{-du}{\sin x} = -\int u^3\,du = -\frac{u^4}{4} + C$

$= \boxed{-\frac{\cos^4 x}{4} + C}$

5. $\int x^4 e^{2x}\,dx$

$+$	x^4	e^{2x}
$-$	$4x^3$	$\frac{e^{2x}}{2}$

$\Rightarrow \boxed{\int x^4 e^{2x}\,dx = \frac{x^4 e^{2x}}{2} - x^3 e^{2x} + \frac{3}{2} x^2 e^{2x} - \frac{3}{2} x e^{2x} + \frac{3e^{2x}}{4} + C}$

$+$

$-\quad 4x^3 \quad \frac{e^{2x}}{2}$

$+\quad 12x^2 \quad e^{2x}/4$

$-\quad 24x \quad e^{2x}/8$

$+\quad 24 \quad e^{2x}/16$

$-\quad 0 \quad e^{2x}/32$

$\Rightarrow \boxed{\int x e \quad 2 \quad - \frac{3}{2} x e^{2x} + \frac{3e^{2x}}{4} + C}$

6. $\int 4x \sec^2(3x^2+2)\,dx$

$\boxed{u = 3x^2+2}$ $\quad du/dx = 6x$ $\quad \boxed{dx = \frac{du}{6x}}$

$= \int 4x \sec^2(u) \frac{du}{6x} = \frac{2}{3}\int \sec^2 u\,du = \frac{2}{3}\tan u + C$

$= \boxed{\frac{2}{3}\tan(3x^2+2) + C}$

7. $\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$

$\frac{5x^2+20x+6}{x^3+2x^2+x} = \frac{5x^2+20x+6}{x(x^2+2x+1)} = \frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$= \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$

$5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$

$5x^2+20x+6 = A(x^2+2x+1) + B(x^2+x) + Cx$

$5x^2+20x+6 = (A+B)x^2 + (2A+B+C)x + A$

$A = 6$

$B = -1$

$C = 9$

$\therefore \int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx = \int\left(\frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}\right)dx = \boxed{6\ln|x| - \ln|x+1| - \frac{9}{x+1} + C}$

8. $\int \frac{5-x^2}{x^3+x}\,dx = ?$

$\frac{5-x^2}{x^3+x} = \frac{5-x^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\Rightarrow 5-x^2 = A(x^2+1) + (Bx+C)x$

$\Rightarrow 5-x^2 = (A+B)x^2 + Cx + A$

$\Rightarrow A+B = -1,\ C = 0,\ A = 5$

$\Rightarrow B = -6$

$\therefore \int \frac{5-x^2}{x^3+x}\,dx = \int\left(\frac{5}{x} + \frac{-6x}{x^2+1}\right)dx = \int \frac{5}{x}\,dx - \int \frac{6x}{x^2+1}\,dx$

$3\ln(x^2+1) + C$

$$\therefore \int \frac{5-x^2}{x^3+x}\,dx = \int \left(\frac{5}{x} + \frac{-6x}{x^2+1}\right) dx$$

$$= 5\ln|x| - 3\ln(x^2+1) + C$$