solutions, Midterm Phy 2323, S2016

1. $\vec{A} = (4x+9y)\hat{x} - 14yz\hat{y} + 8x^2z\hat{z}$

a) $\vec{A}\cdot d\vec{l} = (4x+9y)\,dx - 14yz\,dy + 8x^2z\,dz$

$x = t \Rightarrow dx = dt$

$y = t^2 \Rightarrow dy = 2t\,dt$

$z = t^3 \Rightarrow dz = 3t^2\,dt$

subst. $\rightarrow$ $\int_C \vec{A}\cdot d\vec{l} = \int_{t=0}^{1} [4t + 9t^2 - 28t^6 + 24t^7]\,dt = 4.$

b) Along a straight line from P to Q: $y = x$, $z = x$, $dy = dx$, $dz = dx$.

$\int_C \vec{A}\cdot d\vec{l} = \int_0^1 (13x - 14x^2 + 8x^3)\,dx = 3.833$

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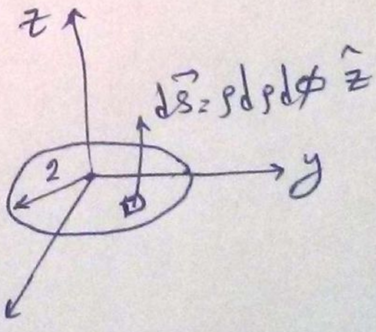
2. $\vec{D} = (2+16\rho^2)\hat{z}$



$\int_S \vec{D}\cdot d\vec{s} = \int_0^2 (2+16\rho^2)\rho\,d\rho \int_0^{2\pi} d\phi$

$= 136\pi$

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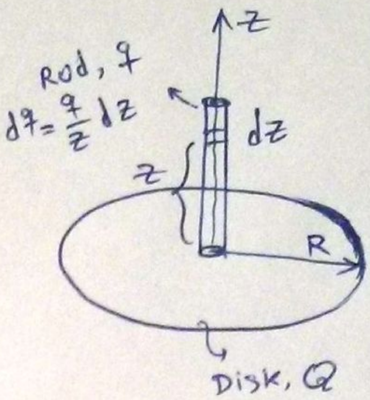
3. we calculate the force exerted on a Point charge on the rod, then move this Point along the rod to calculate the total force.



$$E_Z = \frac{\rho_s}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$$

$$= \frac{Q}{2\pi\epsilon_0 R^2}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)$$

$$dF = E_z\, dq = E_z\left(\frac{q}{R}\, dz\right) = \frac{Q}{2\pi R^2 \epsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)\frac{q}{R}\, dz$$

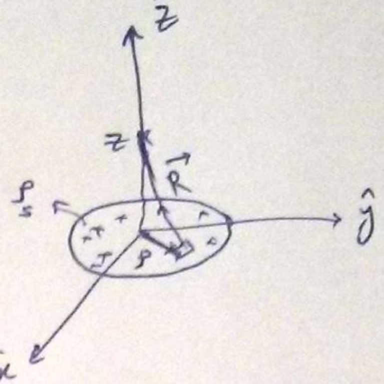
$$F = \int_0^R \frac{Qq}{2\pi R^3 \epsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right) dz$$

$$= \frac{Qq}{2\pi R^3 \epsilon_0}\left[R - \int_0^R \frac{z\, dz}{\sqrt{R^2+z^2}}\right]$$

$$= \frac{Qq}{2\pi R^3 \epsilon_0}\left[R - \sqrt{z^2+R^2}\,\Big|_0^R\right]$$

$$= \frac{Qq}{2\pi\epsilon_0 R^2}\left(2-\sqrt{2}\right) \quad [N]$$

4. $V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho_s\, ds}{R}$



a)

$$V = \frac{\rho_s}{4\pi\epsilon_0}\int_0^b\int_0^{2\pi} \frac{\rho\, d\phi\, d\rho}{\sqrt{\rho^2+z^2}}$$

$$= \frac{\rho_s}{2\epsilon_0}\int_0^b \frac{\rho\, d\rho}{\sqrt{z^2+\rho^2}} = \frac{\rho_s}{2\epsilon_0}\left[\sqrt{\rho^2+z^2}\,\right]_0^b$$

$$= \frac{\rho_s}{2\epsilon_0}\left[\sqrt{b^2+z^2} - z\right]$$

b)

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial z}\hat{z} = -\frac{\rho_s}{2\epsilon_0}\left[\frac{z}{\sqrt{b^2+z^2}} - 1\right]\hat{z}$$

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5. $\nabla^2 V = 0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) = 0 \Rightarrow V = -\frac{C_1}{r} + C_2$

at $r=b$, $V=0 \Rightarrow C_2 = \frac{C_1}{b} \rightarrow V = C_1\left[\frac{1}{b} - \frac{1}{r}\right]$

at $r=a$, $V=V_0 \rightarrow C_1 = \frac{V_0}{\frac{1}{b}-\frac{1}{a}} \rightarrow V = \frac{-V_0}{\frac{1}{b}-\frac{1}{a}}\left(\frac{1}{b}-\frac{1}{r}\right) = \frac{V_0 ab}{(b-a)r} - \frac{V_0 a}{b-a}$

$$\vec{E} = -\vec{\nabla} V = -\frac{\partial V}{\partial r}\hat{r} = \frac{V_0}{\frac{1}{b}-\frac{1}{a}}\cdot\frac{1}{r^2}\hat{r} = \frac{V_0 ab}{(b-a)r}\hat{r} \rightarrow \vec{D} = \frac{V_0 \epsilon ab}{(b-a)r}\hat{r}$$

$$\rho_{s+}\Big|_{r=a} = \frac{V_0 \epsilon b}{a(b-a)} \rightarrow Q_+ = 4\pi a^2 \rho_{s+} = \frac{4\pi ab\,\epsilon V_0}{b-a}$$

$$C = \frac{Q_+}{V_0} = \frac{4\pi\epsilon ab}{b-a} = \frac{4\pi\epsilon}{1/a - 1/b}.$$