Physics 204
Solutions
Assignment 6

Q1

1. A wheel starts from rest and rotates with constant angular acceleration to an angular speed of 12.0 rad/s in 3.0 s. Find (a) the magnitude of angular acceleration of the wheel and (b) the angle in radians through it rotates in this time.

<u>Solution:</u> $\omega_0 = 0$, Angular acceleration, $\alpha = ?$
$\omega = 12.0\ \text{rad/s}$ after $t = 3s$

(a) using the equation, $\omega = \omega_0 + \alpha t$

we get: $12 = 0 + 3\alpha$

Angular acceleration $\rightarrow \alpha = \frac{12}{3} = 4\ \text{rad/s}^2$

(b) Angle θ rotated after time t is given by:

$$\theta = \theta_0 + \frac{1}{2}\alpha t^2$$

In the problem, $\theta_0 = 0$ $\quad \alpha = 4\ \text{rad/s}^2$ $\quad t = 3$

$$\therefore \theta = 0 + \frac{1}{2} \times 4 \times 3^2 = 18\ \text{rad}$$

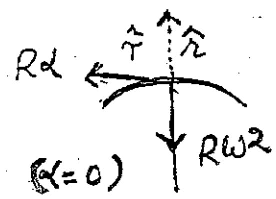
Q.2

A racing car travels in a circular track of radius 250 m. If the car moves with a constant linear speed of 45.0 m/s, find (a) its angular speed and (b) the magnitude and direction of its acceleration.

<u>Solution:</u> Radius of track = 250 m
Linear speed = 45.0 m/s

(a) Angular velocity $\rightarrow \omega = \frac{v}{R} = \frac{45.0}{250.0} = 0.18\ \text{rad/s}.$

(b) $\vec{a} = (R\alpha)\,\hat{\tau} + (-R\omega^2)\,\hat{r}$

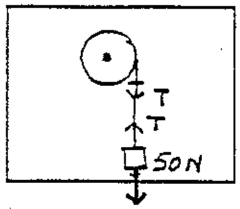


transverse Component of acceleration $\rightarrow a_\tau = R\alpha = 0$ $\quad (\alpha = 0)$

radial Component of acceleration $\rightarrow a_r = -R\omega^2 = -(250)(0.18)^2$

$$= -8.1\ \text{m/s}^2$$

Q3. A weight of 50.0 N is attached to the free end of a light string wrapped around a pulley of radius 0.025 m and a mass 3.0 kg. The pulley is free to rotate in a vertical plane about the horizontal axis passing through its center. The weight is released 6.0 m above the floor. (a) Determine the tension in the string, the acceleration of the mass, and the speed with which the weight hits the floor. (b) Find the speed calculated in part (a) by using the principle of conservation of energy.



Solution: Considering pulley as a uniform disc of radius R, the moment of inertia of the disc about an axis through the Center of pulley is given by: $I = \frac{mR^2}{2}$

(a) In the problem $m = 3\,kg$, $R = 0.025\,m$

$$\therefore \quad I = \frac{3 \times (0.025)^2}{2}\ kg.\,m^2$$

The rotational equation of motion of the pulley is

$$I\alpha = \Sigma \tau$$

$$\text{or} \quad \left[\frac{3 \times (0.025)^2}{2}\right] \alpha = T \times 0.025 \quad (1)$$

where angular acc., $\alpha = \frac{a}{0.025}$, and a is the linear acceleration of 50 N-weight

$\therefore$ (1) Can be written as

$$\left[\frac{3 \times (0.025)^2}{2}\right] \cdot \frac{a}{0.025} = T \times 0.025 \quad (2)$$

The translational equation of motion of 50 N weight is

$$\overset{m}{\left(\frac{50}{9.8}\right)} a = 50 - T \quad (3) \qquad a \rightarrow \text{acceleration of weight}$$

$$\text{From}(2) \quad T = \frac{3 \times (0.025)^2 a}{2 \times (0.025)(0.025)} = 1.5a \quad (4)$$

Putting T in (3) gives

$$\frac{50}{9.8} a = 50 - 1.5a \quad \text{or} \quad a = \frac{50}{6.60} = 7.58\ m/s^2$$

$$\text{From } (4) \quad T = 1.5a = 1.5 \times 7.58 = 11.37\,N.$$

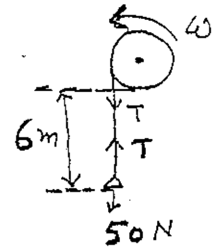
For the motion of weight we use $v^2 - v_0^2 = 2as$

$v_0 = 0$, $a = 7.58\,m/s^2$, $S = 6\,m$ gives $v^2 = 2 \times 7.58 \times 6$

or $v = 9.54\ m/s$.

