In a source-free region (no current): $\vec{\nabla} \times \vec{H} = 0 \Rightarrow \oint_C \vec{H} \cdot d\vec{l} = 0$

if path C does not enclose any current.

→ For conservative field $\vec{E}$: $\vec{E} = -\vec{\nabla} V$

$$V_{ab} = -\int_b^a \vec{E} \cdot d\vec{l}.$$

when $\vec{\nabla} \times \vec{H} = 0 \Rightarrow \boxed{\vec{H} = -\vec{\nabla} f}$ → [A] f = m-scalar Potential or magnetostatic Potential.

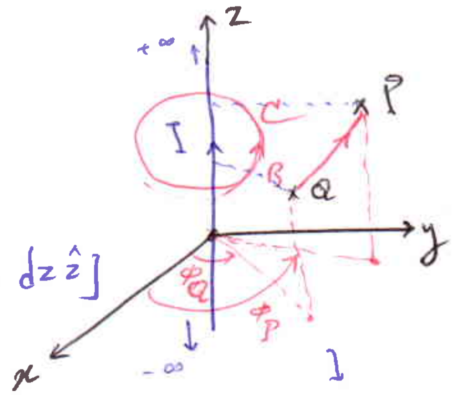
$$\boxed{f_{ab} = f_a - f_b = -\int_b^a \vec{H} \cdot d\vec{l}.}$$

magnetomotive force (mmf): To describe the differences in m-Potential between two Points.

$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{\nabla} \cdot \mu \vec{H} = 0 \Rightarrow \vec{\nabla} \cdot \vec{H} = 0$ (linear, isotropic & homogeneous).

$\Rightarrow \vec{\nabla} \cdot (-\vec{\nabla} f) = 0 \Rightarrow \boxed{\nabla^2 f = 0}$ Laplace's eqn for m-scalar Potential in a current-free region.

EX: A wire, I in z-direction. Find m-Potential differene between two Points in space:



$\oint_C \vec{H} \cdot d\vec{l} = I_{enc.} \Rightarrow H(2\pi\rho) = I \Rightarrow \vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$

$f_{PQ} = -\int_{\phi_Q}^{\phi_P} \vec{H} \cdot d\vec{l}$; $\vec{H} \cdot d\vec{l} = H_\phi \hat{\phi} \cdot [d\rho\,\hat{\rho} + \rho\, d\phi\,\hat{\phi} + dz\,\hat{z}]$
$= \frac{I}{2\pi} d\phi$

$\phi_P > \phi_Q$

Pot. drop (mmf drop).

$P(\rho_P, \phi_P, z)$, $Q(\rho_Q, \phi_Q, z_Q)$:

$$f_{PQ} = -\int_{\phi_Q}^{\phi_P} \frac{I}{2\pi} d\phi = -\frac{I}{2\pi}[\phi_P - \phi_Q] = \frac{I}{2\pi}(\phi_Q - \phi_P)$$

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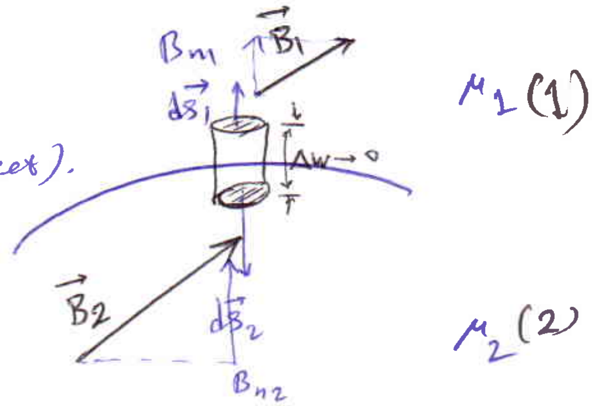
Boundary (Interface) conditions for m-fields: behavior of m-fields at the boundary between two media (regions) with different Permeabilities.

