

Midterm Test, MAST 218, Fall 2012

All problems have equal value. Each is worth 6 marks.

Show all your steps. Write the solutions in the examination booklet only.

Formulas: $\cos(2t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$; $\sin(2t) = 2\sin t \cos t$

Problem 1: The curve is given by the polar equation:

$$r = \sin^3(\theta/3), \quad t \in [0, \pi] .$$

Find the length L of the curve.

Problem 2: Find the foci and vertices and sketch the graph of the conic section:

$$25x^2 + 4y^2 + 50x - 16y = 59,$$

Problem 3: Find the volume V of the parallelepiped whose four adjacent vertices are the points: $A = (1, 0, 1)$, $B = (2, 3, 0)$, $C = (-1, 1, 4)$, $D = (0, 3, 2)$.

Problem 4: Find the equation of the plane that passes through the line of intersection of the planes: $x - y = 1$ and $y + 2z = 3$ and is perpendicular to the plane: $x + y - 2z = 1$.

Problem 5: The curve is given by the parametric equations:

$$x = e^t, \quad y = te^t, \quad z = te^{t^2}$$

Find parametric equations of its tangent line at the point $(1, 0, 0)$.

Problem Bonus: Find equations of the spheres with center $(2, -3, 6)$ that touch

- (a) the xy -plane
- (b) the yz -plane
- (c) the xz -plane

GOOD LUCK !!!