(b) Using the law of Conservation of energy (Since the pulley is friction-less) when the weight hits the floor

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \quad (5)$$



In the problem, $h = 6\ m$

$v =$ velocity when the mass hits the floor

Moment of inertia of pulley $\rightarrow I = \frac{3}{2} \times (0.025)^2$

$\omega = \frac{v}{R} = \frac{v}{0.025}$ $\quad \omega \rightarrow$ angular velocity of pulley when the weight hits the floor

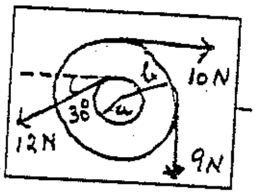
$\therefore$ From (5) $\quad (50) \times 6 = \frac{1}{2} \cdot \frac{50}{9.8} \cdot v^2 + \frac{1}{2} \times \left(\frac{3}{2} \times (0.025)^2\right) \frac{v^2}{(0.025)^2}$

$$300 = 2.55 v^2 + 0.75 v^2$$

$$\therefore v = \sqrt{\frac{300}{3.3}} = 9.53\ m/s$$

Q.4 Find the net torque on the wheel shown in the figure about the axle through o if a=10 cm and b=25 cm.

Solution:



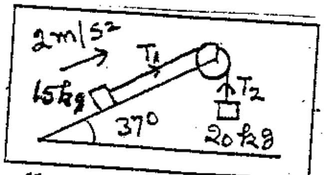
Total torque $\sum \tau_i = \sum F_i \times (\perp \text{distance})$

$$\tau = -(10)b + 12 \times a \cos 30^\circ - (9) b$$

$$= -10 \times 0.25 + 12 \times (0.1)(0.866) - 9 \times .25$$

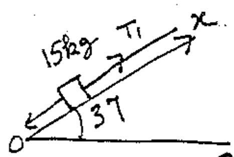
Total torque $\therefore \tau = -3.71\ N.m$ (going into paper.)

Q.5 The blocks shown in the figure are connected by a string of negligible mass passing over a pulley of radius R=0.25 m and moment of inertia I. The block on the incline is moving up with a constant acceleration of magnitude a= 2.0 m/s² (a) Determine T_1 and T_2, the tension in the two parts of the string, and (b) find the moment of inertia of the pulley.



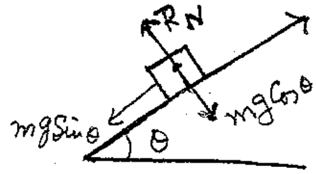
Solution:

Taking x- axis along the plane, the equation of motion of 15kg weight is given by



$$\therefore ma = T_1 - mg \sin\theta \quad (1)$$

given: $a = 2.0\ m/s^2$



$\therefore$ From (1) $\quad (15) \times (2.0) = T_1 - 15 \times 9.8 \times \sin 37^\circ$

or $T_1 = 30.0 + 15 \times 9.8 \times \sin 37^\circ = 118.5\ N$

writing the equation of motion of 20kg wt is

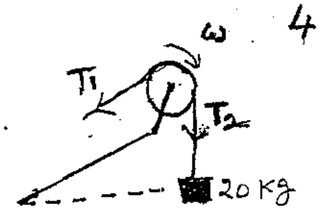
$$(20) \times (2.0) = 20 \times 9.8 - T_2 \quad (2)$$

or $T_2 = -40.0 + 196.0 = 156\ N$

(b) For the pulley

Radius of the pulley = 0.25 m

given, $a = 2.0\ m/s^2$.



$\therefore$ angular acceleration of pulley $= \frac{a}{R} = \frac{2}{0.25} = 8.0\ rad/s^2$ (Clockwise).

writing the rotational equation of motion of the pulley

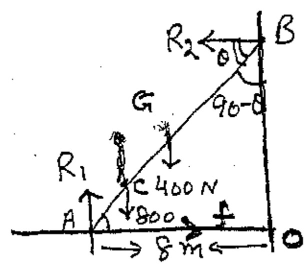
$$I\alpha = (T_2 - T_1) \cdot R$$

$$\therefore I(8.0) = (156 - 118.5)\ 0.25$$

$$\therefore I = \frac{37.5 \times 0.25}{8.0} = 1.17\ kg.m^2$$

Q.6 A ladder of weight 400 N and length 10.0 m is placed against a smooth vertical wall. A person weighing 800 N stands on the ladder 2.0 m from the bottom as measured along the ladder. The foot of the ladder is 8.0 m from the bottom of the wall. Calculate the force exerted by the wall, and the normal force exerted by the floor on the ladder.

