

ECON 421: Answers to Problem Set 2

November, 2017

Question 1) Lets find all SPE using backward induction:

*Reminders: In SPE, everyone plays best responses in each subgame. A SPE is a strategy profile. A strategy for player i specifies an action for each information set in which i could play.

- In the node following R, player 3 will choose A. Therefore, A must be played part of SPE.
- In the node following M, player 3 is indifferent between F and B. Both could be part of SPE.
- In the node following L, player 2 is indifferent between U and D. Both could be part of SPE.
- Player 1 can get 2 when choosing R (outside option). Therefore, M is never a best response. Also, player 1 prefers L when 2 chooses U, and R otherwise.

SPE: (L,U,FA), (L,U,BA), (R,D,FA), (R,D,BA).

Question 2)

(a) Lets find all SPE using backward induction. The node following *game* starts a subgame, represented below:

		2	
		Hockey	Soccer
1	Hockey	3,1	0,0
	Soccer	0,0	1,3

There are two pure strategy NE – (Hockey, Hockey) and (Soccer, Soccer) – and one mixed strategy NE – $[(\frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$. The three NE of this subgame can be part of SPE.

Note that the expected payoff of the mixed strategy NE is $\frac{3}{4} \times \frac{1}{4} \times (1+3) = \frac{12}{16}$ for both players. Now, let's think about 1's decision at the first node for different values of x .

Without knowing x , there are six candidates for SPE:

- $[(g, H), (H)]$ and $[(c, H), (H)]$
- $[(g, S), (S)]$ and $[(c, S), (S)]$
- $[(g, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$ and $[(c, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$

If $x < \frac{12}{16}$, *couch* is never a best response. SPE: $[(g, H), (H)], [(g, S), (S)], [(g, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$.

If $x > 3$, *game* is never a best response. SPE: $[(c, H), (H)], [(c, S), (S)], [(c, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$.

If $\frac{12}{16} < x < 1$: $[(g, H), (H)], [(g, S), (S)], [(c, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$.

If $1 < x < 3$: $[(g, H), (H)], [(c, S), (S)], [(c, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$.

Note that if one of the conditions hold with equality, there are four SPE (e.g. $x = 3$: $[(c, H), (H)], [(c, S), (S)], [(c, \frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4})]$, and $[(g, H), (H)]$).

(b) Representing the game in matrix form:

		2	
		H	S
1	game, H	3,1	0,0
	game, S	0,0	1,3
	couch, H	x,0	x,0
	couch, S	x,0	x,0

For $x \geq 1$, $[(couch, H), (S)]$ is NE, but it is not SPE (because (H,S) is not NE in the subgame).

Question 3)

(a) Two: the entire game and the subgame starting in the node after O_1 .

(b) Let's find all SPE using backward induction. The node following O_1 starts a subgame, represented below (I omit 1's payoff):

		3	
		X	Y
	A	10,0	0,1
2	B	0,5	10,4
	C	3,10	3,10

There is no pure strategy NE in this game. Also, C is dominated by mixture of A and B (eg. play A with prob. 0.5 and B with prob. 0.5 yields payoff $5 > 3$). Lets look for mixed strategy NE mixing (A,B) and (X,Y). Denote the probability that 2 chooses A as p , and the probability that 3 chooses X as q :

$$\begin{aligned} \text{Player 2: } 10q &= 10(1 - q) \implies q = \frac{1}{2} \\ \text{Player 1: } 5(1 - p) &= p + 4(1 - p) \implies p = \frac{1}{2} \end{aligned}$$

Mixed strategy NE: $[(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2})]$.

Now, lets think about player 1's decision. The expected payoff of choosing O_1 is 5. As $5 > 2$, chooses O_1 . SPE: $[O_1, (\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2})]$.

(c) Representing the game in matrix form, where 2 chooses row, 3 chooses column, and 1 chooses matrix:

*Caution: payoffs in order "(Matrix, Row, Column)".

		3				3	
		X	Y			X	Y
	A	5,10,0	5,0,1		A	2,2,-5	2,2,-5
2	B	5,0,5	5,10,4	2	B	2,2,-5	2,2,-5
	C	1,3,10	1,3,10		C	2,2,-5	2,2,-5
		1: O_1				1: O_2	

(O_2, C, X) is NE but it is not SPE, leading to outcome $(2, 2, -5)$. Player 2 "threatens" to choose C if player 1 chooses O_1 , and as a result player 1 chooses O_2 . This is not SPE because choosing C is not a best response once he reaches that subgame.

Question 4)

(a) There are six subgames (including the entire game). Each decision node starts a subgame. Doing backward induction, we find the SPE: (ch, ej, gm). It is unique because players are never indifferent.

(b) (Next page) Represent the game in matrix form, where 1 chooses row, 2 chooses column, and 3 chooses matrix.

(c) (ah, ej, fn) is NE but not SPE (because f is not best response in subgame).