— $\vec{B}$: $\oint_S \vec{B}\cdot d\vec{S} = 0$

S: pillbox

$\Delta w \to 0$ (no effect).



$$\int_{S_1} \vec{B}\cdot d\vec{S} + \int_{S_2} \vec{B}\cdot d\vec{S} = 0$$

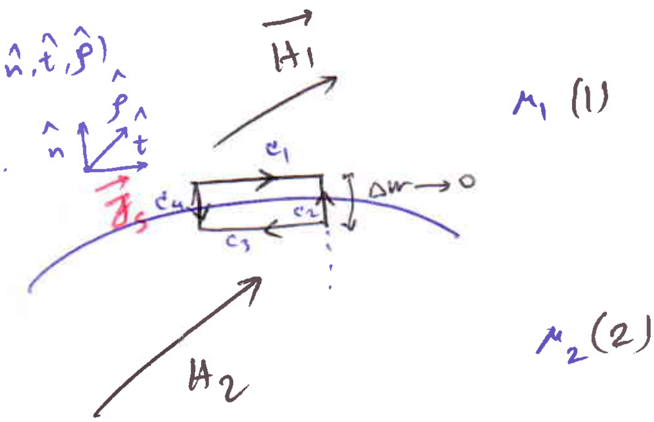
$$\begin{cases} d\vec{S}_1 = dS\,\hat{n} \\ d\vec{S}_2 = dS(-\hat{n}) \end{cases} \rightarrow \int_{S_1} \vec{B}\cdot\hat{n}\,dS + \int_{S_2} \vec{B}\cdot(-\hat{n})\,dS = 0$$

$$\int_S (B_{n1} - B_{n2})\,dS = 0 \Rightarrow \boxed{B_{n1} = B_{n2}}$$

$$\boxed{\hat{n}\cdot(\vec{B}_1 - \vec{B}_2) = 0}$$

— $\vec{H}$: $\oint \vec{H}\cdot d\vec{\ell} = I$, $\Delta w \to 0$

$(\hat{n},\hat{t},\hat{\rho})$ mutually Perpendicular unit Vectors.



$$\int_{c_1} \vec{H}\cdot d\vec{\ell} + \int_{c_3} \vec{H}\cdot d\vec{\ell} = I$$

$$\int_{c_1} \vec{H}_1\cdot\hat{t}\,d\ell + \int_{c_3} \vec{H}_2\cdot(-\hat{t})\,d\ell = I$$

$$= \int_S \vec{J}_v\cdot\hat{\rho}\,d\ell\,\Delta w$$

$\lim_{\Delta w \to 0} \vec{J}_v\,\Delta w = \vec{J}_s$ shrink.

$$\Rightarrow \int_{c_1} (\vec{H}_1 - \vec{H}_2)\cdot\hat{t}\,d\ell = \int_{c_1} \vec{J}_s\cdot\hat{\rho}\,d\ell$$

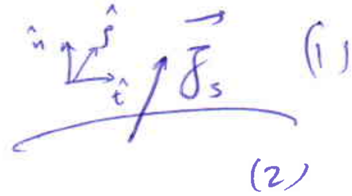
; $\hat{t} = \hat{\rho}\times\hat{n}$

$$\rightarrow \int_{c_1} (\vec{H}_1 - \vec{H}_2)\cdot(\hat{\rho}\times\hat{n})\,d\ell = \int_{c_1} \vec{J}_s\cdot\hat{\rho}\,d\ell$$

using vector identity: $\int_{C_1} \hat{n} \times (\vec{H}_1 - \vec{H}_2) \cdot \hat{\rho}\, dl = \int_{C_1} \vec{J}_s \cdot \hat{\rho}\, dl$

$\rightarrow$ $\boxed{\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s}$ tangential components of $\vec{H}$-field at boundary are discontinuous.

$$\boxed{H_{t1} - H_{t2} = J_s}$$



if $\vec{J}_s = J_s \hat{\rho}$. $\leftarrow H_{t1} > H_{t2} \Leftarrow$

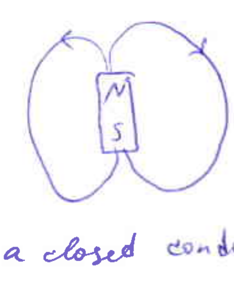
$\Rightarrow$ For two m-media with finite (not ∞) conductivities (σ), $\vec{J}_s = 0$.
If there is any current flow in either medium, it will be in $\vec{J}_v$.

$\rightarrow$ If one of the media is Perfect conductor, m-field inside the Perfect conductor $\Rightarrow$ $\vec{J}_s$ exists on the surface of Perfect conductor.