Solution: The Forces acting on the ladder are



$R_1 \rightarrow$ Reaction of floor on end A of the ladder

$R_2 \rightarrow$ Reaction of wall on the end B of the ladder

$W_1 = 400\ N \rightarrow$ weight of the ladder acting at G, 5 m from A

$W_2 = 800\ N \rightarrow$ weight of man at C, 2 meters from A.

$f \rightarrow$ force of friction on the end A along AO

using the Conditions of equilibrium in the horizontal and vertical sense, we get:

Horiz: $\Sigma X_i = 0$ or $f - R_2 = 0$ (1) or $f = R_2$ (1)

Vertical: $\Sigma Y_i = 0$ or $R_1 - 800 - 400 = 0$ (2) or $R_1 = 1200\ N$ (2)

Taking moments of all the forces about point A, (if θ is the angle made by the ladder with the horizontal)

(about A) $\rightarrow$ $R_2 \times 10 \sin\theta - 800 \times 2 \cos\theta - 400 \times 5 \cos\theta = 0$ where $\cos\theta = \frac{8}{10}$

Since $\cos\theta = \frac{8}{10}$, $\theta = 36.87^\circ$

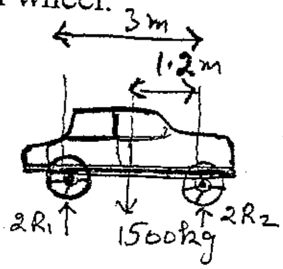
$$\therefore R_2 \times 10 \times 0.6 - 1600 \times 0.8 - 2000 \times .8 = 0$$

or $6 R_2 = 3600 \times .8$

or $R_2 = 480\ N = f \rightarrow$ force of friction

Q.7 A 1500 kg. Automobile has a wheel base (the distance from the axles) of 3 m. The center of mass of the automobile is on the center line at a point 1.2 m behind the front axle. Find the force exerted by the ground on each wheel.

Solution: Let $2R_1$ and $2R_2$ be the reactions of ground on the rear wheels and the front (two) wheels, then from equilibrium condition of forces in the vertical sense we get



$$2R_1 + 2R_2 = 1500 \quad (1)$$

Taking the moments about the front axle

$\Sigma \vec{M_i} = 0$ (Sum of moments must be zero)

$$1500 \times 1.2 - 2R_1(3) = 0$$

$$6R_1 = 1800 \quad \text{or} \quad \underline{R_1 = 300\,kg} = 2940\,N$$

From (1) $600 + 2R_2 = 1500$

$$R_2 = 450\,kg = 4410\,N$$

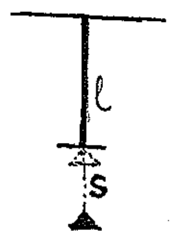
8 A 200 kg. Load is hung on a wire of length 4.0 m, cross-sectional area 0.2 cm^2, and Young's modulus 8.0×10^{10} N/m^2. What is its increase in length?

Solution:

Young's Modulus = $\frac{\text{Stress}}{\text{Strain}}$

where Stress = F/A = $\frac{\text{Force}}{\text{Area of Cross-section}}$

Strain = $\frac{s}{l}$ = $\frac{\text{extension}}{\text{Original length}}$



In the problem: Stress = $\frac{F}{A} = \frac{200 \times 9.8}{0.2 \times 10^{-4}}$

Strain = $\frac{s}{l} = \frac{s}{4} \rightarrow ?$

$$\therefore \quad 8 \times 10^{10} = \left(\frac{200 \times 9.8}{0.2 \times 10^{-4}}\right) \times \frac{4}{s}$$

$$\therefore \text{extension} \rightarrow s = \left(\frac{200 \times 9.8}{0.2 \times 10^{-4}}\right) \frac{4}{8 \times 10^{10}} = 4.9 \times 10^{-3}\,m$$

$$= 4.9\,mm$$

Q.9 When water freezes, it expands about 9%. What would be the pressure increase inside your automobile engine block if the water in it freezes? (The bulk modulus of ice is 2.0×10^9 N/m^2)

Solution: Bulk modulus of elasticity $\rightarrow B = \frac{\Delta P}{\frac{\Delta V}{V}}$

