Keep in mind:

a) em-fields inside a perfect conductor ($\sigma = \infty$) $= 0$

so on the surface of a perfect conductor both ρ_s & $\vec{J}_s$ can exist.

b) Time-varying fields can exist inside a conductor ($\sigma < \infty$).

so $\vec{J}_s = 0$, but ρ_s can exist at the boundary of a conductor & a perfect dielectric.

c) At the interface of 2 perfect dielectrics $\vec{J}_s = 0$; $\rho_s = 0$ unless we put a charge there physically.

EX: $\vec{E} = C\cos(\omega t - \beta z)\hat{x}$ [V/m] in a source-free dielectric medium is given.
under what condition this field can exist? find the other quantities?

A field can exist if & only if it satisfies all maxwell's eqns.

Assume the given $\vec{E}$ can exist in a source-free ($\rho_v = 0, \sigma = 0, \vec{J} = 0$) dielectric medium.

$\Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow -\frac{\partial}{\partial z}[E_x]\hat{y} = \frac{\partial \vec{B}}{\partial t} \Rightarrow \frac{\partial \vec{B}}{\partial t} = -C\beta \sin(\omega t - \beta z)\hat{y}$.

$\int \frac{\partial \vec{B}}{\partial t} dt \Rightarrow \vec{B} = \frac{C\beta}{\omega}\cos(\omega t - \beta z)\hat{y}$ [T].

$\vec{B} = \mu \vec{H} \Rightarrow \vec{H} = \frac{C\beta}{\mu\omega}\cos(\omega t - \beta z)\hat{y}$ [A/m].

$\vec{D} = \epsilon \vec{E} \Rightarrow \vec{D} = \epsilon C \cos(\omega t - \beta z)\hat{x}$ [C/m²].

Gauss' law check: $\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0$ ✓

$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 + 0 + 0 = 0$ ✓

Last eqn: $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \rightarrow -\frac{\partial}{\partial z}\left[\frac{C\beta}{\mu\omega}\cos(\omega t - \beta z)\right]\hat{x} = \frac{\partial}{\partial t}\left[\epsilon C \cos(\omega t - \beta z)\right]\hat{x}$

$$\Rightarrow -\frac{c\beta^2}{\mu\omega}\sin(\omega t-\beta z)\hat{x} = -c\omega\epsilon\sin(\omega t-\beta z)\hat{x}$$

$$\Rightarrow \frac{c\beta^2}{\mu\omega} = c\omega\epsilon \rightarrow \boxed{\beta = \pm\omega\sqrt{\mu\epsilon}}$$ the condition for existing the given E-field.

Poynting's theorem: The expressions for energy density associated with static e- & m-fields are valid for time-varying fields.

- An expression for propagation of energy in a medium.

q is moving with $\vec{u}$ in a time-varying em-fields: $\vec{F} = q(\vec{E} + \vec{u}\times\vec{B})$

If q moves $d\vec{l}$ by $\vec{F}$: $dW = q(\vec{E} + \vec{u}\times\vec{B})\cdot d\vec{l}$

$$P\,dt = q(\vec{E} + \vec{u}\times\vec{B})\cdot\vec{u}\,dt$$

Power supplied by the field $\boxed{P = q\vec{u}\cdot\vec{E}}$

$(\vec{u}\times\vec{B})\cdot\vec{u} = 0 \rightarrow$ time-varying m-field does not supply any energy to a charged particle.

→ Power supplied by e-field to an infinitesimal charge $\rho_v\,dv$:

$$dP = \rho_v\,dv\,\vec{E}\cdot\vec{u} = (\vec{E}\cdot\rho_v\vec{u})\,dv = \vec{E}\cdot\vec{J}\,dv$$

$$\Rightarrow \boxed{\frac{dP}{dv} = p = \vec{J}\cdot\vec{E}}$$ we had this for static fields using conservation of energy.

$$\Rightarrow \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \Rightarrow \vec{J}\cdot\vec{E} = \vec{E}\cdot(\vec{\nabla}\times\vec{H}) - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}$$

vector identity:

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{H})$$

$$\vec{J}\cdot\vec{E} = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{H}) - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot(\vec{E}\times\vec{H}) + \vec{J}\cdot\vec{E} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = 0}$$

Poynting's theorem.

$\vec{S} = \vec{E} \times \vec{H}$ ≡ Poynting vector: Power density $[\frac{W}{m^2}]$.

↘ instantaneous flow of power per unit area.

