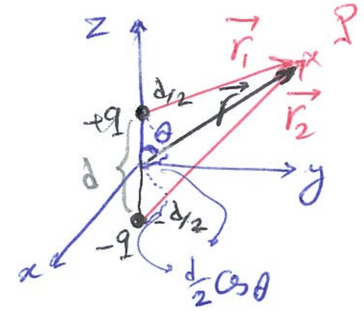
Electric Dipole: Two equal charges of opposite signs that are very close to gesher.



$$V(P(x,y,z)) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

If the charges are symmetrically along the z-axis; $r \gg d$:

$$r_1 \cong r - \frac{d}{2}\cos\theta \quad , \quad r_1 r_2 = r^2 - \left(\frac{d}{2}\cos\theta\right)^2 \simeq r^2$$
$$r_2 \simeq r + \frac{d}{2}\cos\theta$$

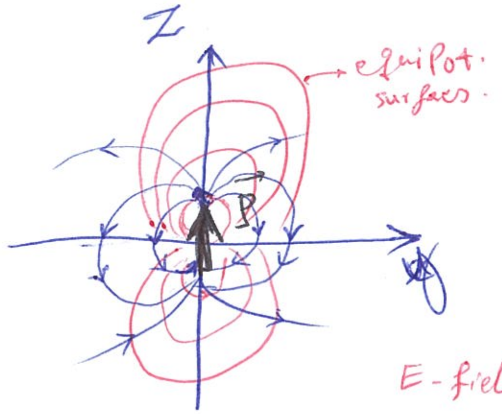
$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{d\cos\theta}{r^2}\right] \Rightarrow \text{if } \theta = 90° \Rightarrow V = 0 \Rightarrow \text{for a plane } \theta = 90°.$$

→ No energy expenditure if q moves in this plane.

$\vec{P}$ = dipole moment vector $= q\vec{d}$ directed from $-q$ to $+q$.

$$\vec{P} = (qd)\hat{z} \Rightarrow V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2} = \frac{\vec{P}\cdot\hat{r}}{4\pi\epsilon_0 r^2} \rightarrow V \propto \frac{1}{r^2} \text{ for dipole!}$$

$* \ V_{\text{Point charge}} \propto \frac{1}{r}$.



Equipotential surfaces: $\frac{\cos\theta}{r^2} = \text{const.}$

E-fields: $\vec{E} = -\vec{\nabla}V = \frac{P}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$

in spherical system

$E \propto \frac{1}{r^3}$

$= 3\cos\theta\,\hat{r} - (\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$

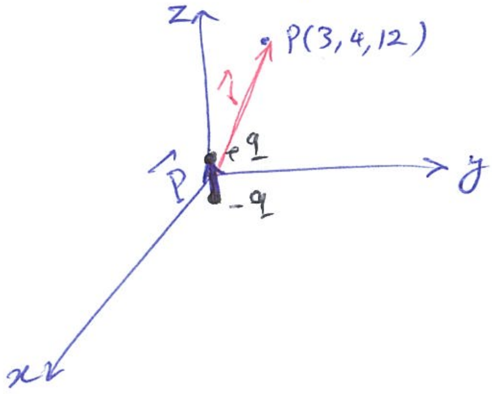
$= 3\cos\theta\,\hat{r} - \hat{z}$

$$\rightarrow \boxed{\vec{E} = \frac{3(\vec{P}\cdot\vec{r})\vec{r} - r^2\vec{P}}{4\pi\epsilon_0 r^5}}$$

$$\begin{cases} \theta = \pm\frac{\pi}{2} \Rightarrow \vec{E} = E\hat{\theta} = -E\hat{z} \rightarrow \vec{E} = -\frac{\vec{P}}{4\pi\epsilon_0 r^3} \\ \theta = 0, \pi \Rightarrow \vec{E} \parallel \vec{P} \end{cases}$$

useful for: explaining dielectric in an E-field.

