CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	208/4		All except EC
Examination	**Date**	**Time**	**Pages**
Final	April 2016	3 Hours	3

Instructors
D. Sen, E. Duma, H. Greenspan, L. Aggoun, M. Padamadan
S. Giri, S. White

Course Examiner
D. Sen

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\,\frac{(1+i)^n - 1}{i}, \quad PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[10] **1.** Given the quadratic function $f(x) = 1.2 - 0.12x^2 + 0.96x$

(A) Find x and y intercepts algebraically.

(B) Find the vertex form of f.

(C) Find the vertex and the maximum or minimum.

(D) Find the range of f.

[10] **2.** Solve for x in the following equations:

(A) $2e^{5x+2} = 8$

(B) $\left(\frac{3}{4}\right)^x = \frac{256}{81}$

(C) $\log x = \log(x+3) - \log(x-1)$

(D) $\log_8(x-6) = 2 - \log_8(x+15)$

(E) $\log x^2 = (\log x)^2$

[10] **3.** For $f(x) = 24x - 5$ and $g(x) = 5(1.4)^x$ find the following by only using a proper formula:

(A) $\sum_{k=1}^{50} f(k) = f(1) + f(2) + f(3) + \cdots + f(50)$.

(B) $\sum_{h=0}^{39} g(h) = g(0) + g(1) + g(2) + \cdots + g(39)$.

[10] **4.** At the end of each quarter, a 50–year old woman deposits \$1200 in a retirement account that pays 7% interest compounded quarterly.

(A) What will be the amount when she reaches at age 60?

(B) At the age of 60, she withdraws the entire amount and places it in a mutual fund that pays 9% interest compounded monthly. How much amount does she have there at the age of 65?

(C) She additionally deposits \$300 at the end of each month from the age of 60 to 65 in the same mutual fund that pays 9% interest compounded monthly. How much is in the account when she reaches age 65?

[10] **5.** An ordinary annuity pays 7.44% compounded monthly. A person deposits \$100 monthly for 30 years and then makes equal monthly withdrawals for the next 15 years, reducing the balance to zero.

(A) What are the monthly withdrawals? How much interest is earned during the entire 45-year process?

(B) If the person wants to make withdrawals of \$2,000 per month for the last 15 years, how much must be deposited monthly for the first 30 years?

[10] **6.** A chemical manufacture wants to lease a fleet of 24 railroad tank cars with a combined carrying capacity of 520,000 gallons. Tank cars with three different carrying capacities are available: 8,000 gallons, 16,000 gallons, and 24,000 gallons.

(A) Write the linear system of equations in terms of x, y and z; x, y and z being the number of 8,000 gallon tank cars, 16,000 gallon tank cars, and 24,000 gallon tank cars respectively.

(B) Solve this system of equations by using Gauss-Jordan Elimination.

No other method of solving these systems of equations will be accepted!

(C) The cost of leasing an 8,000 gallon tank car is \$450 per month, a 16,000 gallon tank car is \$650 per month, and a 24,000 gallon tank car is \$1,150 per month. Which of the solutions would minimize the monthly leasing cost?

[10] **7.** An economy is based on three sectors, shipping, agriculture, and mining. Production of a dollar's worth of shipping requires an input of \$0.20 from the shipping sector, \$0.20 from the agriculture sector and \$0.20 from mining sector. Production of a dollar's worth of agriculture requires an input of \$0.40 from the shipping sector, \$0.10 from the agriculture sector and \$0.10 from mining sector. Production of a dollar's worth of mining requires an input of \$0.30 from the shipping sector, \$0.10 from the agriculture sector and \$0.10 from mining sector.

(A) Write the technological matrix M for this economy.

(B) If a final demand of \$20 million for shipping, \$30 million for agriculture, and \$40 million for mining is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

[10] **8.** Extremize $P(x,y) = 40x + 20y$ subject to

$$10x + 15y \geq 150,\ 4x + 2y \leq 52,\ -6x + 15y \leq 102,\ x \geq 0,\ y \geq 0.$$

[10] **9.** A county park system rates its 20 golf courses in increasing order of difficulty as bronze, silver, or gold. There are only two gold courses and twice as many bronze as silver courses.

(A) If a golfer decides to play a round at a silver or gold course, how many selections are possible?

(B) If a golfer decides to play one round per week for 3 weeks, first on a bronze course, then silver, then gold, how many combined selections are possible?

[10] **10.** In a country, 50% of marriages are consanguineous (between first cousins or people even more closely related). A recent study in that country showed that 16% of children from unrelated marriages died by age 10, while 21% of children from consanguineous marriages died by age 10. Find the following probabilities:

(A) A child survives at age 10.