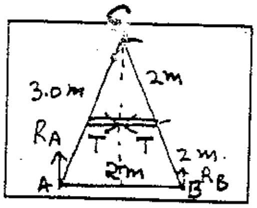
Stress $\rightarrow \Delta P = ?$

$B(\text{ice}) = 2.0 \times 10^9\ N/m^2$

$$\therefore \Delta P = B \cdot \frac{\Delta V}{V} = \frac{(2.0 \times 10^9) \times (0.09) V}{V}$$

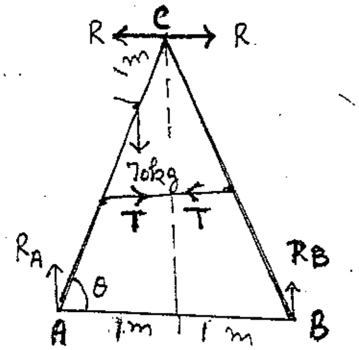
$$= 1.8 \times 10^8\ N/m^2$$

Q10 The step ladder of negligible weight is constructed as shown in the figure. A painter of mass 70.0 kg. Stands on the ladder 3.0 m from the bottom. Assuming the floor is frictionless, find (a) the tension in the horizontal bar connecting the two halves of the ladder. (b) The normal forces at A and B, © the components of the reaction force at the hinge C that the left half of the ladder exerts on the right half.



Solution:

Considering the equilibrium of the whole ladder, we get from equilibrium Conditions.



$\Sigma X_i = 0$ give No forces. (1)

$\Sigma Y_i = 0$ give $R_A + R_B - 70 \times 9.8 = 0$

or $R_A + R_B = 686$ N (2)

ΣM_i (about A) $= 0$ give $-R_A \times 0 + R_B \times 2 - 686 \times (3 \cos\theta) = 0$ (3)

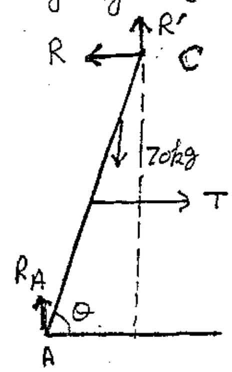
$\cos\theta = \frac{1}{4}$ $\therefore \theta = 75.5^\circ$

From (3) $\therefore 2R_B = 686 \times 3 \times \frac{1}{4}$

$\therefore R_B = 257$ N

From (2) $R_A = 686 - R_B = 686 - 257 = 429$ N

Now Considering the equilibrium of each half of the ladder separately. Again applying the Condition of =m to left half of the ladder



$\Sigma X_i = 0$, $T - R = 0$ (4)

$\Sigma Y_i = 0$, $R' + R_A - 70 = 0$

or $R' + R_A = 686$ (5)

ΣM_i (about C) $= 0$ give

$-4 R_A \cos 75.5 + 2T \sin 75.5 + 686.0 \cos 75.5 = 0$ (6)

$-4 \times 429 \times \frac{1}{4} + 2T \times 0.97 + 686.0 \times \frac{1}{4} = 0$

$2T \times 0.97 = 429 - 686 \times .25$

$\therefore T = \frac{257.5}{2 \times 0.97} = \underline{133\ N}$

From (4) $R = T = 133$ N

From (5) $R' = 686 - R_A = 686 - 429 = 257$N

Q11 The displacement of a particle at any time t is given by the expression $x=4\ \mathrm{Cos}\ (3t+\pi)$, where x is in meters and t is in seconds. Determine (a) the frequency and period of motion, (b) the amplitude of the motion © the phase constant, and (d) the displacement of the particle at t=0.25 s.

Solution: The displacement is given by

$$x = 4 \cos(3t + \pi) \text{ --- (1)}$$

A Simple harmonic motion is given by a displacement given by

$$x = A \sin(\omega t + \phi) \quad \text{or} \quad x = A \cos(\omega t + \phi)$$

where $A =$ amplitude, $\omega =$ angular velocity

$\phi =$ phase angle, frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$

Time period $T = \frac{2\pi}{\omega}$

(a) Period of motion in (1) is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3} = 2.09\ \mathrm{s}$$

Frequency $f = \frac{1}{T} = \frac{3}{2\pi} = 0.477\ \mathrm{Hz}$

(b) amplitude $A = 4\ \mathrm{m}$

(c) phase Constant $= \pi$ rad.