Ex: Hydrogen atom (1P, 1e): An $\bar{e}$ & a Proton separated by 10^{-11} m, symmetrically arranged along z-axis with z=0 as its disecting plane. $\vec{E}(3,4,12) = ?$



$$\begin{cases} \vec{r} = 3\hat{x} + 4\hat{y} + 12\hat{z} \\ r = 13 \text{ m} \end{cases}$$

$$\vec{P} = qd = (1.6\times10^{-19})(10^{-11})\hat{z} = 1.6\times10^{-30}\hat{z}$$

$$V = \frac{\vec{P}\cdot\vec{r}}{4\pi\epsilon_0 r^3} = \frac{(9\times10^{9})(1.6\times10^{-30}\times12)}{13^3} = 7.865\times10^{-23}\ V$$

$$\vec{E} = \frac{3(\vec{P}\cdot\vec{r})\vec{r} - r^2\vec{P}}{4\pi\epsilon_0 r^5} = \frac{9\times10^{9}}{13^5}\left[3(12\times1.6\times10^{-3})(3\hat{x}+4\hat{y}+12\hat{z}) - 13^2(1.6\times10^{-30})\hat{z}\right]$$

$$= 10^{-24}\left[4.189\hat{x} + 5.585\hat{y} + 10.2\hat{z}\right]\ V/m.$$

Materials in an E-fild (Conductors, Semiconductors, insulators):

A. Conductors in E-fields:

free electron: (a) loosely associated with its nucleus
(b) free to wander throughout the conductor
(c) responds to almost an infinitesimal E-field
(d) continues to move as long as it experiences a force.

Conductor (metal): has a relltively large # of free $\bar{e}$.

Valance electrons: In the space latice of metal crystals, there are 1, 2, or 3 valance electrons per atom that are normally free from nucleus.

→ Thermal motion $\simeq 10^{6}$ m/s randomly. ; No net drift ~~...~~ in any direction in an isolated conductor (No V across this conductor).

↳ These are electrons in charge of current when we have E-field within a

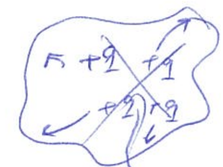
Conductivity: Instead of describing a material in terms of # of free electron we prefer to describe in terms of Conductivity.

conductivity $\propto$ ~~[illegible]~~ / # of valance electrons.

Isolated conductor: # of $+q$ = # of $-q$. (neutral).

excess charge inside a conductor have mutual repulsion till their repulsion is balanced by surface barrier forces. $\rightarrow$ Q goes to surface outside.

$\simeq 10^{-14}$ sec. for copper.



in conductor $\sum Q_{\text{inside}} = 0$

$\downarrow$

$\boxed{\rho_v = 0}$ inside cond.

Isolated conductor in E-field:

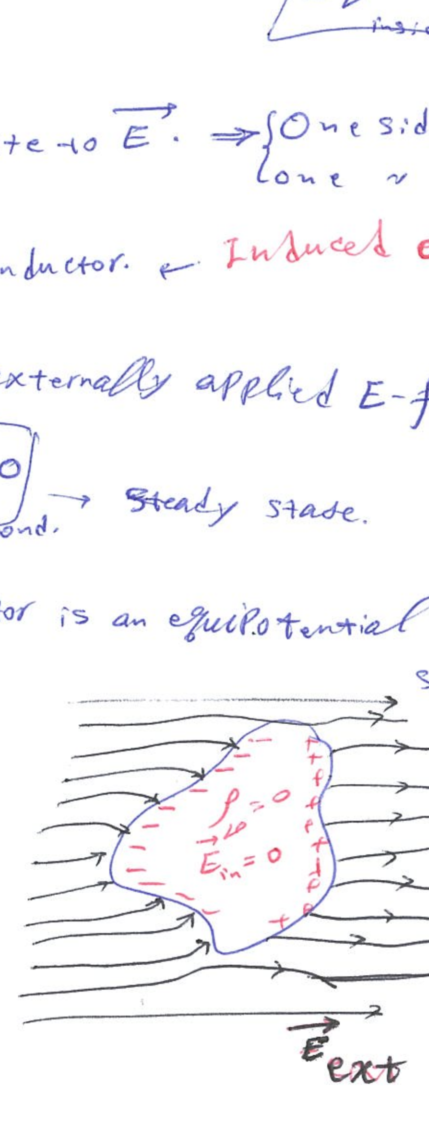
$\vec{F}_e = -|e|\vec{E}$ $\rightarrow$ move $\bar{e}$ in a direction opposite to $\vec{E}$. $\Rightarrow$ One side +, one side − } $\leftarrow$ Induced charge.

$\Leftarrow$ No contact with conductor.

create an E-field equal & opposite to the externally applied E-field.