P3 – fm:

		P2					
		dj	dk	dl	ej	ek	el
P1	ah	3,2,8	3,2,8	3,2,8	4,3,6	4,3,6	4,3,6
	ai	0,0,7	0,0,7	0,0,7	4,3,6	4,3,6	4,3,6
	bh	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	bi	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	ch	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0
	ci	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0

P3 – fn:

		P2					
		dj	dk	dl	ej	ek	el
P1	ah	3,2,8	3,2,8	3,2,8	4,3,6	4,3,6	4,3,6
	ai	5,1,3	5,1,3	5,1,3	4,3,6	4,3,6	4,3,6
	bh	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	bi	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	ch	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0
	ci	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0	0,5,0

P3 – gm:

		P2					
		dj	dk	dl	ej	ek	el
P1	ah	3,2,8	3,2,8	3,2,8	4,3,6	4,3,6	4,3,6
	ai	0,0,7	0,0,7	0,0,7	4,3,6	4,3,6	4,3,6
	bh	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	bi	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	ch	5,5,5	8,4,0	3,0,6	5,5,5	8,4,0	3,0,6
	ci	5,5,5	8,4,0	3,0,6	5,5,5	8,4,0	3,0,6

P3 – gn:

		P2					
		dj	dk	dl	ej	ek	el
P1	ah	3,2,8	3,2,8	3,2,8	4,3,6	4,3,6	4,3,6
	ai	5,1,3	5,1,3	5,1,3	4,3,6	4,3,6	4,3,6
	bh	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	bi	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6	2,5,6
	ch	5,5,5	8,4,0	3,0,6	5,5,5	8,4,0	3,0,6
	ci	5,5,5	8,4,0	3,0,6	5,5,5	8,4,0	3,0,6

Question 5) The subgame starting with Player 1's offer in the second period is the bargaining game of alternating offers seen in class. The outcome of the SPE in this subgame is that Player 1 begins proposing x^* and Player 2 accepts it, with $x^* = (\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$.

Therefore, Player 2 only accepts Player 1's offer at $t = 1$, x^1 , if $x^1 \geq \frac{\delta^2}{1+\delta}$. Outcome: in $t = 1$, Player 1 proposes $x^{**} = (\frac{1+\delta(1-\delta)}{1+\delta}, \frac{\delta^2}{1+\delta})$ and Player 2 accepts it.

SPE:

- Player 1 proposes x^{**} in $t = 1$. Other than that, always proposes x^* . Accepts a proposal y iff $y_1 \geq \frac{\delta}{1+\delta}$.
- Player 2 accepts x^1 in $t = 1$ if $x^1 \geq x^{**}$. For $t > 1$, always proposes y^* and accepts a proposal x iff $x_2 \geq \frac{\delta}{1+\delta}$.

Where $y^* = (\frac{\delta}{1+\delta}, \frac{1}{1+\delta})$.

Question 6) Lets find the SPE using backward induction. The node following L starts a subgame, represented below:

		2	
		l	r
1	x	2,1	1,2
	y	1,2	2,1

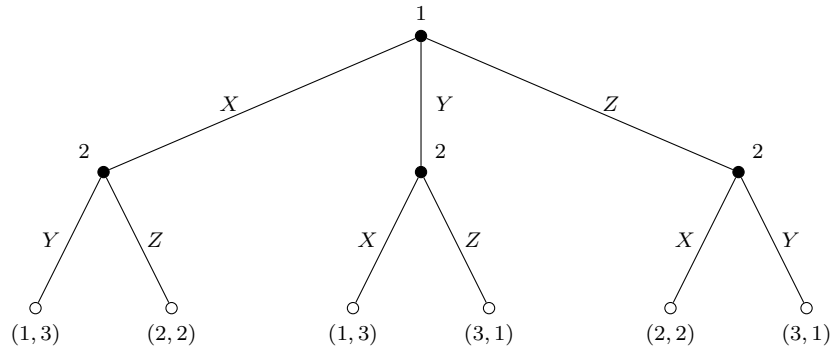
There is no pure strategy NE in this game. Mixed strategy NE: $[(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})]$. Thus, player 1's expected payoff of choosing L is 1.5.

Now, lets move up and find the NE in the whole game, but restricting the strategies players can choose so that they play the NE in the subgame we solved above.

		2	
		(1/2,1/2), a	(1/2,1/2), b
1	M, (1/2,1/2)	0,0	1,1
	R, (1/2,1/2)	3,3	0,0
	L, (1/2,1/2)	1.5,1.5	1.5,1.5

Both $[(R, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, a)]$ and $[(L, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, b)]$ are NE. Furthermore, we know they are NE in each subgame (since we imposed this restriction). Therefore, they are SPE.

Question 8)



Where players 1 and 2 choose which policy to veto – X, Y, or Z. Using backward induction: SPE is (Z,YXX). Finding NE:

		2							
		YXX	YXY	YZX	YZY	ZXX	ZXY	ZZX	ZZY
1	X	1,3	1,3	1,3	1,3	2,2	2,2	2,2	2,2
	Y	1,3	1,3	3,1	3,1	1,3	1,3	3,1	3,1
	Z	2,2	3,1	2,2	3,1	2,2	3,1	2,2	3,1

NE: (Z,YXX) and (Z,ZXX). (b) Yes, (Z,ZXX) is not SPE.