(d) x at $t = 0.25$ s

$$x(2.5) = 4 \cos(\underbrace{3 \times 0.25 + \pi}_{\text{radians.}})$$

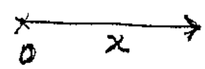
$$= 4 \cos\left(\frac{0.75 \times 180}{\pi} + 180^\circ\right)$$

$$= -4 \cos 43^\circ = -2.93\ \mathrm{m}$$

Q.12 A particle moving along the x-axis in simple harmonic motion starts from the origin at t=0 and moves to the right . if the amplitude of its motion is 2.0 cm and the frequency is 1.5 Hz, show that its displacement is given by $x=(2\ \mathrm{cm})\ \mathrm{Sin}\ (3\pi t)$. Determine (b) the maximum speed and the earliest time at which the particle has this speed, © the maximum acceleration and the earliest time at which the particle has this acceleration, and (d) the total diatance traveled between t=0 and t=1.

Solution: amplitude $A = 2\ \mathrm{cm} = 0.02\ \mathrm{m}$

frequency $f = 1.5\ \mathrm{Hz}$



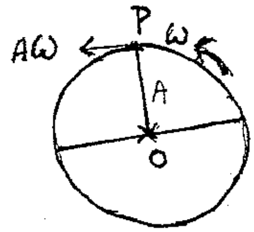
In the equation $x = A \sin(\omega t + \phi)$

$\phi = 0$ Since at $t = 0$ $x = 0$

$\therefore$ The equation of motion is

$$x = (2\ \mathrm{cm}) \sin(2\pi \times \tfrac{3}{2} t + 0)$$

$$x = (2\ \mathrm{cm}) \sin(3\pi t)$$



(b) The maximum speed will take place at P in the figure.

$\therefore$ Maximum Speed $= A\omega = 0.02 \times (2\pi \times \frac{3}{2})$

$= 0.19\ m/s$

Maximum speed will be attained at the origin. It will attain maximum speed after time $\frac{T}{2} = \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$ Sec.

(c) Maximum acc will when x is maximum. i.e when $x = A$

i.e $\quad ma = -kx$

$\therefore m a_{max} = -kA$

$\therefore |a_{max}| = |-\mu A| = \omega^2 A = 0.02 \times \left(2\pi \times \frac{3}{2}\right)^2$

$= 0.02 \times 9\pi^2 = 0.18\pi^2\ m/s^2$

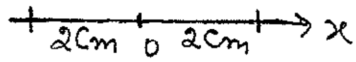
Maximum acc. will be attained at the end of amplitude after time $\frac{T}{4} = \frac{1}{6}$ Sec.

(d) $\quad x = (2cm) \sin(3\pi t)$

At $t=0 \quad x(t=0) = 0$

At $t=1 \quad x(t=1) = (2\ cm) \sin 3\pi$

Time period $T = \frac{2}{3}$ s.



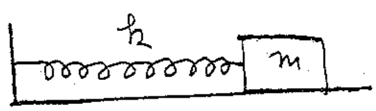
in $\frac{2}{3}$ s it travels $\quad 2 \times 4 = 8$ cm.

$\therefore$ Total distance travelled in 1 S $= 8 \times \frac{3}{2} = 12$ cm.

Q.13 a 0.5 kg mass attached to a spring of force constant 8.0 N/m vibrating in simple harmonic motion with an amplitude of 10 cm. Calculate (a) the maximum value of its speed and acceleration, (the speed and acceleration when the mass is 6.0 cm from the equilibrium position, and © the time it takes the mass to move from x=0 to x=8.0 cm.

Solution:

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8.0}{0.50}} = 4.0\ s^{-1}$



Position is given by

$x = (10\ cm) \sin[4.0\, t]$

$v = \frac{dx}{dt} = 40 \cos 4t \quad (1) \quad v_{max} = 40\ cm/s$

$a = \frac{dv}{dt} = -160 \sin 4t \quad (2) \quad a_{max} = 160\ cm/s^2$

(b) $x = 6.0$ Cm , $A = 10$ Cm.

From $x = A \sin \omega t$

we get $\frac{x}{A} = \sin \omega t$

or $\boxed{t = \frac{1}{\omega} \sin^{-1} \frac{x}{A}}$ (3)

Putting $x = 6.0$ Cm in (1)

gives $t = \frac{1}{4} \sin^{-1} \frac{6}{10} = \frac{1}{4} \times (\cdot 644) = 0.16$ S.

From (1) in part (a): $v = 40 \cos(4 \times 0.16) = 32$ Cm/s.