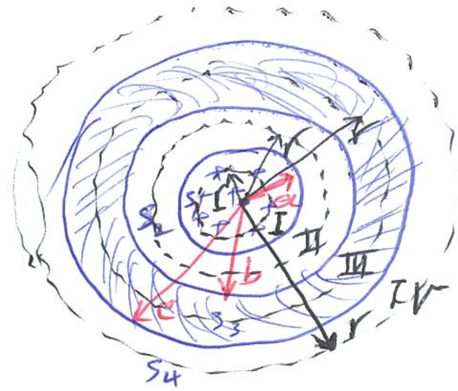
within conductor: $\vec{E}_{ind} = -\vec{E}_{ext}$ $\Rightarrow$ $\boxed{\vec{E}_{net} = 0}$ inside cond. $\rightarrow$ Steady state.

$\vec{E} = -\vec{\nabla} V$ $\rightarrow$ $\boxed{V = \text{const.}}$ inside cond. $\Rightarrow$ Conductor is an equipotential region of space.



EX: A spherical region of radius $\underline{a}$ with uniformly distributed charge ρ_v is in a conducting concentric spherical shell with inner radius b & outer radius $\underline{c}$.

Find $\vec{E}$ intensity in all regions.



I) $r<a$: for a sphere of r: $Q = \rho_v v = \rho_v \frac{4}{3}\pi r^3$

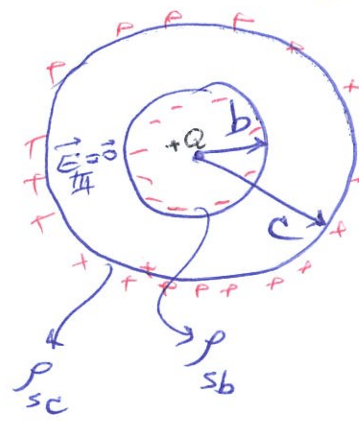
S_I: Gaussian surface $\oint_{S_1} \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0} = \frac{4\rho_v \pi r^3}{3\epsilon_0}$

$\Rightarrow E(4\pi r^2) = \frac{4\pi \rho_v r^3}{3\epsilon_0} \Rightarrow \boxed{\vec{E}_I = \frac{\rho_v r}{3\epsilon_0}\hat{r}}$; $0<r<a$

II) $a\leq r<b$: $\oint_{S_2} \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{\rho_v v}{\epsilon_0} = \frac{\rho_v \frac{4}{3}\pi a^3}{\epsilon_0} \Rightarrow \boxed{\vec{E}_{II} = \frac{\rho_v a^3}{3\epsilon_0 r^2}\hat{r}}$

$a\leq r<b$

III) $b\leq r\leq c$: conductor: $\boxed{\vec{E}_{III} = 0} \Rightarrow$



$Q_{r=b} = -Q_{0<r<a} = \frac{4}{3}\pi a^3 \rho_v$

$4\pi b^2 \rho_{sb} \Rightarrow \boxed{\rho_{sb} = -\frac{a^3}{3b^2}\rho_v}$

IV) $r\geq c$: outer side of conductor has $+Q$. $\Rightarrow Q_{r=c} = -Q_{r=b}$

$\Rightarrow 4\pi c^2 \rho_{sc} = +4\pi b^2 \rho_{sb} \Rightarrow \boxed{\rho_{sc} = \frac{b^2}{c^2}\cdot\frac{a^3}{3b^2}\rho_v = \frac{a^3}{3c^2}\rho_v}$

$\oint_{S_4} \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{\frac{4}{3}\pi a^3 \rho_v}{\epsilon_0} \Rightarrow \boxed{\vec{E}_{IV} = \frac{a^3\rho_v}{3\epsilon_0 r^2}\hat{r}}$

$r\geq c$.

Dielectrics in an E-field: Ideal dielectric (insulator): is a material with no free-electrons in its lattice structure.

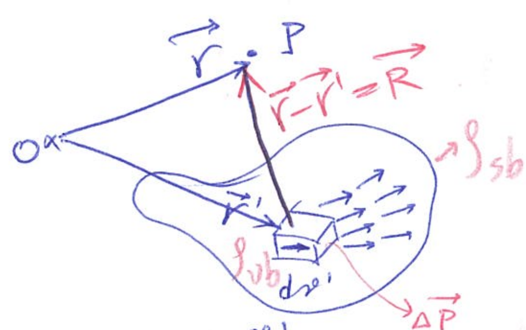
→ No random movements for electrons. ⇒ (+) & (−) charges are so sternly bound that they are inseparable.