Energy in m-field: $w_e = \frac{1}{2}\vec{D}\cdot\vec{E} \rightarrow W_e = \frac{1}{2}\int_v \vec{D}\cdot\vec{E}\, dv$

$w_m = \frac{1}{2}\vec{B}\cdot\vec{H} \rightarrow W_m = \frac{1}{2}\int_v \vec{B}\cdot\vec{H}\, dv$

$= \frac{1}{2}\mu H^2 = \frac{B^2}{2\mu} \rightarrow W_m = \int_v w_m\, dv$.



magnetic circuits:

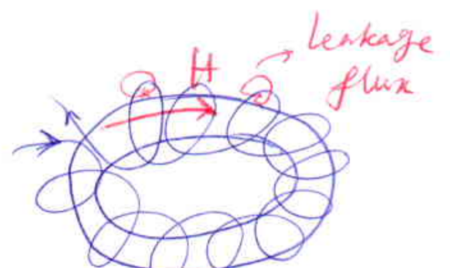
$\rightarrow$ magnetic flux lines form a closed path and m-flux entering a boundary is the same as the m-flux leaving a boundary $\rightarrow$ Like ~~a~~ current in a closed conducting circuit.

→ The m-flux cannot be completely confined to follow a given Path in a magnetic material as good a ~~the~~ currents but if the Permeability of the magnetic material is very high compared to that of surrounding (such as free space) most of the flux will be confined to highly Permeable material.

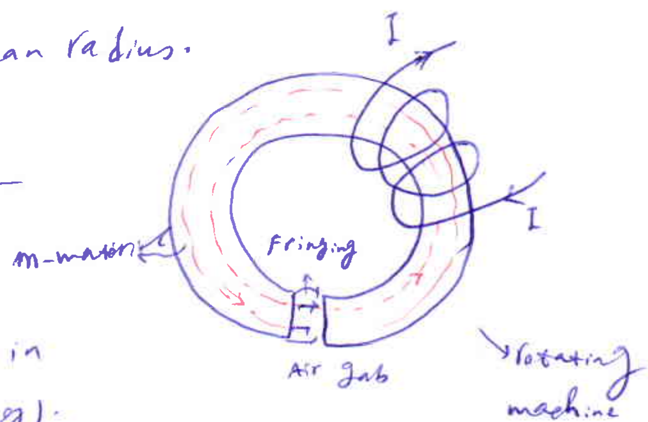
magnetic shielding: m-flux will be concentrated within a magnetic material.

→ channeling of the flux ≡ current through a conductor.

→ $\boxed{\text{magnetic circuit}}$



→ we assume that the m-flux density is uniform within the m-material, and its magnitude is equal to the m-flux density at the mean radius.



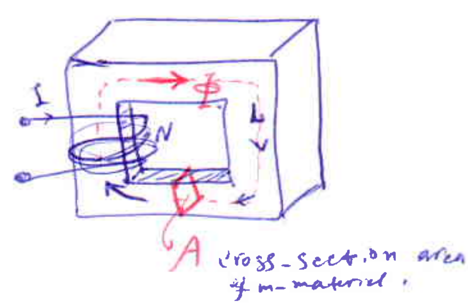
Highly Permeable material is in series with a low permeable material. →

flux are equal ; spread of m-flux in air gap. (fringing).

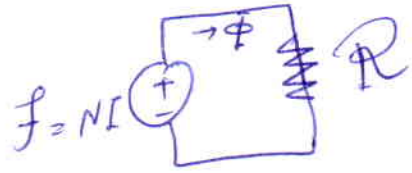
Ampere-turn [A.t]

$$f = NI = \oint_C \vec{H}\cdot d\vec{l} \ \overset{H \text{ is uniform}}{=\!=} \ HL$$

L: length of magnetic Path.



$$B = \mu H = \frac{\mu N I}{L} \quad ; \mu = \text{Permeability of the m-material.}$$



$$\Phi = \int_S \vec{B}\cdot d\vec{S} = BA = \frac{\mu N I A}{L} = \frac{NI}{L/\mu A} = \frac{f}{L/\mu A}$$

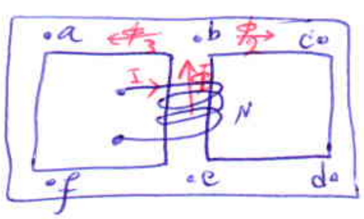
$\frac{L}{\mu A} \equiv \mathcal{R}$ "reluctance" of m-circuit. [A·t ...]