From (2) in part (a): $a = -(160) \sin(4 \times 0.16) = -96$ Cm/s^2

(C) Using (3) At $x = 0$, $t = 0$

when $x = 8$ Cm. $t = \frac{1}{4} \sin^{-1} \frac{8}{10} = 0.23$ S

$\therefore \Delta t = 0.23$ S.

Q.14 A particle executes simple harmonic motion with an amplitude of 3.0 cm. At what displacement from the midpoint of its motion does its speed equal one half of its maximum speed?

Solution:

From energy principle (Conservation)

$\omega^2 x^2 + v^2 = \omega^2 A^2$ (1)

$v_{max} = A\omega$

Half of the speed $\rightarrow v = \frac{v_{max}}{2} = \frac{A\omega}{2}$

From (1) $\omega^2 x^2 + \left(\frac{A\omega}{2}\right)^2 = \omega^2 A^2$

$\therefore x^2 = \frac{3A^2}{4}$

Given: Amplitude $A = 3$ Cm

$\therefore x^2 = \frac{3 \times 9}{4} = \frac{27}{4}$

$x = \sqrt{\frac{27}{4}} = 2.60$ Cm.

Q15 A centrifuge in a medical laboratory rotates at an angular speed of 3600 rev/min. When switched off, it rotates 50.0 times before coming to rest. Find the constant angular acceleration of the centrifuge. (-2.26×10^2 rad/s)

Solution: Angular Speed Initially $\rightarrow \omega_i = \frac{(3600)(2\pi)}{60} = 3.77\times10^2 \frac{rad}{s}$

Total angular distance Covered in 50 revolutions $\rightarrow \theta$ is given by:

$$\theta = (50)(2\pi) = 3.14\times10^2 \text{ rad.}$$

After Switch is turned off $\omega_f = 0$

$\therefore$ using $\omega_f^2 - \omega_i^2 = 2\alpha\theta$, we get

$$0 - (3.77\times10^2)^2 = 2\alpha(3.14\times10^2)$$

$\therefore$ Angular Acceleration $\rightarrow \alpha = \frac{-(3.77\times10^2)^2}{2\times(3.14\times10^2)} = -2.26\times10^2 \frac{rad}{s^2}$

Q16 A rotating wheel requires 3.00 s to rotate through 37.0 revolutions Its angular speed at the end of 3.00 s interval is 98.0 rad/s. What is the constant angular acceleration of the wheel? (13.7 rad/s^2)

Solution: Total angular distance Covered in 3.0S,

$$\theta = (37.0)(2\pi) = 2.32\times10^2 \text{ rad.} \quad (1)$$

Angular speed after 3.0S $\rightarrow \omega_f = 98.0$ rad/S

$\therefore$ Initial angular Speed $\rightarrow \omega_0 = \omega_f - \alpha t = 98.0 - 3\alpha$ (2)

To find α, we use: $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$,

$\therefore$ From (1) and (2): $2.32\times10^2 = (98.0-3\alpha)3 + \frac{1}{2}\alpha(3)^2$

$$= 294 - 9\alpha + 4.5\alpha$$

$$\therefore 4.5\alpha = 294 - 232 = 62$$

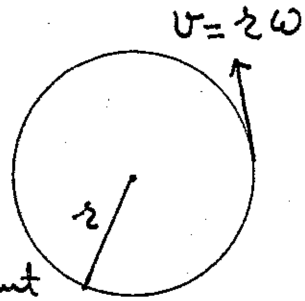
$$\therefore \alpha = \frac{62}{4.5} = 13.8 \frac{rad}{s^2}$$

Q17 A racing car travels on a circular track of radius 250 m. If the car moves with a constant linear speed of 45.0 m/s, find (a) its angular speed and (b) the magnitude and direction of its acceleration.

(0.180 rad/s, 8.10 m/s^2 to center)

Solution: Radius of the track = 250m

(a) Linear speed → $v = r\omega = 45.0\ m/s$



∴ Angular speed → $\omega = \frac{v}{r} = \frac{45.0}{250}$

$= 0.18\ \frac{rad}{s}$

(b) Since the car is moving with constant angular speed, the tangential component of acceleration is zero. The radial acceleration a_r is given by:

$$a_r = -\frac{v^2}{r} = -\frac{(45)^2}{250} = -8.1\ m/s^2 \text{ (toward the Center)}$$

Q18 A car accelerates uniformly from rest and reaches a speed of 22.0 m/s in 9.00 s. If the diameter of the tire is 58.0 cm, find (a) the number of revolutions the tire makes during the motion, assuming that no slipping occurs. (b) What is the final angular speed of a tire in revolutions per second?

