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assignment 7 (mat1320)

Score: **10/10.0** Duration: 16 hrs 58 min
 Started: 12/2/17 2:24:01 AM GMT
 Finished: 12/2/17 7:22:11 PM GMT

- Haard Trivedi

Login: uottawa2017fa0617 Assignments
 Email: htriv050@uOttawa.ca Completed: 12 Active: 2
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Question

Q1

1/1.0

Let $f(x) = \tan(x/10)$ for $\pi \leq x \leq 3\pi$. We wish to estimate

$$\int_{\pi}^{3\pi} f(x) dx$$

by the Midpoint Rule.

In (a), (b) and (c), you must provide the exact values (as formulas). The approximate decimal values are not accepted.

a) Divide the domain of f into 5 sub-intervals of equal length. Calculate their common length Δx .

Your response	Correct response
(2/5)*pi	

✔ Grade: 1/1.0

b) What are the midpoints m_1, m_2, m_3, m_4 and m_5 ? List below their values, in ascending order, separating them by commas.

Your response	Correct response
1.2*pi, 1.6*pi, 2*pi, 2.4*pi, 2.8*pi	

✔ Grade: 1/1.0

c) List below the corresponding values $f(m_1), f(m_2), f(m_3), f(m_4)$ and $f(m_5)$, separating them by commas.

Your response	Correct response
tan(.12*pi), tan(.16*pi), tan(.2*pi), tan(.24*pi), tan(.28*pi)	

✔ Grade: 1/1.0

d) Using the information found in (a), (b) and (c), find the approximation of

$$\int_{\pi}^{3\pi} f(x) dx$$

that is produced by the Midpoint Rule with 5 sub-intervals. Give the answer with 3 decimal places.

Your response	Correct response
4.800	

✔ Grade: 1/1.0

✔ Total grade: $1.0 \times 1/4 + 1.0 \times 1/4 + 1.0 \times 1/4 + 1.0 \times 1/4 = 25\% + 25\% + 25\% + 25\%$
 Comment:

(a) We have $\Delta x = \frac{3\pi - \pi}{5} = \frac{2\pi}{5}$.

(b) The 5 intervals have as endpoints π , $(7/5)\pi$, $(9/5)\pi$, $(11/5)\pi$, $(13/5)\pi$ and 3π . Their midpoints are thus

$$m_1 = \frac{\pi + (7/5)\pi}{2} = 1.2\pi ,$$

$$m_2 = \frac{(7/5)\pi + (9/5)\pi}{2} = 1.6\pi ,$$

$$m_3 = \frac{(9/5)\pi + (11/5)\pi}{2} = 2\pi ,$$

$$m_4 = \frac{(11/5)\pi + (13/5)\pi}{2} = 2.4\pi$$

et

$$m_5 = \frac{(13/5)\pi + 3\pi}{2} = 2.8\pi .$$

(c) We have

$$f(m_1) = \tan(0.12\pi) ,$$

$$f(m_2) = \tan(0.16\pi) ,$$

$$f(m_3) = \tan(0.2\pi) ,$$

$$f(m_4) = \tan(0.24\pi)$$

et

$$f(m_5) = \tan(0.28\pi) .$$

(d) Finally, we obtain

$$\int_{\pi}^{3\pi} f(x) dx \cong \sum_{k=1}^5 f(m_k) \Delta x$$

$$= (\tan(0.12\pi) + \tan(0.16\pi) + \tan(0.2\pi) + \tan(0.24\pi) + \tan(0.28\pi)) (2/5)\pi$$

$$\cong 4.8005 .$$

Q2

1/1.0

Question
Let $f(x) = \sqrt{x^2 + x}$ for $1 \leq x \leq 7$. We wish to estimate

$$\int_1^7 f(x) dx$$

by the Trapezoidal Rule.

a) Divide the domain of f into 4 sub-intervals of equal length. Calculate their common length Δx (exact value).