The direction of power flow is ⊥ to the plane containing $\vec{E}$ & $\vec{H}$.

For linear, homogeneous & isotropic medium:

$\vec{B} = \mu\vec{H}$, $\vec{D} = \epsilon\vec{E}$

$$\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}[\vec{B}\cdot\vec{H}] = \frac{1}{2}\frac{\partial}{\partial t}[\mu H^2]$$

$$\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}[\vec{D}\cdot\vec{E}] = \frac{1}{2}\frac{\partial}{\partial t}[\epsilon E^2]$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{S} + \vec{J}\cdot\vec{E} + \frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2\right] + \frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon E^2\right] = 0}$$

diff. form of Poynting theorem

$\frac{\partial}{\partial t}[\frac{1}{2}\mu H^2]$: The rate of change of energy density in the m-field.
$w_m = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{1}{2}\mu H^2$

$\frac{\partial}{\partial t}[\frac{1}{2}\epsilon E^2]$: the rate of change of energy density in the e-field.
$w_e = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\epsilon E^2$

$$\int_v \vec{\nabla}\cdot\vec{S}\, dv + \int_v \vec{J}\cdot\vec{E}\, dv + \int_v \frac{\partial}{\partial t}(w_m)\, dv + \int_v \frac{\partial}{\partial t}(w_e)\, dv = 0$$

; $\underline{v}$ is bounded by surface $\underline{S}$.

$$\Rightarrow \boxed{\oint_s \vec{S}\cdot d\vec{s} + \int_v \vec{J}\cdot\vec{E}\, dv + \frac{d}{dt}\int_v w_m\, dv + \frac{d}{dt}\int_v w_e\, dv = 0}$$

∫ form of Poynting theorem.

$\oint_s \vec{S}\cdot d\vec{s}$: Power crossing the closed surface $\underline{S}$ bounding volume v.
+ : Power flowing out
− : Power flowing into the $\underline{v}$.

$\int_v \vec{J}\cdot\vec{E}\, dv$: Power supplied to the charged particles by the field.
+ : field is doing work on q.
− : external force is doing work to move q against the field.
* in conductive medium $\vec{J} = \sigma\vec{E}$, $\int \vec{J}\cdot\vec{E}\, dv$ = Power dessipation (ohm power loss

$\frac{d}{dt}\int_v w_m\, dv$: the rate of change of stored m-energy.
+ : external source is supplying energy to m-field. → m-energy ↑
− : m-energy extracted from m-field → m-field decay.

$\frac{d}{dt}\int_v w_e\, dv$: the rate of change of stored e-energy.
+ : external soure supplying energy to e-field → e-energy ↑
− : e-energy extracted fro e-field → e-field decay.

usually:

$$-\oint_s \vec{S}\cdot d\vec{s} = \int_v \vec{J}\cdot E\, dv + \frac{d}{dt}\int (w_m + w_e)\, dv$$

net power must flow into v to account for: the power dissipation in the region as heat + increase in energy stored in both e- & m- fields.

For static fields: $\boxed{-\oint_s \vec{S}\cdot d\vec{s} = \int_v \vec{J}\cdot\vec{E}\, dv}$

Power flowing through s into v = Power dissipation in that volume v.

EX: E-field intensity in a perfect dielectric medium is $\vec{E} = E\cos(\omega t - kz)\hat{x}$ [V/m]. a) m-field intensity in the region? b) Direction of power flow c) Average power density?

a) * check if $\vec{E}$ can exist in that medium:

$E_x = E\cos(\omega t - kz)$ [V/m].

$\to D_x = \epsilon E_x = \epsilon E\cos(\omega t - kz)$ $[\frac{C}{m^2}]$

$\to \rho_v = \vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}[\epsilon E\cos(\omega t - kz)] = 0$ ✓ $\Rightarrow \vec{\nabla}\cdot\vec{D} = 0$ free charge density in a perfect dielectric medium = 0.

$$\frac{\partial\vec{B}}{\partial t} = -\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial z}(E_x)\hat{y} = -Ek\sin(\omega t - kz)\hat{y}$$

$$\Rightarrow \boxed{B_y = \frac{Ek}{\omega}\cos(\omega t - kz)\ [T]}$$

$\vec{H} = \frac{\vec{B}}{\mu} \Rightarrow H_y = \frac{Ek}{\mu\omega}\cos(\omega t - kz)$ [A/m].