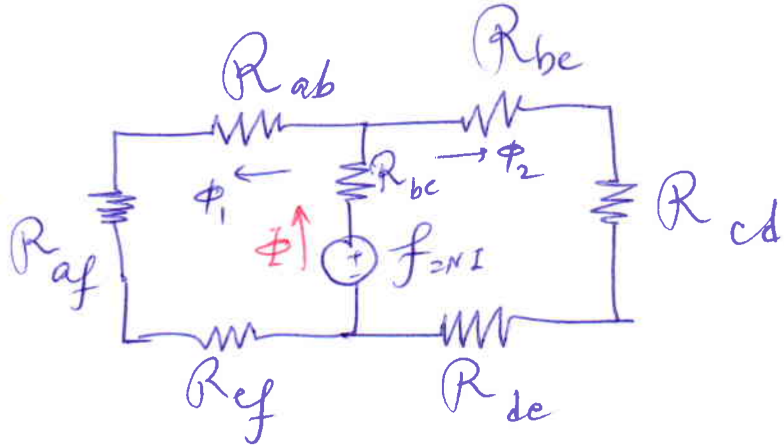
$\Rightarrow \mathcal{R} = \frac{L}{\mu A} \Rightarrow \boxed{\Phi \mathcal{R} = NI}$ ohm's law for the m-circuit.

$R = \frac{L}{\sigma A} \Rightarrow \mu \equiv \sigma \Rightarrow \mu\uparrow \Rightarrow \mathcal{R}\downarrow$.

For the same applied magnetomotive force (mmf), the flux in highly permeable material will be higher than that in a low permeable material.

In a circuit containing many m-materials, each part can be represented by its' reluctance $\equiv$ resistances.





H_i = m-field intensity in the i^{th} section of a m-circuit

L_i = mean length of i^{th} section.

Total mmf drop in the m-circuit = mmf applied

$$\boxed{\sum_{i=1}^{n} H_i L_i = NI}$$

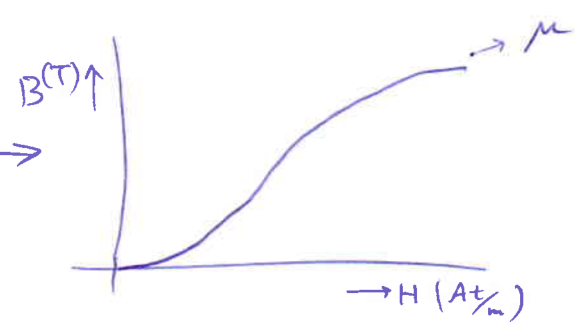
(analogous) Kirchhoff's voltage law for an e-circuit

* for linear m-materials (constant permeability) we can use analogous circuit.

Nonlinear m-materials: magnetization characteristic (B-H curve) $\rightarrow$

μ is dep. to m-flux density

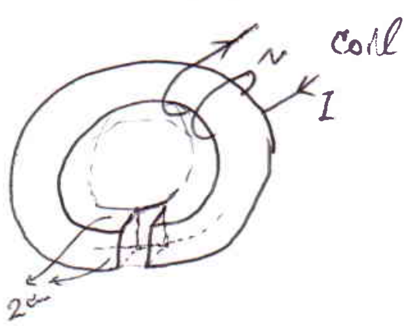
Ferromagnetic materials (iron).



two types of Problems: (a) m-flux density is given → Find applied mmf
(b) applied mmf is given → calculate m-flux density → flux in m-circuit.

solving problems:
- Linear m-material → equivalant circuit approach
- Non linear:
  - (a) calculate flux density in each m-section → H from B-H curve → mmf drops across each m-section → sum = mmf applied.
  - (b) → using iterative technique: educated guess for mmf drop in one of the m-regions → obtain the total mmf requirements → compare the results with the given mmf → another educated guess if we are off. → continue till the error between calculated mmf & the applied mmf is within Permisible limits. ($\pm 2\%$).

EX! An electromagnet of square cross section, with 1500 turns. radii inner 10 cm & outer 12 cm. length of airgap 1 cm. $I = 4A$, $\mu_r = 1200$, Find flux density in the magnetic circuit.



μ_r = const, Applied mmf is known. → reluctance method → flux density in the core.

$$r_{mean} = \bar{r} = \frac{r_{in} + r_{out}}{2} = 11\,cm$$

l_m = mean length of m-Path $= 2\pi\bar{r} - 1 = 2\pi(11) - 1 = 68.12$ cm

Neglecting the effect of fringing, cross-section area of m-Path is the same as gap → $A_m = A_g = 2\times2 = 4\,cm^2$.