⇒ Distortion: under the influence of an e-force the molecules of a dielectric sense ~~these~~ a distortion ~~that~~ their centre of (+) charge no longer coincides with the center of a (−) charge ⟹ Polarization.

⇓

In a Polarized material there are lots of dipoles.

Potential outside a Polarized dielectric:



Polarization Vector = $\dfrac{\#\text{ of dipole moments}}{\text{Volume}}$

$$\vec{P} = \lim_{\Delta v \to 0} \frac{\Delta \vec{p}}{\Delta v} = \frac{d\vec{p}}{dv}$$

$\rightarrow d\vec{p} = \vec{P}\, dv'$ (Pol. vector) $\Rightarrow$ $d V_{\text{pot}} = \dfrac{d\vec{p} \cdot \hat{R}}{4\pi\epsilon_0 R^2} = \dfrac{\vec{P} \cdot \hat{R}}{4\pi\epsilon_0 R^2}\, dv'$ (Pol. vector)

$\vec{R} = \vec{r} - \vec{r}' = |\vec{r} - \vec{r}'|\hat{R} = R\hat{R}$, $\vec{\nabla}'\left(\dfrac{1}{R}\right) = \dfrac{1}{R^2}\hat{R}$

$\rightarrow d V_{\text{pot}} = \dfrac{\vec{P} \cdot \vec{\nabla}'\left(\frac{1}{R}\right)}{4\pi\epsilon_0}\, dv'$; $\vec{P} \cdot \vec{\nabla}'\left(\dfrac{1}{R}\right) = \vec{\nabla}' \cdot \left(\dfrac{\vec{P}}{R}\right) - (\vec{\nabla}' \cdot \vec{P})/R$

$$\rightarrow dV = \frac{1}{4\pi\epsilon_0}\left[\vec{\nabla}' \cdot \left(\frac{\vec{P}}{R}\right) - \frac{\vec{\nabla}' \cdot \vec{P}}{R}\right] dv' \xrightarrow{\int}$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_{v'} \vec{\nabla}' \cdot \left(\frac{\vec{P}}{R}\right) dv' - \int_{v'} \frac{\vec{\nabla}' \cdot \vec{P}}{R}\, dv'\right]$$

by divergence theorem:

$$\rightarrow V = \frac{1}{4\pi\epsilon_0} \oint_{s'} \frac{\vec{P}\cdot\hat{n}}{R}\, ds' - \frac{1}{4\pi\epsilon_0}\int_{v'} \frac{\vec{\nabla}'\cdot\vec{P}}{R}\, dv'$$

surface term — volume term

$\rho_{sb} = \vec{P}\cdot\hat{n}$ → bound surface charge density

$\rho_{vb} = -\vec{\nabla}\cdot\vec{P}$ → bound volume ~ ~

$$\rightarrow \boxed{V = \frac{1}{4\pi\epsilon_0}\left[\oint_{s'} \frac{\rho_{sb}}{R}\, ds' + \int_{v'} \frac{\rho_{vb}}{R}\, dv'\right]}$$

→ Polarization of a dielectric results in bound charges.
Not free, created by separating the charge pairs.

If a dielectric region contains free charge + bound charges:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_v + \rho_{vb}}{\epsilon_0} = \frac{\rho_v - \vec{\nabla}\cdot\vec{P}}{\epsilon_0} \Rightarrow \boxed{\vec{\nabla}\cdot(\epsilon_0\vec{E} + \vec{P}) = \rho_v}$$

→ free charge density

e-field inside the dielectric

we had for free charge that $\vec{\nabla}\cdot\vec{D} = \rho_v \Rightarrow \boxed{\vec{D} = \epsilon_0\vec{E} + \vec{P}}$

effect of Polarization in a dielectric.

→ $\vec{\nabla}\cdot\vec{D}$ will always represent the free charge density of any medium.

→ electric susceptibility.

Linear dielectric: if $\vec{P}$ or $\vec{P} \propto \vec{E} \Rightarrow \boxed{\vec{P} = \epsilon_0 \chi \vec{E}}$

Isotropic dielec : If the electrical properties of the dielectric are indep. of the direction.

Homogeneous ~ ~: all portions of the material are identical.