Your response	Correct response
3/2	

✔ Grade: 1/1.0

b) Find the approximation of

$$\int_1^7 f(x) dx$$

that the Trapezoidal Rule produces with 4 sub-intervals. Give the answer with 4 decimal places.

Your response	Correct response
26.7871	

✔ Grade: 3/3.0

✔ Total grade: $1.0 \times 1/4 + 1.0 \times 3/4 = 25\% + 75\%$

Comment:

(a) We have $\Delta x = \frac{7-1}{4} = 3/2$.

(b) The endpoints of the 4 intervals are

$$\begin{aligned} x_0 &= 1, & x_1 &= 1 + 3/2 = 5/2, & x_2 &= 1 + 2(3/2) = 4, \\ x_3 &= 1 + 3(3/2) = 11/2 & \text{and} & & x_4 &= 1 + 4(3/2) = 7. \end{aligned}$$

Hence, the Trapezoidal Rule gives

$$\begin{aligned} \int_1^7 f(x) dx &\cong \frac{1}{2} \left(f(x_0) + 2 \sum_{k=1}^3 f(x_k) + f(x_4) \right) \Delta x \\ &= \frac{1}{2} (1.414214 + 2((2.95804) + (4.472136) + (5.97913)) + 7.483315)(3/2) \\ &\cong 26.7871. \end{aligned}$$

Question

Q3

1/1.0

Let $f(x) = \sqrt{x^2 - 4x}$ for $4 \leq x \leq 13$. We wish to estimate

$$\int_4^{13} f(x) dx$$

by Simpson's Rule.

a) Divide the domain of f into 6 sub-intervals of equal length. Calculate their common length Δx (exact value).

Your response	Correct response
3/2	

✔ Grade: 1/1.0

b) Find the approximation of

$$\int_4^{13} f(x) dx$$

that Simpson's Rule produces with 6 sub-intervals. Give the answer with 3 decimal places.

Your response	Correct response
54.425	

✔ Grade: 3/3.0

✔ Total grade: $1.0 \times 1/4 + 1.0 \times 3/4 = 25\% + 75\%$

Comment:

(a) We have $\Delta x = \frac{13 - 4}{6} = 3/2$.

(b) The endpoints of the 6 intervals are

$$\begin{aligned} x_0 &= 4, & x_1 &= 4 + 3/2 = 11/2, & x_2 &= 4 + 2(3/2) = 7, \\ x_3 &= 4 + 3(3/2) = 17/2, & x_4 &= 4 + 4(3/2) = 10, \\ x_5 &= 4 + 5(3/2) = 23/2, & x_6 &= 4 + 6(3/2) = 13. \end{aligned}$$

Hence, Simpson's Rule gives

$$\begin{aligned} \int_4^{13} f(x) dx &\cong \frac{1}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)) \Delta x \\ &= \frac{1}{3} (0 + 4\sqrt{8.25} + 2\sqrt{21} + 4\sqrt{38.25} + 2\sqrt{60} + 4\sqrt{86.25} + \sqrt{117}) (3/2) \\ &\cong 54.4249 \end{aligned}$$

Q4

1/1.0

Question

The midpoint rule does not compute an integral $\int_a^b f(x) dx$ exactly. The **error** is the difference between the midpoint rule estimate M_n and the actual value; we cannot find the actual error unless we can evaluate the integral exactly. We have a formula for an **error bound** on the midpoint rule, depending on f, a, b and n ; we can calculate the error bound without evaluating the integral exactly. The important theorem is that the error on the estimate will always be less than or equal to the error bound, in absolute value.

Consider $f(x) = (x - 1)^5$ on the interval $[0, 3]$.

Compute $f''(x) =$

Your response	Correct response
20*(x-1)^3	

✔ Grade: 1/1.0

What is the maximum value of $|f''(x)|$ on the interval $[0, 3]$?

Your response	Correct response
160	

✔ Grade: 1/1.0

The error bound on M_n , as a function of n , is therefore $B(n) =$

Your response	Correct response
180/n^2	

✔ Grade: 1/1.0

Therefore, to be sure that M_n is within 10^{-3} of the true value of the integral $\int_0^3 (x - 1)^5 dx$, we need $B(n) \leq 10^{-3}$, which means we must choose $n =$

Your response	Correct response
425	

✔ Grade: 1/1.0

(Round up to the nearest integer.)