(B) A child from a consanguineous marriage survives.

COPYRIGHT

The present document and the contents thereof are the property and copyright of the professor(s) who prepared this exam at Concordia University. No part of the present document may be used for any purpose other than research or teaching purposes at Concordia University. Furthermore, no part of the present document may be sold, reproduced, republished or re-disseminated in any manner or form without the prior written permission of its owner and copyright holder.

Math 208, Final April 2016 (1)

1) $f(x) = 1.2 - 0.12x^2 + 0.96x$

$f(x) = -0.12x^2 + 0.96x + 1.2$

A) y intercept $= \boxed{1.2}$

x intercept: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-0.96 \pm \sqrt{0.96^2 - 4(-0.12)(1.2)}}{2(-0.12)}$

$x = \frac{-0.96 \pm \sqrt{1.4976}}{-0.24} = \frac{-0.96 \pm 1.223764683}{-0.24} = \boxed{\begin{matrix} x = -1.099 \\ x = 9.099 \end{matrix}}$

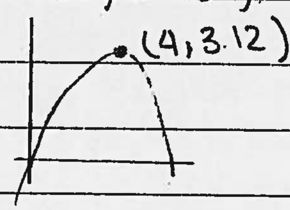
B) Find $h = \frac{-b}{2a} = \frac{-0.96}{2(-0.12)} = 4$ | Y-int $\Rightarrow x = 0 \Rightarrow 1.2$.

Find $k = -0.12(4)^2 + 0.96(4) + 1.2 = 3.12$

vertex form $= f(x) = -0.12(x-4)^2 + 3.12$

c) vertex $= (h, k) = (4, 3.12)$

max $= 3.12$



d) $[-\infty, 3.12]$

2) A) $2e^{5x+2} = 8$

B) $(3/4)^x = \frac{256}{81}$

$3/4^x = 3/4^{-4}$

$\boxed{x = -4}$

$e^{5x+2} = 4$

$5x + 2 = \ln 4$

$x = \frac{\ln 4 - 2}{5} = -0.1227$.

2

c) $\log x = \log(x+3) - \log(x-1)$

$\log x = \log \dfrac{(x+3)}{x-1}$

$x(x-1) = x+3.$

$x^2 - x = x+3$

$x^2 - x - x - 3 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$\cancel{x = -1}$ $\boxed{x=3}$

D) $\log_8(x-6) = 2 - \log_8(x+15)$

$\log_8(x-6) + \log_8(x+15) = 2$

$(x-6)(x+15) = 8^2$

$x^2 - 6x + 15x - 90 = 64$

$x^2 + 9x - 154 = 0.$

$\dfrac{-9 \pm \sqrt{9^2 - 4(1)(-154)}}{2.1} = \dfrac{-9 \pm 26.40075756}{2.}$

$x = -17.7$ ← not valid

$x = 8.7$

E) $\log x^2 = (\log x)^2$

$2\log x = (\log x)^2$

$0 = (\log x)^2 - 2\log x$

$0 = \log x(\log x - 2)$

$\left.\begin{array}{l}\log x = 0 \\ x = 10^0;\ x = 1\end{array}\right\}$

$\log x - 2 = 0$

$\log x = 2$

$x = 10^2 = 100$

3a) $F(x) = 24x - 5$

$f(1) = 24 - 5 = 19$; $f(2) = 24(2) - 5 = 43$

$f(3) = 24(3) - 5 = 67$

$f(3) - f(2) = f(2) - f(1) = 24 = d$; A.P

$f(50) = a_1 + (n-1)d = 19 + 45(24) = 1195$

$\sum_{k=1}^{50} f(k) = S_{50} = \frac{50}{2}(a_1 + a_{50})$

$= \frac{50}{2}(19 + 1195) = 30350$

3 B) $g(0) = 5(1.4)^0 = 5 = a_1$ $r = 1.4$ g.p.

$S_n = \frac{a_1 \cdot r^n - 1}{r-1} = S_{40} = \frac{5 \cdot (1.4^{40} - 1)}{1.4 - 1}$.

$\boxed{S_{40} = 8,750,458.71}$

4. pmt = 1200 every 3 months. $i = 0.07/4 = 0.0175$

60 - 50 = 10 $n = 10 \text{ yearx} \times 4 = 40$.

A) $FV = pmt\left[\frac{(1+i)^n - 1}{i}\right] = 1200\left[\frac{(1.0175)^{40} - 1}{0.0175}\right]$

$\boxed{FV = 68,680.96}$

B) 65 - 60 = 5 years. $i = \frac{0.09}{12} = 0.0075$

$n = 5 \times 12 = 60$.

