CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
MATH	203	All
Examination	**Date**	**Pages**
Final	April 2017	3
Instructors		**Course Examiners**
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Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.

MARKS

[11] **1.** (a) Suppose $f(x) = x + \dfrac{1}{x}$ and $g(x) = \dfrac{x+1}{x+2}$. Find $f \circ g$ and $g \circ f$. What is the domain of $g \circ f$?

(b) Find the inverse of the function $f(x) = e^{x^3} - 1$. Determine the domain and range of f and f^{-1}.

[10] **2.** Evaluate the limits:

a) $\displaystyle\lim_{x\to 2} \frac{\sqrt{3x+10}-4}{x^4-4x^2}$ b) $\displaystyle\lim_{x\to\infty} \frac{3x^2(\sqrt{x}+1)^3}{(2x+1)^3(\sqrt{x}-1)}$.

Do not use L'Hôpital's rule.

[6] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{x\sqrt{9x^2+1}-2x^2}{x^2-25}$$

[15] **4.** Find the derivatives of the following functions (you do not need to simplify the answers)

(a) $f(x) = \dfrac{1+\sqrt{x}-3x-2x\sqrt[3]{x^2}-2x\sqrt{x}}{2x\sqrt{x}}$;

(b) $f(x) = (x^2 - xe^{\pi})2^x$;

(c) $f(x) = \ln^2(1+\cos(5x))$;

(d) $f(x) = \dfrac{\sin^2(x)+\sin(x^2)}{\ln\sqrt{1+x}}$

(e) $f(x) = (\arctan x)^{1+x^2}$

[15] **5.** (a) The equation of a curve is $y^2 \cos x = xy^5 + y + 2$. and defines y implicitly as a function of x. Verify that the point $(0, -1)$ belongs to this curve. Find an equation of the tangent line to the curve at this point.

(b) Let $f(x) = \dfrac{(1+x^2)^2}{x^3}$. Find $f'''(x)$.

(c) Use L'Hôpital's rule to evaluate $\lim\limits_{x\to 0} \dfrac{\sin^2(x)}{x \ln(1+3x)}$

[6] **6.** Verify that $f(x) = x^3 - 3x^2 + 2x + 5$ satisfies the conditions of Rolle's Theorem on the interval $[0, 1]$, and find all numbers c in $[0, 1]$ that satisfy the conclusion of Rolle's Theorem.

[11] **7.** Given the function $f(x) = \dfrac{2x}{x+2}$,

(a) Find the derivative $f'(x)$ using the definition of the derivative as the limit of the difference quotient.

(b) Use the appropriate differentiation rule(s) to verify your answer in part (a).

(c) Find the linear approximation $L(x)$ to $f(x)$ at $a = 2$.

(d) Use the above linear approximation (or use differentials) to estimate the value of $f(2.4)$.

[10] **8.** (a) A particle is moving along the curve $y^2 - 6x^4 = y$. At the moment when $x = -1$ the x-coordinate is increasing at the rate of 5 cm/sec. If the y-coordiante is negative at this moment, is y increasing or decreasing? How fast?

(b) A box with an open top and a square base must have a volume of 32,000 cm^2. Find the dimensions of the box that minimizes the amount of material used.

[16] **9.** Given the function $f(x) = x^2e^x$,

(a) Find the domain and check for symmetry. Find all asymptotes (if there are any).

(b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if there are any).

(c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward, and points of inflection (if there are any).

(d) Sketch the graph of the function.

[5] **Bonus Question**

Suppose f is an odd function that is differentiable everywhere. Prove that for every positive number b, there is a number c in the interval $(-b, b)$ such that $f'(c) = \dfrac{f(b)}{b}$.