(54.3 rev, 12.1 rev/s)

Solution: Speed of Car after 9 s → $v_f = 22.0\ m/s$

∴ Average speed of Car $= \frac{0+22}{2} = 11\ m/s$

∴ Distance Covered in 9 s $= (11)(9) = 99\ m.$

Circumference of tire $= \pi(0.58) = 0.58\pi$

(a) ∴ Number of revolutions tire makes $= \frac{99}{0.58\pi} = 54.3\ rev$

(b) Final angular speed in rev. → $\frac{22}{0.58\pi} = 12.1\ rev/s.$

Q19 A grinding wheel is in the form of a uniform solid disc of radius 7.00 cm and mass 2.00 kg. It starts from rest and accelerates uniformly under the action of the constant torque of 0.600 N.m that the motor exerts on the wheel. (a) How long does the wheel take to reach its final operating speed of 1200 rev/min? (b) Through how many revolutions does it turn while accelerating? (1.03 s, 10.3 rev)

Solution: Moment of inertia of the grinding wheel I is:

$$I = m\left(\frac{a^2}{2}\right) = (2.00)\frac{(0.07)^2}{2} = 4.9\times10^{-3}\ kg.m^2$$

From: $I\alpha = T$ we get the angular acceleration of the wheel i.e. $\alpha = \frac{T}{I} = \frac{0.6}{(4.9\times10^{-3})} = 122\ rad/s^2$

(a) Final operating speed $\rightarrow \omega_f = \frac{(1200)(2\pi)}{60} = 40\pi\ \frac{rad}{s}$

Time taken to reach final speed $= \frac{40\pi}{122} = 1.03s.$

(b) Angle covered in 1.03 s is given by:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(122)(1.03)^2 = 64.7\ rad.$$

$\therefore$ No. of revolutions $= \frac{64.7}{2\pi} = 10.3\ rev.$

20 A cylinder of mass 10.0 kg rolls without slipping on a horizontal surface. At the instant its center of mass has a speed of 10.0 m/s, determine (a) the translational kinetic energy of its center of mass, (b) the rotational kinetic energy about its center of mass, and (c) its total energy. (500 J, 250 J, 750 J)

Solution: Since the cylinder is rolling,

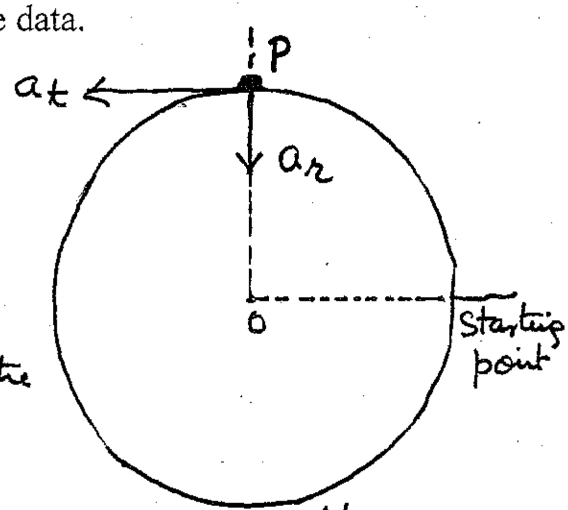
$$v_c = r\omega = 10.0\ m/s$$

(a) Translational K.E $\rightarrow T = \frac{1}{2}mv^2 = \frac{1}{2}(10.0)(10.0)^2 = 500J$

(b) Rotational K.E $\rightarrow T_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{mr^2}{2}\right)\omega^2 = \frac{1}{4}(10)(10)^2 = 250J$

(C) Total Energy $\rightarrow E = T + T_R = 500J + 250J = 750J$

21 .A car traveling on a flat (unbanked) circular track accelerates uniformly from rest with a tangential acceleration of 1.70 m/s^2. The car makes it one quarter of the way around the circle before it skids off the track. Determine the coefficient of static friction between the car and track from these data.



Solution:

Let R be the radius of the circular track and m be the mass of the car.