$A = P(1+i)^n = 68680.96(1.0075)^{60}$

$\boxed{A = 107532.48}$

C) $FV = 300\left[\frac{(1.0075)^{60} - 1}{0.0075}\right] = 22627.24$

$107532.48 + 22627.24 = \boxed{130159.72}$ in account

when retire

5. A) $i = \frac{0.0744}{12} = 0.0062$ $\quad pmt = 100 \quad n = 30 \times 12 = 360$

Find $FV = 100\left[\frac{(1.0062)^{360} - 1}{0.0062}\right] = 133\,136.99$
$= \$133\,137$

Find pmt

$pmt = 133137\left[\frac{0.0062}{1-(1.0062)^{-180}}\right]$ $\quad 15 \times 12 = 180 = n$

$\boxed{pmt = 1229.66}$ ← withdrawals

$133\,137 - (100 \times 360) = 97\,137$ +
$(1229.66 \times 180) - 133\,137 = 88\,201.8$

$\boxed{\$185\,338.80 \text{ in interest}}$

B)

$PV = 2000\left[\frac{1-(1.0062)^{-180}}{0.0062}\right] = \$216\,542.54$ ← must be in account

Find monthly deposits.

$pmt = 216\,542.54\left[\frac{0.0062}{(1.0062)^{360} - 1}\right] = 162.646$

$\boxed{= \$162.65}$ must be deposited every month for 30 years

5

6 A) $x + y + z = 24$

$8000 + 16000 + 24000 = 520000$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 24 \\ 8000 & 16000 & 24000 & 520000 \end{array}\right]$$

B) $$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 24 \\ 8000 & 16000 & 24000 & 520000 \end{array}\right] \xrightarrow{-8000R_1+R_2 \Rightarrow R_2} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 24 \\ 0 & 8000 & 16000 & 328000 \end{array}\right]$$

$$\xrightarrow{8000 R_2 \to R_2} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 24 \\ 0 & 1 & 2 & 41 \end{array}\right] \xrightarrow{-1R_2+R_1 \to R_1} \left[\begin{array}{ccc:c} 1 & 0 & -1 & -17 \\ 0 & 1 & 2 & 41 \end{array}\right]$$

$x - z = -17$ $\quad$ $x = -17 + z$

$y + 2z = 41$ $\quad$ $y = 41 - 2z$

$z = t$

$17 \leq t \geq 20$

C) $I = 450x + 650y + 1150z$

$I = 450(-17+t) + 650(41-2t) + 1150t$

$= -7650 + 450t + 26650 - 1300t + 1150t$

$I = 19000 + 300t$

minimize

$\boxed{t=17 \quad 19000 + 300(17) = 24100}$ ← minimize monthly cost when $t = 17 = z$

$t=18 \quad 19000 + 300(18) = 24400$

$t=19 \quad 19000 + 300(19) = 24700$

$t=20 \quad 19000 + 300(20) = 25000$

$x = 0, y = 7, z = 17$

OR - $z = 17, x = 0 \quad y = 7 \to \text{cost} = 1150(17) + 650(7)$

$= 24100$

x	y	z
0	7	17
1	5	18
2	3	19

7. A) M:

	Shipping	Agriculture	Mining.
Shipping	0.2	0.4	0.3
Agriculture	0.2	0.1	0.1
Mining	0.2	0.1	0.1

$$M: \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix}$$

B) $X = MX + D$

$X = (I-M)^{-1} \cdot D$

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \begin{matrix} \text{shipping} \\ \text{agriculture} \\ \text{mining} \end{matrix} \qquad D: \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix} \text{ million}$$

C) $$X = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & -0.4 & -0.3 \\ -0.2 & 0.9 & -0.1 \\ -0.2 & -0.1 & 0.9 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 0.8 & -0.4 & -0.3 & 1 & 0 & 0 \\ -0.2 & 0.9 & -0.1 & 0 & 1 & 0 \\ -0.2 & -0.1 & 0.9 & 0 & 0 & 1 \end{array}\right] \times 10 \left[\begin{array}{ccc|ccc} 8 & -4 & -3 & 10 & 0 & 0 \\ -2 & 9 & -1 & 0 & 10 & 0 \\ -2 & -1 & 9 & 0 & 0 & 10 \end{array}\right] R_1 \times \frac{1}{8}$$