✔ Total grade: $1.0 \times 1/4 + 1.0 \times 1/4 + 1.0 \times 1/4 + 1.0 \times 1/4 = 25\% + 25\% + 25\% + 25\%$

Comment:

We have $f'(x) = 5(x - 1)^4$ so $f''(x) = 20(x - 1)^3$.

Since $f'''(x) = 60(x - 1)^2$, the only critical point of f'' is $x = 1$, where $f''(1) = 0$. Therefore the maximum K of $|f''(x)|$ on $[a, b] = [0, 3]$ will occur at an endpoint of the interval. We find $f''(0) = -20$ and $f''(3) = 160$ so the answer is $K = 160$.

The error bound is $B(n) = \frac{K(b - a)^3}{24n^2} = \frac{160(3 - 0)^3}{24n^2} = 180 \times \frac{1}{n^2}$.

Solving $B(n) \leq 10^{-3}$ yields $180 \times \frac{1}{n^2} \leq 10^{-3}$. Multiplying both sides by $10^3 n^2$ and simplifying gives $180 \times 10^3 \leq n^2$.

So we should choose our integer $n \geq \sqrt{180 \times 10^3} \approx 424.264069$. Namely, we should choose $n = 425$.

Q5

1/1.0

Consider the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$$f(x) = x^6 (1 - x)^5.$$

(a) Provide a list of all the critical numbers of f . Separate the values by a semi-colon should there be more than one.

Input the word "none" (without quotes or capitals) should there be none.

$x =$

Your response	Correct response
0;1;6/11	

✔ Grade: 1/1.0

(b) Provide a list of all the local minima of f (following the same instruction).

$x =$

Your response	Correct response

0 Question

Grade: 1/1.0

(c) Provide a list of all the local maxima of f (following the same instruction).
 $x =$

Your response	Correct response
6/11	

Grade: 1/1.0

Total grade: $1.0 \times 1/3 + 1.0 \times 1/3 + 1.0 \times 1/3 = 33\% + 33\% + 33\%$

Comment:

As f is a polynomial, it is everywhere differentiable, and its only critical numbers are the solutions of $f'(x) = 0$. We have

$$\begin{aligned} f'(x) &= 6x^5(1-x)^5 - 5x^6(1-x)^4 \\ &= (6(1-x) - 5x)x^5(1-x)^4 \\ &= (6 - 11x)x^5(1-x)^4. \end{aligned}$$

The critical numbers of f are thus 0, $6/11$ and 1. The table below shows the sign of each of the factors of $f'(x)$ and that of $f'(x)$ on each of the sub-intervals defined by these points. On the last line, we indicate the behavior of f ("↗" for increasing and "↘" for decreasing).

x	$-\infty$	0	$6/11$	1	∞		
$6 - 11x$	+	+	0	-	-		
x^5	-	0	+	+	+		
$(1 - x)^4$	+	+	+	0	+		
$f'(x)$	-	0	+	0	-	0	-
$f(x)$		↘	↗	↘	↘		

We deduce that f has a local maximum at $x = 6/11$ and a local minimum at $x = 0$.

Q6

1/1.0

Consider the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by

$$f(x) = \sqrt{x^4 + 35} - \frac{x^3}{9}.$$

(a) Provide a list of all the critical numbers of f . Separate the values by a semi-colon should there be more than one.

Input the word "none" (without quotes or capitals) should there be none.