* check if $\vec{B}$ or $\vec{H}$ field can exist: $\vec{\nabla}\cdot\vec{B} = \frac{\partial}{\partial y}B_y = \frac{\partial}{\partial y}\left[\frac{Ek}{\omega}\cos(\omega t - kz)\right] = 0$ ✓

$$\vec{J} = \vec{\nabla}\times\vec{H} - \frac{\partial\vec{D}}{\partial t} = -\frac{\partial}{\partial z}(H_y)\hat{x} - \frac{\partial}{\partial t}(D_x)\hat{x} = \left[\omega\epsilon - \frac{1}{\omega\mu}k^2\right]E\sin(\omega t - kz)\hat{x}.$$

In a perfect dielectric medium $J = 0$ $\Rightarrow \omega\epsilon - \frac{1}{\omega\mu}k^2 = 0 \to \boxed{k = \pm\omega\sqrt{\epsilon\mu}}$ k is not an arbitrary constant if it wants to exist.

b) Instantaneous Power density (Poynting vector): $\vec{S} = \vec{E} \times \vec{H}$

$$= \frac{k}{\omega\mu} E^2 \cos^2(\omega t - kz)\, \hat{z} \quad [W/m^2].$$

Power flow in Z-direction.

c) Average Power density in z-direction:

$$\langle S_z \rangle = \frac{1}{T}\int_0^T S\, dt = \frac{1}{T}\int_0^T \frac{k}{\omega\mu} E^2 \cos^2(\omega t - kz)\, dt \quad ; \; T = \frac{2\pi}{\omega}$$

$$\cos(2\omega t - 2kz) = 2\cos^2(\omega t - kz) - 1$$

$$\Rightarrow \langle S_z \rangle = \frac{1}{2T}\int_0^T \frac{k}{\omega\mu} E^2\, dt + \underbrace{\frac{1}{2T}\int_0^T \frac{k}{\omega\mu} E^2 \cos(2\omega t - 2kz)\, dt}_{0}$$

$$= \frac{1}{2T}\left[\frac{k}{\omega\mu} E^2 T\right] = \frac{k}{2\omega\mu} E^2 \quad [W/m^2].$$

Applications of em-fields:

- When maxwell's eqns are satisfied, each quantity ($\vec{E}$ or $\vec{H}$) separately satisfies a "wave eqns" or "Helmholtz eqn".

→ many possible solutions → uniform plane-wave solution. → the field components are in a plane normal to the direction of propagation of the wave., and the magnitude of each field component is constant (uniform) in that plane. (Transverse em waves, TEM] → $v_{propagation} = c$.

→ TEM wave can exist where there are 2 or more conductors insulated from one another called → guided wave: it propagates along the length of the conductors

→ Parallel-wire transmission lines
→ coaxial cables

→ TEM mode or principal mode.

→ Transmit power or information (signals).

Waveguide: using one hollow conductor to guide a wave.
[cylindrical & rectangular]:

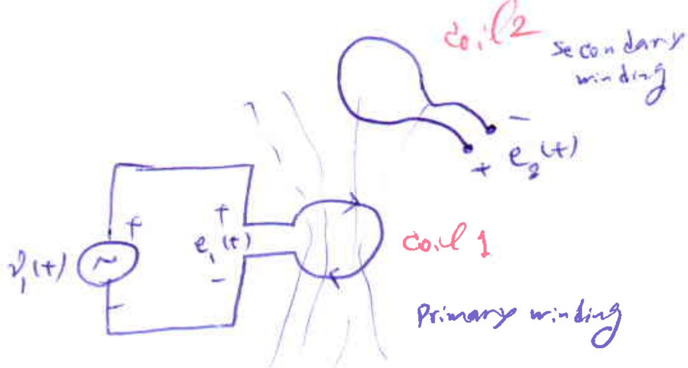
Antennas: to generate the em-fields that can propagate through an unbounded region, along a transmission line, or through the medium enclosed by a waveguide.

⇒ Applications of Maxwell's eqns under various boundary conditions.

The Transformer:

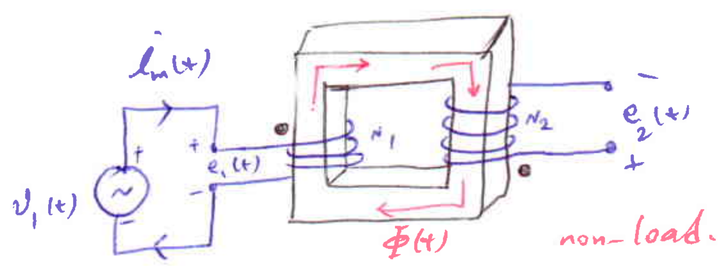
Two electrically isolated coils, time-varying m-flux of one coil links the other coil and induces an emf in it. (magnetically coupled)

→ two-winding transformer.



air-core transformer: two coils isolated in free space or wound over a nonmagnetic material.