7 e) Inverse $= (I - m)^{-1}$

$$= \begin{bmatrix} 1.6 & .78 & .62 \\ .4 & 1.32 & .28 \\ .4 & .32 & 1.28 \end{bmatrix}$$

$$X = (I - m)^{-1} D$$

$$= \begin{bmatrix} 80.2 \\ 58.8 \\ 68.8 \end{bmatrix}$$

OR:

$$(I - m) X = D.$$

$$\begin{bmatrix} 0.8 & -0.4 & -0.3 \\ -0.2 & 0.9 & -0.1 \\ -0.2 & -0.1 & 0.9 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$$

Solve by Gauss-Jordan Elimination

$$\left[\begin{array}{ccc|c} 0.8 & -0.4 & -0.3 & 20 \\ -0.2 & 0.9 & -0.1 & 30 \\ -0.2 & -0.1 & 0.9 & 40 \end{array}\right]$$ → Do Row Reduction

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 80.2 \\ 0 & 1 & 0 & 58.8 \\ 0 & 0 & 1 & 68.8 \end{array}\right]$$

8

8. $P(x,y) = 40x + 20y \qquad x \geq 0 \quad y \geq 0.$

① $10x + 15y \geq 150 \qquad (0,10)\ (15,0)$

② $4x + 2y \leq 52 \qquad (0,26)\ (13,0)$

③ $-6x + 15y \leq 102 \qquad (0,6.8)\ (-17,0)$

$$\begin{array}{r} 10x + 15y = 150 \quad \times -4 \\ 4x + 2y = 52 \quad \times 10 \\ \hline -40y = -80 \end{array}$$

(12,2) $y = 2$

$x = 12$

$$\begin{array}{r} 10x + 15y = 150 \quad (-1) \\ -6x + 15y = 102 \\ \hline -16x = 48 \end{array}$$

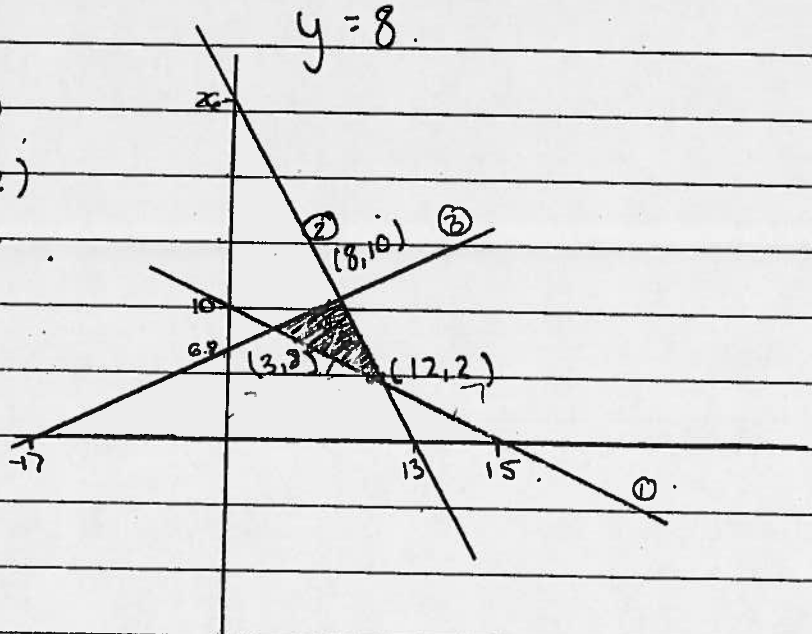
$x = 3 \quad (3,8)$

$y = 8$

$$\begin{array}{r} 4x + 2y = 52 \quad (3) \\ -6x + 15y = 102 \quad (2) \\ \hline 36y = 360 \end{array}$$

(8,10) $y = 10$

$x = 8$



(3,8) → $40(3) + 20(8) = 280$ = min

(8,10) → $40(8) + 20(10) = 520$ = max

(12,2) → $40(12) + 20(2) = 520$

9) 20 golf courses.

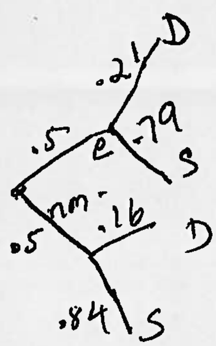
Bronze Silver Gold

12 6 2 = 20

a) $8C_1 = \frac{8!}{(8-1)!\,1!} = 8$ selections.

b) $12C_1 \cdot 6C_1 \cdot 2C_1 = 12(6)(2) = 144$

10)



A) prob. $= .5(.79) + .5(.84) = 81.5\%$

B) $.5(.79) = 39.5\%$