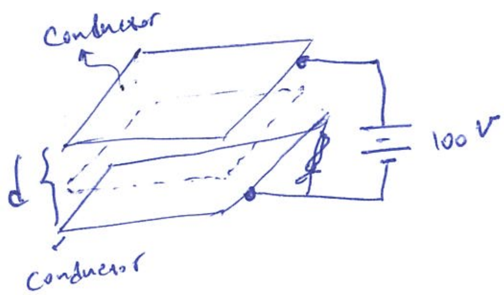
11

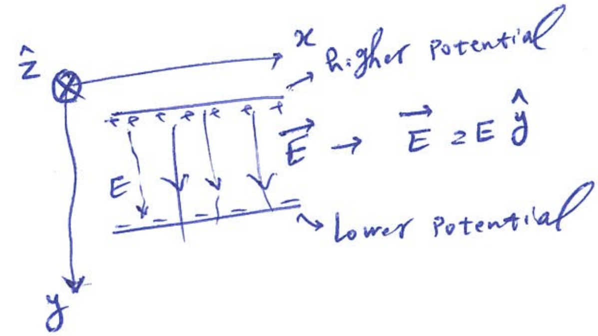
Field: A field is a function that describes a physical quantity at all points in space. physical quantities are scalar or vector (→ van de graaff)

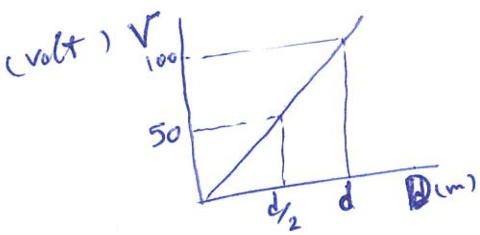
⇒ Fields are scalar or vector.

scalar fields: specified by a single number at each point.

"temperature & pressure of a gas", "altitude above sealevel", & "electric potential"


→ Potential in the space between two conductors is specified with only a number.

Vector fields: specified by both a "magnitude" & a "direction" at each point in space.

"velocity, acceleration, force, electric field, ..."

Static fields: fields does NOT vary with time. ⇒ time-invariant fields

fields produced by "stationary charges" → (electrostatics)

— — — "steady motion of charges → (magnetostatics). → a wire, battery + compass

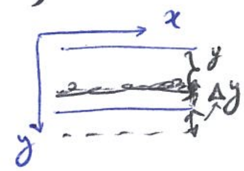
Time-varying fields: fields varies with time. ⇒

"time-varying electromagnetic fields".

## Vector calculus:

Derivative of a scalar (field) function: $f = f(s)$ (indep. variable) , $f = f(u(s), v(s))$ (dep. variable, indep. variable)

$$\frac{df}{ds} = \lim_{\Delta s \to 0} \frac{f(s+\Delta s) - f(s)}{\Delta s} \quad \Longrightarrow \text{exp: e. Pot. } \frac{dV}{dy} = \lim_{\Delta y \to 0} \frac{V(y+\Delta y) - V(y)}{\Delta y}$$




$$\frac{df}{ds} = \frac{\partial f}{\partial u}\frac{du}{ds} + \frac{\partial f}{\partial v}\frac{dv}{ds}$$

$$\frac{\partial f}{\partial u} = \lim_{\Delta u \to 0} \frac{f(u+\Delta u, v) - f(u, v)}{\Delta u}$$

$$\frac{\partial f}{\partial v} = \lim_{\Delta v \to 0} \frac{f(u, v+\Delta v) - f(u, v)}{\Delta v}$$

Derivative of a vector field: $\vec{F} = \vec{F}(s)$ ; $\vec{F} = \vec{F}(x, y, z)$

$$\frac{d\vec{F}}{ds} = \lim_{\Delta s \to 0} \frac{\vec{F}(s+\Delta s) - \vec{F}(s)}{\Delta s}$$

$$\frac{\partial \vec{F}}{\partial x} = \lim_{\Delta x \to 0} \frac{\vec{F}(x+\Delta x, y, z) - \vec{F}(x, y, z)}{\Delta x}$$

$$\frac{\partial \vec{F}}{\partial y} = \lim_{\Delta y \to 0} \frac{\vec{F}(x, y+\Delta y, z) - \vec{F}(x, y, z)}{\Delta y}$$

$\vdots$

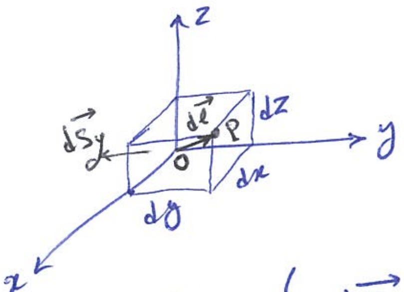
→ If a scalar or a vector field can be differentiated, it is Integrable as w

Divergence: the physical interpretation: integration of a vector over a surface.

to perform this $\int$ ⟹ we need to define the differential surface element.

