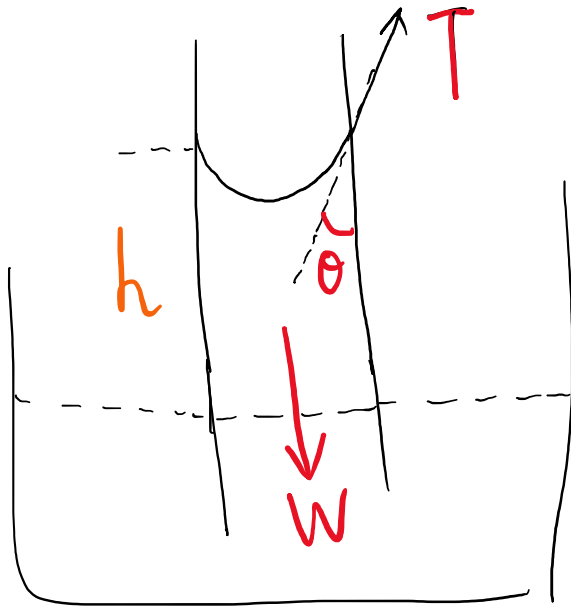


Homework #7

Problem #7.1 Whole blood has a surface tension of 0.058 N/m and a density of 1050 kg/m^3 . To what height can whole blood rise in a capillary blood vessel that has a radius of $2.0 \times 10^{-6} \text{ m}$? Assume the tangent angle $\theta = 0^\circ$ when blood contacts the wall of the capillary.



$$W = mg = \rho(\pi r^2 h) g$$

$$W = T \cos \theta = (\gamma \cdot 2\pi r) \cos \theta$$

$$\therefore \rho(\pi r^2 h) g = (2\pi r \cdot \gamma) \cos \theta$$

$$\therefore h = \frac{2\gamma \cos \theta}{\rho g r} = \frac{2\gamma \cos 0^\circ}{\rho g r} = \frac{2\gamma}{\rho g r}$$

$$h = \frac{2 \cdot (0.058 \text{ N/m})}{(1050 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.0 \times 10^{-6} \text{ m})} = 5.6 \text{ m}$$

Problem #7.2 With the nose above the water, about 95% of the body is submerged. Calculate the power expended by a 50-kg person treading water in this position. Assume that the average density of the body is about the same as water and that the area A of the limbs acting on the water is about 600 cm^2 . (You can use results derived from class lecture.)

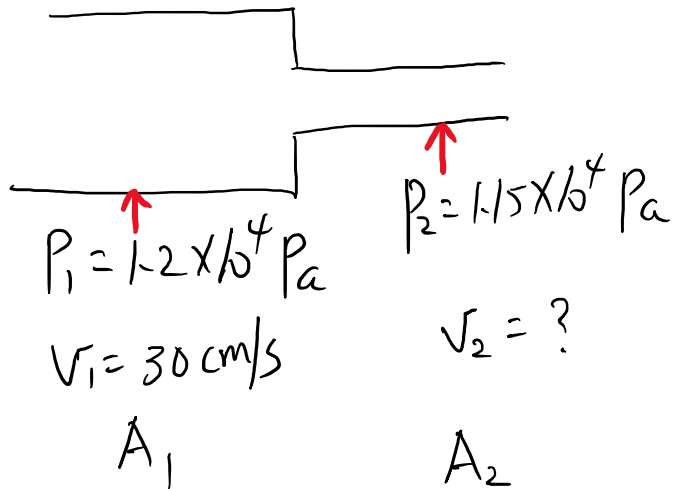
$$W = mg = (50 \text{ kg})(9.8 \text{ m/s}^2) = 490 \text{ N}, \quad \rho \approx \rho_w$$

$$\text{Power} = \frac{1}{2} \sqrt{\frac{[W(1 - f\rho_w/\rho)]^3}{A\rho_w}} = \frac{1}{2} \sqrt{\frac{[W(1 - f)]^3}{A\rho_w}}$$