The tangential acceleration of the car is → $a_t = 1.70\,\hat{t}$

∴ Angular acceleration → $\alpha = \frac{a_t}{R} = \frac{1.70}{R}$ rad/s^2

The angular distance covered by the car is → $\theta = \pi/2$

At $t = 0$, $\omega_0 = 0$

∴ using the equation $\omega^2 - \omega_0^2 = 2\alpha\theta$ gives the angular velocity at point P when the car skids.

$$\omega^2 - 0 = 2\left(\frac{1.70}{R}\right)\left(\frac{\pi}{2}\right) = \frac{1.70\pi}{R}$$

∴ Centripetal acceleration → $a_r = R\omega^2$

$$= -\left(\frac{1.70\pi}{\cancel{R}}\right)\cancel{R}\,\hat{r}$$

∴ The resultant acceleration a is given by

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(1.70)^2 + (1.70\pi)^2} = (1.70)\sqrt{1+\pi^2}$$

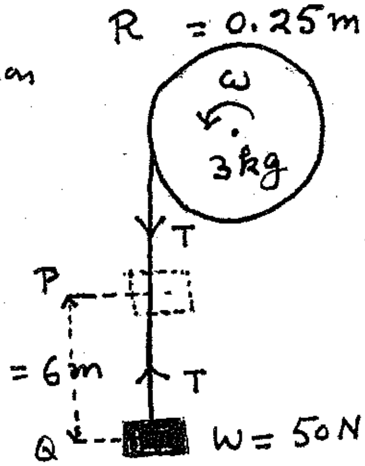
$$= 5.605 \text{ m/s}^2$$

If μ is the coefficient of friction, then

$$\mu mg = ma \quad \text{or} \quad \mu = \frac{a}{g} = \frac{5.605}{9.8} = 0.572$$

22 An object with a weight of 50.0 N is attached to the free end of a light string wrapped around a reel of radius 0.250 m and mass 3.00 kg. The reel is a solid disc, free to rotate in a vertical plane about the horizontal axis passing through its center. The suspended object is released 6.00 m above the floor. (a0 Determine the tension in the string, the acceleration of the object, and the speed with which the object hits the floor. (b0 Verify your last answer by using the principle of conservation of energy to find the speed with which the object hits the floor.

(9.5 m/s)



Solution: (a) The translational equation of motion of $\left(\frac{50.0}{g}\right)$ kg mass is:

$$\left(\frac{50.0}{g}\right) a = 50.0 - T \quad (1)$$

a is the acceleration of the object downward.

The rotational equation of motion of the reel is:

$$I\alpha = T \quad \text{or} \quad \left(\frac{1}{2} m R^2\right)\alpha = TR \quad (2)$$

using the relation $a = \alpha R$, we can eleminate a and α from eq.s (1) and (2)

From (2) $\left(\frac{1}{2} m R^2\right)\frac{a}{R} = TR$ or $a = \frac{2T}{m}$ (3)

∴ From (1) $\left(\frac{50.0}{9.8}\right)\cdot\frac{2T}{3.0} = 50 - T$ or $3.40T = 50 - T$

∴ $4.4T = 50.0$ or $\boxed{T = 11.36\text{ N}}$

From (3) $a = \frac{2T}{m} = \frac{2 \times 11.36}{3} = 7.57\ m/s^2$

∴ Acceleration of the weight $= \boxed{7.57\ m/s^2}$

If v_f is the speed with which the weight hits the floor

$v_f^2 - 0 = 2ah = 2 \times 7.57 \times 6$ or $\boxed{v_f = 9.53\ m/s}$

(b) Since there is no loss of energy, the energy of the weight and pulley before and after the weight hits the ground must be equal.

Energy before when the weight is at P is:

$$E_i = mgh = 50 \times 6 = 300\,J \quad (4)$$

The energy of pulley and the weight when the weight hits ground is:

$$E_f = \text{K.E of weight (Trans)} + \text{Rotational K.E of pulley}$$

$$= \frac{1}{2}\left(\frac{50}{9.8}\right) v_f^2 + \frac{1}{2} I \omega_f^2$$

$$= \frac{1}{2}\left(\frac{50}{9.8}\right) v_f^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v_f}{R}\right)^2$$

$$= \frac{1}{2}\cdot\left(\frac{50}{9.8}\right)\left(v_f\right)^2 + \frac{1}{4}(3)\left(v_f\right)^2 \quad (5)$$

From Conservation of energy principle we have

$$E_i = E_f$$

or

$$300 = \frac{1}{2}\left(\frac{50}{9.8}\right) v_f^2 + \frac{3}{4} v_f^2$$

$$= 3.30\, v_f^2$$

or

$$v_f = \sqrt{\frac{300}{3.3}} = \boxed{9.53\ m/s}$$