Reluctance of each region: $\mathcal{R}_m = \dfrac{L}{\mu_r \mu_0 A} = \dfrac{68.12\times10^{-2}}{(1200)(4\pi\times10^{-7})(4\times10^{-4})}$

$= 1.29\times10^{6}\ \frac{At}{Wb}$

$$\mathcal{R}_g = \frac{L_g}{\mu_o A} = \frac{1\times10^{-2}}{(4\pi\times10^{-7})(4\times10^{-4})} = 19.894\times10^{6}\ At/_{Wb}$$

$$\mathcal{R} = \mathcal{R}_m + \mathcal{R}_g = 21.023\times10^{6}\ \frac{At}{Wb}.$$

Flux in the m-circuit $\Phi = \dfrac{NI}{\mathcal{R}} = \dfrac{(1500)(4)}{21.023\times10^{6}} = 285.402\times10^{-6}\ Wb$

Flux density: $B_m = B_g = \dfrac{\Phi}{A} = \dfrac{285.402\times10^{-6}}{4\times10^{-4}} = 0.714\ T.$

Problems: 5.2, 5.7, 5.11, 5.20, 5.25, 5.31, 5.35, 5.39,

7: Time-varying electromagnetic fields

summary so far: a) static e-fields are created by charges.

b) " m-fields are produced by charges in motion or steady currents.

c) " e-field is a conservative field ($\vec{\nabla}\times\vec{E}=0$)

d) " m-field is continuous because ($\vec{\nabla}\cdot\vec{B}=0$).

e) " e-field can exist even when there is no static m-field

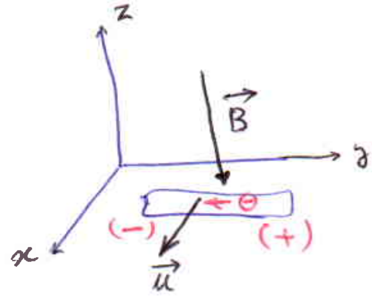
and vice versa.

- In this chapter: we show that a time-varying m-field → produces a time-varying e-field, which is called "induced e-field" or "an emf-producing e-field".

- Induced e-field is not a "conservative" field.
- Induced emf: $\oint \vec{E}_{ind}\cdot d\vec{l}$. (electro-motive force).
- If there exists a t-varying e-field $\overset{\text{in a region}}{\longleftrightarrow}$ there exist a t-varying m-field. in that reg...
- maxwell's eqn: describes the relations between e-field & m-field.

Motional electromotive force:

A conductor moving with a uniform velocity $\vec{u}$ in $\hat{x}$ direction; $\vec{B}=-B\hat{z}$ $\Rightarrow \vec{F}_{\text{on each free electron}} = q_e\vec{u}\times\vec{B} = q_e uB\hat{y}$



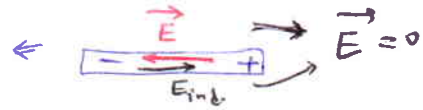
Barnett exp: cut the conductor while it was moving.
The left part had (−) charge, & the right part had (+) charge.

$\Rightarrow \dfrac{\vec{F}}{q_e} = uB\hat{y} = \vec{u} \times \vec{B} = \vec{E} \longrightarrow$ induced electric field (motional e-field).

induced e-field is a nonconservative field.

Boundary condition:

- induced e-field is tangential to the surface of the conductor. As the tangential component of e-field just above the surface $= 0 \Rightarrow$ e-field just beneath the surface of conductor $= 0$

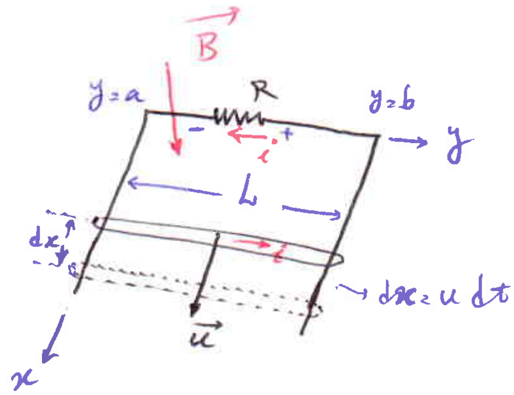


$\Rightarrow \vec{E}$ due to separation of charges $= \vec{E}_{induced} \Rightarrow \vec{E}_{net} = 0$

- sliding conductor: If we make a closed loop, the accumulation of charges will not happen. $\rightarrow$ induced current. in the same direction as induced e-field.

The moving conductor acts like an emf source

$\rightarrow$ Induced emf.