Iron-core transformer: wound over a magnetic material with high permeability to form a common m-circuit.



⇒ $i_m(t)$ = magnetization current makes

- $\Phi(t)$
- mmf drop in m-circuit due to reluctance
- Power loss in the Primary winding & the m-loss in the core.

Ideal Transformer:

a) ∞ Permeability ($\mu = \infty$)

b) R = 0 winding resistance

c) NO magnetic losses

$\rightarrow$ $-i_m(t)\big|_{\text{non load}} \simeq 0$

all time-varying flux will follow the m-Path without any leakage.

$$\begin{cases} e_1(t) = \text{emf induced in the primary winding} = N_1 \dfrac{d\Phi}{dt} \\ e_2(t) = \text{" " " secondary "} = N_2 \dfrac{d\Phi}{dt} \end{cases}$$

$\rightarrow$ minus sign replaced by Posting (+/−) Polarity in the figure.

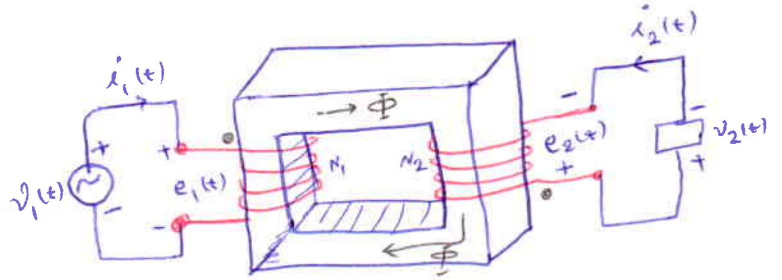
dot in the figure: induced emf is (+) with respect to non dotted end when the flux is increasing with time.

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}$$

$\rightarrow$ if we excite the secondary and get induced e_1 $\rightarrow$ we would get the same expression.

$\xrightarrow[\text{conditions}]{\text{ideal}}$ $\boxed{\dfrac{v_1}{v_2} = \dfrac{N_1}{N_2} = a}$ = (a-ratio) or (Ratio of transformation).

Loaded secondary ~~coil~~ Transformer:



- current in secondary winding produces its own flux, opposes the original flux. $\rightarrow$

Φ ~~in~~ in the core & emf in each winding $\overset{\text{try}}{<}$ no-load value. $\rightarrow$ but

when $e_1 \downarrow$, it causes a current in the primary winding to nullify the decrease in the flux & induced emf. $\Rightarrow$ the increase in the current continues until the flux in the core & thereby emfs in two windings are restored to their no-load values. $\Rightarrow$ Source supplies power to the primary winding, and the secondary winding delivers that power to the load.

- m-flux acts as a medium in the Power transfer process.

$$P_{in} \overset{\text{ideal}}{=} P_{out} \Rightarrow v_1 i_1 = v_2 i_2 \Rightarrow \boxed{\frac{i_2}{i_1} = \frac{v_1}{v_2} = \frac{e_1}{e_2}}$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = \frac{i_2}{i_1} \Rightarrow N_1 i_1 - N_2 i_2 = 0 \longrightarrow$$ under ideal conditions the net mmf needed to excite the transformer is Zero.

→ m-material has $\mu = \infty$ or $\mathcal{R}_{circut} = 0$.

Autotransformer :- Two-winding transformer: electrically insulated, they are coupled together magnetically by a common core.

→ m-induction is doing the energy transfer.

- Autotransformer : when two windings of a transformer are interconnected electrically. → may have single continuous winding ; or consist of 2 or more distinct coils wound on the same core.

→ A part of energy is transferred by conduction. (electrical connection.)