→ class A dielectric. $\Rightarrow \boxed{\vec{D} = \epsilon_0(1+\chi)\vec{E}}$ $\epsilon_r = 1+\chi$ = relative permittivi[ty] or dielectric constan[t]

$\boxed{\Rightarrow \vec{D} = \epsilon_0\epsilon_r\vec{E} = \epsilon\vec{E}}$ Permitti[vity] of ... di[electric]

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dielect. const. ϵ_r	dielect. strength kV/m
$\epsilon_{r_{air}} = 1$	3000
$\epsilon_{r_{Porcelain}} = 5$	11000
$\epsilon_{r_{Rubber}} = 2.5-3$	25000
$\epsilon_{r_{pure\ water}} = 81$?	low

$$\rightarrow \begin{cases} \vec{\nabla} \times \vec{E} = 0 \\ \vec{\nabla} \cdot \vec{D} = \rho_v \\ \vec{D} = \epsilon \vec{E} \end{cases}$$

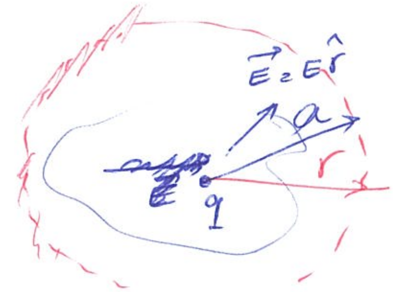
; for all medium.

ρ_v : free volume charge density in the medium.

breakdown: if we increase $\vec{E}$ field to a level at which it pulls electrons completely out of the molecules. $\rightarrow$ ~~becomes~~ act like a conductor.

dielectric strength: E_{max} that the dielectric can withstand before break down.

EX: A point charge q is enclosed in a linear, isotropic, homogeneous dielectric medium of infinite extent. (ϵ_r) calculate $\vec{E}$, $\vec{D}$, Polarization vector $\vec{P}$, bound surface charge density ρ_{sb} & bound volume charge density ρ_{vb}.



In linear medium: $\vec{E} \parallel \vec{D} \parallel \vec{P}$. ; $\vec{E} = E\hat{r}$

$\oint \vec{D} \cdot d\vec{S} = q$ Gauss's law

$4\pi r^2 D = q \Rightarrow \boxed{\vec{D} = \frac{q}{4\pi r^2}\hat{r}}$ → unchanged

$\vec{E} = \frac{\vec{D}}{\epsilon} = \frac{q}{4\pi\epsilon r^2}\hat{r} = \boxed{\frac{q}{4\pi\epsilon_0\epsilon_r r^2}\hat{r}}$ → changed by ϵ_r.

$\vec{P} = \vec{D} - \epsilon_0\vec{E} = \epsilon_r\epsilon_0\vec{E} - \epsilon_0\vec{E} = \epsilon_0(\epsilon_r - 1)\vec{E} = \boxed{\frac{q}{4\pi\epsilon_r r^2}(\epsilon_r - 1)\hat{r}}$ $0 < r < \infty$

surface $r \to 0$

$\vec{\nabla} \cdot \vec{P} = 0 = -\rho_{vb} \Rightarrow \rho_{vb} = 0 \Rightarrow Q_{vb} = 0$; $\rho_{sb} = \vec{P} \cdot (-\hat{r}) = \frac{q}{4\pi\epsilon_r r^2}(1 - \epsilon_r)$

$\infty \rightarrow Q_{sb_\infty} =$

$Q_{sb} = \lim_{a \to 0}[4\pi a^2 \rho_{sb}] = \lim_{a \to 0}[4\pi a^2 \vec{P} \cdot (-\hat{r})] = \frac{-q(\epsilon_r - 1)}{\epsilon_r}$

... $\vec{E}$ field ... $\boxed{Q_{...} = q + Q_{sb} = \frac{q}{\epsilon_r}}$ → $|\vec{E}|$ decreases by $\epsilon_r \uparrow$.

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Semiconductors in an e-field:

semiConductor: a small fraction of the total # of valance electrons are free to move about randomly within space lattice. has → some conductivity

→ If we place some excess charge inside a semicond. it will move toward its outer surface due to repulsive forces, but slower rate than conductor