$x =$

Your response	Correct response
0;1;sqrt(35)	

Grade: 1/1.0

Question	
(b) Provide a list of all the local minima of f (following the same instruction). $x =$	
Your response	Correct response
1	

Grade: 1/1.0

(c) Provide a list of all the local maxima of f (following the same instruction). $x =$	
Your response	Correct response
sqrt35	

Grade: 1/1.0

Total grade: $1.0 \times 1/3 + 1.0 \times 1/3 + 1.0 \times 1/3 = 33\% + 33\% + 33\%$
Comment:

As $x^4 + 35 > 0$ for all $x \in \mathbf{R}$, the function f is defined for all \mathbf{R} . It is even differentiable for all \mathbf{R} , with

$$f'(x) = \frac{4x^3}{2\sqrt{x^4+35}} - \frac{x^2}{3} = \frac{x^2(6x - \sqrt{x^4+35})}{3\sqrt{x^4+35}}$$

The critical numbers of f are thus the zeros of $f'(x)$, that is to say, the solutions of

$$x^2(6x - \sqrt{x^4+35}) = 0$$

- $\Leftrightarrow x = 0$ ou $6x = \sqrt{x^4+35}$
- $\Leftrightarrow x = 0$ ou $(x > 0 \text{ et } 36x^2 = x^4 + 35)$
- $\Leftrightarrow x = 0$ ou $(x > 0 \text{ et } (x^2 - 1)(x^2 - 35) = 0)$
- $\Leftrightarrow x = 0, 1 \text{ ou } \sqrt{35}$.

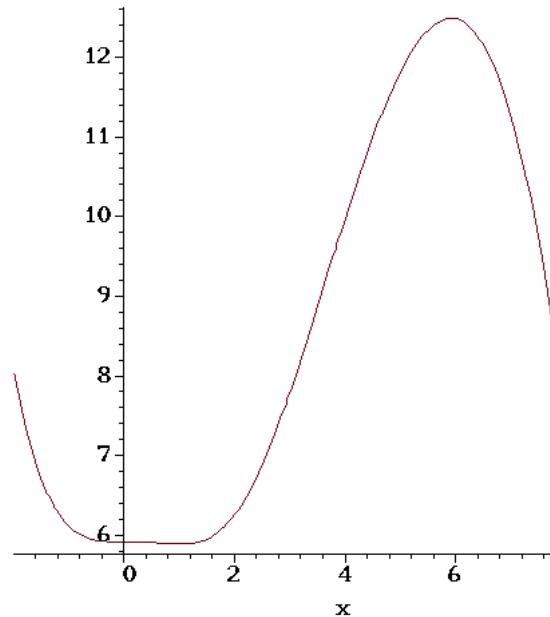
Thus the critical numbers of f are $x = 0, 1$ and $\sqrt{35}$. Moreover, as f' is continuous, the sign of $f'(x)$ is the same for each of the 4 intervals of \mathbf{R} defined by these points, equivalent to the sign of $6x - \sqrt{x^4+35}$. We obtain the table below, with the last line indicating the behavior of $f(x)$ (" / " for increasing and " \ " for decreasing).

x	$-\infty$	0	1	$\sqrt{35}$	∞		
$f'(x)$	-	0	-	+	-		
$f(x)$	\	5.9	\	5.9	/	12.5	\

We deduce that $x = 1$ is a local minimum of f and that $x = \sqrt{35}$ is a local maximum of f . However, $x = 0$ is not a local extremum.

Question

Here to illustrate these calculations, the graph of the function f .



Q7

1/1.0

Consider the function $f(x) = \frac{x^2 + 1}{x}$, defined for all $x \neq 0$.

(i) Provide a list of all the local minima of f separating the values by a semi-colon should there be more than one.
Input the word "none" (without quotes or capitals) should there be none.
 $x =$

Your response	Correct response
1	

Grade: 1/1.0

(ii) Provide a list of all the local maxima of f following the same instruction.
 $x =$

Your response	Correct response
-1	

Grade: 1/1.0

Total grade: $1.0 \times 1/2 + 1.0 \times 1/2 = 50\% + 50\%$
Comment:

As $f(x) = x + \frac{1}{x}$, on trouve

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} = \frac{(x - 1)(x + 1)}{x^2}$$

Thus, excluding the point $x = 0$ which is not in the domain of the function, the function f is everywhere differentiable. Therefore, the critical numbers of f are the zeros $x = \pm 1$ of its derivative.