$$= \frac{1}{2} \sqrt{\frac{[490 \text{ N}(1 - 0.95)]^3}{(600 \times 10^{-4} \text{ m}^2)(1 \times 10^3 \text{ kg/m}^3)}}$$

$$= 7.8 \text{ J/s}$$

Problem #7.3 A medical technician is trying to determine what percentage of a patient's artery is blocked by plaque. To do this, she measures the blood pressure just before the region of blockage and finds that it is $1.20 \times 10^4 \text{ Pa}$, while in the region of blockage it is $1.15 \times 10^4 \text{ Pa}$. Furthermore, she knows that blood flowing through the normal artery just before the point of blockage is traveling at 30.0 cm/s . What percentage of the cross-sectional area of the patient's artery is blocked by the plaque? Blood density is 1060 kg/m^3 . (Hint: use Bernoulli's equation.)



$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$\therefore \frac{1}{2} \rho v_2^2 = P_1 - P_2 + \frac{1}{2} \rho v_1^2 = 548 \text{ Pa}$$

$$\therefore v_2 = \sqrt{\frac{2 \times 548 \text{ Pa}}{1060 \text{ kg/m}^3}} = 1.02 \text{ m/s}$$

$$A_1 \cdot v_1 = A_2 \cdot v_2 \Rightarrow \frac{A_2}{A_1} = \frac{v_1}{v_2}$$

$$\therefore \frac{A_1 - A_2}{A_1} = 1 - \frac{A_2}{A_1} = 1 - \frac{v_1}{v_2} = 1 - \frac{0.3 \text{ m/s}}{1.02 \text{ m/s}} = 0.706 = 70.6\%$$

Problem #7.4 Viscous blood is flowing through an artery partially clogged by cholesterol. A surgeon wants to remove enough of the cholesterol to **double the flow rate** of blood through this artery. If the original diameter of the artery is D , what should be the new diameter (in terms of D) to accomplish this for the same pressure difference?

$$Q_1 = \frac{\pi R_1^4 \Delta P}{8 \eta \cdot L}, \quad Q_2 = \frac{\pi R_2^4 \Delta P}{8 \eta \cdot L}$$

$$\frac{Q_2}{Q_1} = \frac{R_2^4}{R_1^4} = 2.0$$

$$\therefore \frac{R_2}{R_1} = (2.0)^{1/4} = 1.189$$

$$\therefore R_2 = 1.189 R_1$$

$$\therefore D_2 = 1.189 D_1 = 1.189 D$$

$$Q_2 = 2 Q_1$$

Problem #7.5 (a) Show that if the pressure drop remains constant, reduction of the radius of the arteriole from 0.10 to 0.08 mm decreases the blood flow by more than a factor of 2. (b) Calculate the decrease in the radius required to reduce the blood flow by 90%.

$$(a) \quad Q_1 = \frac{\pi R_1^4 \Delta P}{8 \eta L}, \quad Q_2 = \frac{\pi R_2^4 \Delta P}{8 \eta L}$$

$$\frac{Q_2}{Q_1} = \frac{R_2^4}{R_1^4} = \left(\frac{R_2}{R_1}\right)^4 = \left(\frac{0.08}{0.10}\right)^4 = 0.41$$

$$\therefore \frac{Q_1}{Q_2} = \frac{1}{0.41} = 2.4 > 2.0$$

$$(b) \quad Q_2 = (1 - 90\%) Q_1 = 0.1 Q_1 \quad \therefore \frac{Q_2}{Q_1} = 0.1$$

$$\left(\frac{R_2}{R_1}\right)^4 = 0.1$$

$$\therefore \frac{R_2}{R_1} = (0.1)^{1/4} = 0.562$$

$$\therefore R_2 = R_1 \times 0.562 = (0.1 \text{ mm})(0.562) = 0.0562 \text{ mm}$$