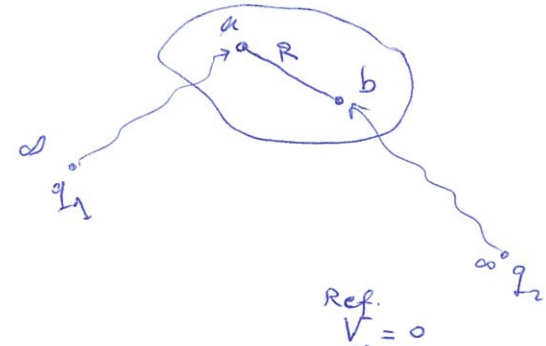
like: Silican, Germanium

semiCond. in an e-field: The motion of free electrons will cancel the external applied e-field by producing $\vec{E}_{in}$. $\Rightarrow \vec{E}_{net} = 0 = \vec{E}_{ext} + \vec{E}_{int}$

→ $\boxed{\text{Electrostatic: conductors \& dielectrics.}}$

Energy stored in an e-field:

Two methods: - Sources
- Field quantities



$$\begin{cases} W_1 = 0 \\ W_2 = q_2 V_{b,a} = \dfrac{q_1 q_2}{4\pi \epsilon R} \end{cases} \Rightarrow W_{tot} = W_1 + W_2 = \frac{q_1 q_2}{4\pi \epsilon R}$$

↓ mutual Pot. energy of Two Point charges (d) in any medium ϵ.

$$\begin{cases} W_1 = q_1 V_{a,b} = \dfrac{q_1 q_2}{4\pi \epsilon R} \\ W_2 = 0 \end{cases}$$

$$\begin{cases} W_1 = 0 \\ W_2 = q_2 V_{b,a} \\ W_3 = q_3 (V_{c,a} + V_{c,b}) \end{cases} \Rightarrow W = \frac{1}{4\pi\epsilon}\left[\frac{q_2 q_1}{R_{21}} + \frac{q_3 q_1}{R_{31}} + \frac{q_3 q_2}{R_{32}}\right]$$

$$\begin{cases} W_3 = 0 \\ W_2 = q_2 (V_{b,c}) \\ W_1 = q_1 (V_{a,c} + V_{a,b}) \end{cases} \Rightarrow W = \frac{1}{4\pi\epsilon}\left[\frac{q_2 q_3}{R_{23}} + \frac{q_1 q_3}{R_{13}} + \frac{q_1 q_2}{R_{12}}\right]$$

$$\Rightarrow W = \frac{1}{2}\left[q_1 (V_{a,c} + V_{a,b}) + q_2 (V_{b,a} + V_{b,c}) + q_3 (V_{c,a} + V_{c,b})\right]$$

$$\Rightarrow W = \frac{1}{2}\left[q_1 V_1 + q_2 V_2 + q_3 V_3\right] = \frac{1}{2}\sum_{i=1}^{3} q_i V_i$$

Pot. at Point a, " b, " " c

system of n Point charges $\Rightarrow W = \frac{1}{2}\sum_{i=1}^{n} q_i V_i$ → electrostatic Pot. energy for a group of Point charges in their mutual field.

$$\boxed{W = \frac{1}{2}\int_v \rho_v V \, dv}$$

ρ_v → vol. charge density

→ $W = \frac{1}{2}\int_s \rho_s V \, ds$

$W = \frac{1}{2}\int_\ell \rho_\ell V \, dl$

EX: A metallic sphere of radius 10 cm has a $\rho_s = 10 \frac{nC}{m^2}$ → energy stored in this system:

Pot. on the surface of the sphere: $V = \int_s \frac{\rho_s ds}{4\pi\epsilon_0 R}$ ($ds \to R^2 \sin\theta \, d\theta d\phi$) $= (9\times10^{9})(10\times10^{-9})(0.1)\int_0^{\pi}\sin\theta\, d\theta \int_0^{2\pi} d\phi$

$\Rightarrow V = 113.1^{V} =$ const.

$$W = \frac{1}{2}\int_s \rho_s V \, ds = \frac{1}{2} Q_t V = \frac{1}{2}\left[\underbrace{4\pi R^2 \rho_s}_{1.257 \text{ nC}}\right](113.1) = 71.08\times10^{-9} \text{ J}$$

$= 71.08$ nJ.

$W = \frac{1}{2}\int_v V(\underbrace{\vec{\nabla}\cdot\vec{D}}_{\rho_v})\, dv$; $V(\vec{\nabla}\cdot\vec{D}) = \vec{\nabla}\cdot(V\vec{D}) - \vec{D}\cdot\vec{\nabla}V$

$$= \frac{1}{2}\left[\int_v \vec{\nabla}\cdot(V\vec{D})\, dv - \int_v \vec{D}\cdot(\vec{\nabla}V)\, dv\right]$$

$$= \frac{1}{2}\left[\oint_s V\vec{D}\cdot d\vec{s} - \int_v \vec{D}\cdot(\vec{\nabla}V)\, dv\right] \Rightarrow \boxed{W = +\frac{1}{2}\int_v \vec{D}\cdot\vec{E}\, dv}$$

→ $\int$ over all space $R\to\infty$.