The table below shows the sign of the factors of $f'(x)$ and that of $f'(x)$ on each of the intervals defined by these points and the number $x = 0$. The last line gives the corresponding behavior of f .

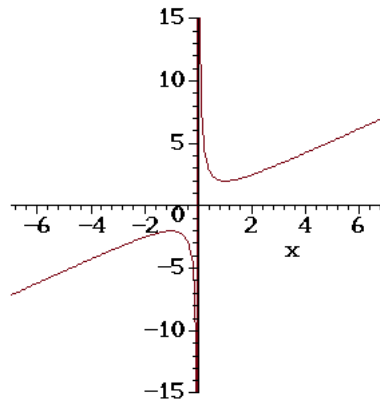
x	$-\infty$	-1	0	1	∞
$x - 1$	-	-	-	0	+
$x + 1$	-	0	+	+	+

□

Question

x^2	+		+	0	+	+	
$f'(x)$	+	0	-		-	0	+
$f(x)$	↗	-2	↘		↘	2	↗

From the table, we observe that f has a local maximum at $x = -1$ and a local minimum at $x = 1$.



□

Q8

1/1.0

Question
Consider the function $f : \mathbf{R} \rightarrow \mathbf{R}$ given by

$$f(x) = x^4 - 32x^2.$$

Find the minimum and maximum values (absolute) of f over the interval $[2, 5]$. Provide the exact values.

Minimum value =

Your response	Correct response
-256	

✔ Grade: 1/1.0

Maximum value =

Your response	Correct response
-112	

✔ Grade: 1/1.0

✔ Total grade: $1.0 \times 1/2 + 1.0 \times 1/2 = 50\% + 50\%$

Comment:

As f is continuous and differentiable everywhere on \mathbf{R} , it attains its absolute minimum on $[2, 5]$ at one of the extremities of this interval, or at a point of this interval where its derivative is zero. The same phenomenon applies to its absolute maximum. We find

$$\begin{aligned} f'(x) &= 4x^3 - 64x \\ &= 4x(x^2 - 4^2) \\ &= 4x(x - 4)(x + 4) \end{aligned}$$

The only zero of $f'(x)$ over $[2, 5]$ is $x = 4$. Therefore f attains its absolute minimum, just as its absolute maximum, at one of the points 2, 4 or 5. As

$$f(2) = -112, \quad f(4) = -256 \quad \text{et} \quad f(5) = -175,$$

we conclude that the minimum value of f is -256 attained at $x = 4$ and that its maximum value is -112 attained at $x = 2$.

Q9

1/1.0

Consider the function $f : \mathbf{R} \rightarrow \mathbf{R}$ given by

$$f(x) = x^3 e^{-3x}.$$

Estimate the minimum value and maximum value (absolute) of f over the interval $[3, 4]$, with precision ± 0.001 .

Minimum value =

Your response	Correct response
4E-4	

✔ Grade: 1/1.0

Maximum value =

Your response	Correct response
0.0033	

✔ Grade: 1/1.0

✔ Total grade: $1.0 \times 1/2 + 1.0 \times 1/2 = 50\% + 50\%$

Comment:

As f is continuous and differentiable everywhere on \mathbf{R} , it attains its absolute minimum on $[3, 4]$ at one of the extremities of this interval, or at a point of this interval where its derivative is zero. The same phenomenon applies to its absolute maximum. We find

$$f'(x) = 3x^2 e^{-3x} + x^3 (-3e^{-3x}) = (3 - 3x)x^2 e^{-3x}.$$

Hence the only zeros of the derivative are $x = 0$ and $x = 1$. Therefore $f'(x)$ does not reach zero on $[3, 4]$. More precisely, we have $f'(x) < 0$ for $x > 1$.

Thus, f is decreasing on $(1, \infty)$ and therefore is decreasing on $[3, 4]$. On this interval, its absolute maximum is $f(3) \cong 0.0033$ and its absolute minimum is $f(4) \cong 4E-4$.