Lorentz force: m-force on moving conductor: $\vec{F}_m = i\vec{L} \times \vec{B} = -Bil\hat{x}$ (opposes the motion of conductor).

$$\vec{F}_{external} = -\vec{F}_m = Bil\hat{x} = i\vec{l} \times \vec{B}$$

$$dW_{ext} = \vec{F} \cdot d\vec{x} = BLi\,dx = BLiu\,dt \quad ; \quad dq = i\,dt$$

$$dW_{ext} = BLu\,dq \Rightarrow e = \frac{dW_{ext}}{dq} = Blu$$

induced emf; motional emf.

"flux cutting action" of a conductor in m-field.

$[Volt] = [J/C]$. $[T = \frac{wb}{m^2}]$ $[m]$ $[m/s]$.

Generally:

$$\vec{F}_{ext} = -\int_c i\,d\vec{l}_c \times \vec{B}$$

$c \rightarrow$ Path of integration in i_{ind}-direction

$d\vec{l}_c \rightarrow$ elemental length of conductor in i direction.

$$\rightarrow dW = \vec{F}_{ext} \cdot d\vec{l} = -i\,d\vec{l} \cdot \int_c d\vec{l}_c \times \vec{B}$$

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$\vec{u}$ does not vary along the length of conductor.

$$i = \frac{dq}{dt}, \quad \vec{u} = \frac{d\vec{\ell}}{dt} \Rightarrow e = \frac{dw}{dq} = -\vec{u} \cdot \int_c d\vec{\ell}_c \times \vec{B}$$

$$= -\int_c \vec{u} \cdot (d\vec{\ell}_c \times \vec{B}) = \int_c \vec{u} \cdot (\vec{B} \times d\vec{\ell}_c)$$

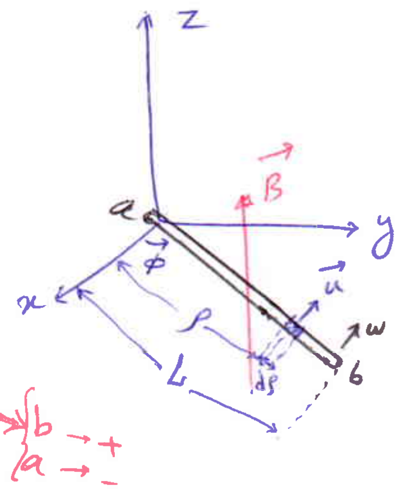
vector identity.

emf$_{ind.}$

$$\boxed{e = \int_c \underbrace{(\vec{u} \times \vec{B})}_{H_{ind} \equiv \text{induced E-field intensity}} \cdot d\vec{\ell}_c}$$

$i_{ind.}$ due to motional emf is in the direction of the E_{ind}.

EX: A copper strip of length $\underline{L}$ pivoted at one end is rotating freely with an angular velocity $\underline{\omega}$ in a uniform m-field. What is induced emf between the two ends of the strip? $\vec{B} = B\hat{z}$

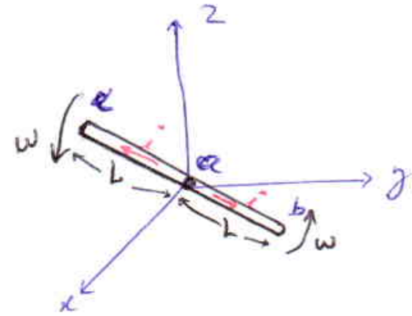


$$e_{ba} = \int_c (\vec{u} \times \vec{B}) \cdot d\vec{\ell}_c$$

$$\vec{u} = \rho\omega\hat{\phi} \Rightarrow \vec{E} = \vec{u} \times \vec{B} = \rho\omega B(\hat{\phi} \times \hat{z})$$
$$= \rho\omega B\hat{\rho} \quad \text{radial direction.}$$

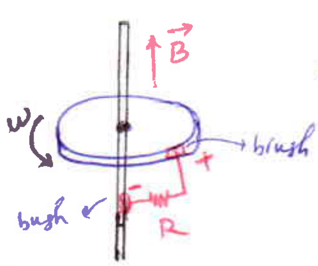
b → +
a → −

$$\rightarrow e_{ba} = \omega B \int_0^L \rho\, d\rho = \frac{1}{2} B\omega L^2.$$



$$\Rightarrow e_{cb} = e_{ca} + e_{ba} = 0.$$

Homopolar Generator: If we use many rotating rod (2L) (each supplying 2I), we can increase the current through $\underline{R}$ considerably. A disc acts like a constant (dc) voltage source.



(Faraday's disc).