$\oint_s$ → arbitrary surface that bound v ⇒ V, D small → $\oint \to 0$

energy density ← $w = \frac{1}{2}\vec{D}\cdot\vec{E}$ energy Per unit Volume.

$= \frac{1}{2}\epsilon E^2 = \frac{1}{2\epsilon}D^2$, $W = \int_v w\, dv$ → $\boxed{\frac{1}{2}\rho_v V = w}$ energy dens

$W = \frac{1}{2}\rho_v V$ suggest that energy density is Non Zero only where charges exist.

EX: Resolve last example: A metallic sphere with $\rho = 10\ \frac{nC}{m^2}$. Energy stored in the system:

$$\oint \vec{D}\cdot d\vec{S} = Q_t \Rightarrow \vec{D} = \frac{Q_t}{4\pi r^2}\hat{r} = \frac{0.1\times 10^{-9}}{r^2}\hat{r}\ \left[\frac{C}{m^2}\right]$$

energy density:

$$w = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{(0.1)^2\times 10^{-18}}{2\epsilon_0 r^4} \rightarrow W = \int_{0.1}^{\infty}\frac{(0.1)^2\times 10^{-18}}{2\epsilon_0 r^4}r^2 dr \int_0^{\pi}\sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$\rightarrow W = \left(\frac{(0.1)^2\times 10^{-18}}{2\epsilon_0}\left(-\frac{1}{r}\right)\Bigg|_{0.1}^{\infty}\right)\underbrace{\left(-\cos\theta\Big|_0^{\pi}\right)}_{-(-1-1)=+2}(2\pi)$$

$$= \frac{4\pi(0.01)\times 10^{-18}}{2\epsilon_0}\left(-\frac{1}{\infty}+\frac{1}{0.1}\right) = \frac{(4\pi)(2\pi)(0.01)\times 10^{-18}}{4\pi\epsilon_0}(0+10)$$

$$= 8\pi^2(0.01)(10)\times 9\times 10^{9}\times 10^{-18} = 71.06\ nJ.$$

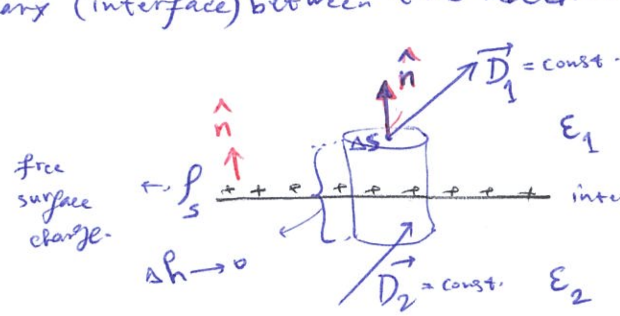
Boundary Conditions:

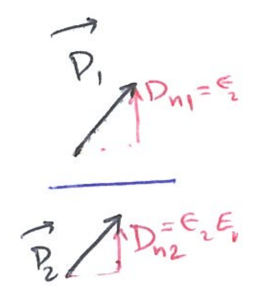
Behavior of E-fields at boundary (interface) between two Media.

Normal component of $\vec{D}$:

$\oint \vec{D}\cdot d\vec{S} = Q_f$

$\Delta q = \Delta S\,\rho_s$.





$\vec{D_1}\cdot\hat{n}\Delta S - \vec{D_2}\cdot\hat{n}\Delta S = \rho_s\Delta S$

$\Rightarrow \hat{n}\cdot(\vec{D_1}-\vec{D_2}) = \rho_s$

$\Rightarrow \boxed{D_{n1}-D_{n2}=\rho_s}$ → The normal components of the e-flux density are discontinuous if a free surface charge density exists at the interface.

$\vec{D} = \epsilon\vec{E} \Rightarrow \hat{n}\cdot(\epsilon_1\vec{E_1}-\epsilon_2\vec{E_2}) = \rho_s \Rightarrow \boxed{\epsilon_1 E_{n1}-\epsilon_2 E_{n2}=\rho_s}$