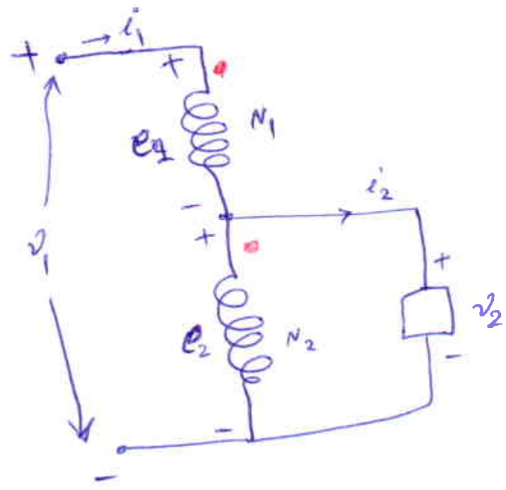
→ 〃 〃 〃 〃 Induction. (magnetic coupling)

Advantages: 1. les expensive than 2-winding.

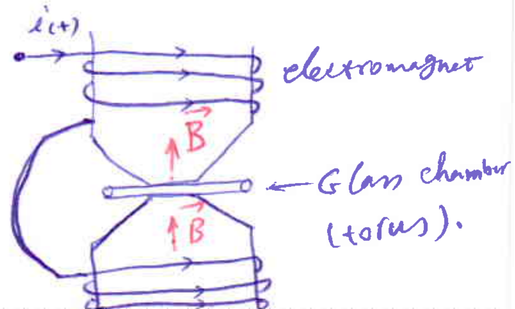
2. deliver more power than -- with the same physical dimention

3. more efficient for a similar power rating

4. need lower excitation current to stablish the same flux in the core.



The betatron: A charged particle revolves in an evacuated glass chamber (torus) at a constant radius.



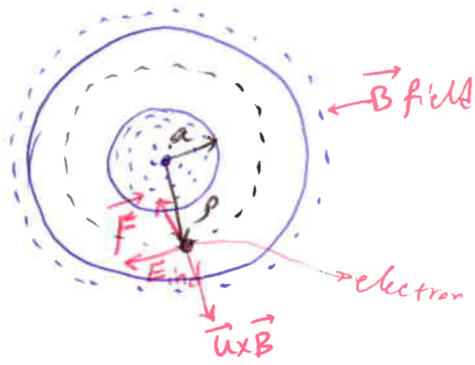
→ Time-varying m-field by an electromagnet.

- the gap between the pole faces of the electromagnet increases ↑ radially outward to control the strength of B-field.

electron: $\cancel{u_0} = 0$ $\Rightarrow$ $B\uparrow = B\hat{z}\uparrow \Rightarrow$ induced $\vec{E}$-field. (closed loop)

$\vec{B}_0 = 0$

maxwell's eqn $\displaystyle\oint_C \vec{E}\cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$



"Plane of torus"

- $\vec{B} = \vec{B}(r,t)$ a function of space & time.

- $|\vec{B}| = \text{const at } \underline{\underline{\rho}} \Rightarrow |\vec{E}| = \text{const at } \underline{\rho}$.

- for $\rho = a$: $\displaystyle E_\varphi = -\frac{1}{2\pi a}\frac{d\Phi}{dt}$; $\displaystyle \Phi = \int_0^a B\rho\, d\rho \int_0^{2\pi} d\varphi$ total flux passing $\underline{s}$ bounded by circle $\rho = a$.

- the force exerted by $\vec{E}$ on electron: $\displaystyle F_\varphi = -eE_\varphi = \frac{\overset{1.602\times10^{-19}C}{e}}{2\pi a}\frac{d\Phi}{dt}$

- Newton's 2nd law: the rate of change of momentum = impressed force

$$\frac{dP}{dt} = F_\varphi = \frac{e}{2\pi a}\frac{d\Phi}{dt}$$

- $t=0, u_0=0 \Rightarrow$ $\displaystyle P = \frac{e\Phi}{2\pi a}$. momentum at any time t.

- As soon as electron starts revolving in a circular path ($\rho=a$), it experiences Lorentz force $\vec{F} = -e(\vec{u}\times\vec{B})$ → move e^- toward center.

= centripetal force. (scape). to stay in the same $\underline{\rho}$.

$\displaystyle \Rightarrow eBu = \frac{mu^2}{a} \Rightarrow mu = eBa$; m = relativistic mass of $e = km_0$

$P = mu$ momentum of $\bar{e}$.

$P = \frac{e\Phi}{2\pi a} = eBa \Rightarrow B = \frac{\Phi}{2\pi a^2}$; $B_0 = \frac{\Phi}{\pi a^2}$ = space average flux density (over surface area bounded by the orbit).

$\Rightarrow B = \frac{1}{2} B_0$. $\Longrightarrow$ the poles of the electromagnet are tapered to create a B-field that decreases in the outward radial direction.

$\rightarrow$ "400 MeV betaterons" industry.