⟶ Differential elements of length, surface & volume:

Differential elements in Rectangular c.s. : $d\vec{l}$, $d\vec{S}$, $dv$



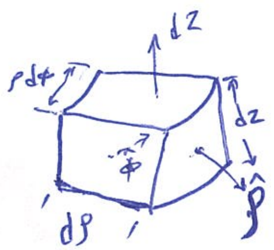

$dv = dx\,dy\,dz \xrightarrow{\text{diff.}}$ volume element $\rightarrow$ enclosed by 6 diff. surfaces.

$\rightarrow$ Each surface is defined by a unit vector normal to that surface. ($\pm\hat{n}$).

diff. surface elements.
$$\begin{cases} d\vec{S}_x = dy\,dz\,\hat{x} \\ d\vec{S}_y = dx\,dz\,\hat{y} \\ d\vec{S}_z = dx\,dy\,\hat{z} \end{cases}$$

; $d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

Differential elements in cylindrical c.s. : $d\vec{l}$, $d\vec{S}$, $dv$

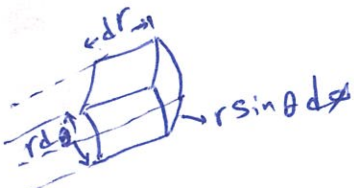



$dv = \rho\,d\rho\,d\phi\,dz$

$$\begin{cases} d\vec{S}_\rho = \rho\,d\phi\,dz\,\hat{\rho} \\ d\vec{S}_\phi = d\rho\,dz\,\hat{\phi} \\ d\vec{S}_z = \rho\,d\rho\,d\phi\,\hat{z} \end{cases}$$

$d\vec{l} = d\rho\,\hat{\rho} + \rho\,d\phi\,\hat{\phi} + dz\,\hat{z}$

Differential elems in spherical c.s. : $d\vec{l}$, $d\vec{S}$, $dv$




$dv = r^2\,dr\,\sin\theta\,d\theta\,d\phi$

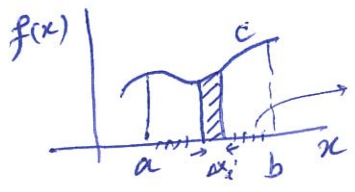
$$\begin{cases} d\vec{S}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{r} \\ dS_\theta = r\,dr\,\sin\theta\,d\phi\,\hat{\theta} \\ dS_\phi = r\,dr\,d\theta\,\hat{\phi} \end{cases}$$

$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

Integration over a line, surface or volume: $V = \int_C \vec{E}\cdot d\vec{l}$ , $I = \int_S \vec{J}\cdot d\vec{A}$

- Line integral: $f(x)$ = a continuous, single valued function between a, b:

1D

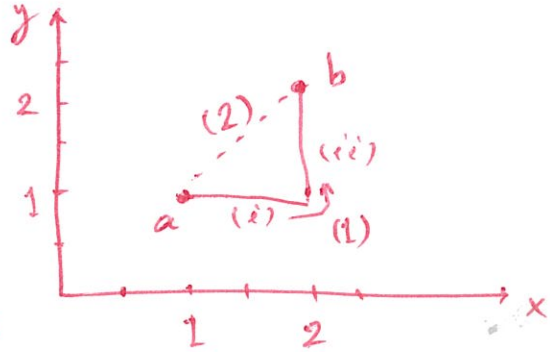



$\rightarrow$ n small segment

$$\int_a^b f(x)\,dx = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f_i\,\Delta x_i$$

$f_i \rightarrow$ value of $f(x)$ for segment $\Delta x_i$, $\Delta x_i \to 0$.

Example: calculate the line integral of the function $\vec{v} = y^2 \hat{x} + 2x(y+1)\hat{y}$ from the point $\vec{a} = (1,1,0)$ to the point $\vec{b} = (2,2,0)$ along the paths (1) and (2) in figure below.




what is $\oint \vec{v} \cdot d\vec{l}$ for the loop that goes from $\vec{a}$ to $\vec{b}$ along (1) and returns to a along (2)?

$d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$\Rightarrow$ Path (1) consists of two parts: $\begin{cases} (i)\ dy = dz = 0 \\ (ii)\ dx = dz = 0 \end{cases}$

(i) $d\vec{l} = dx\,\hat{x}$, $y = 1 \Rightarrow \vec{v} \cdot d\vec{l} = y^2\,dx = dx \rightarrow \int \vec{v} \cdot d\vec{l} = \int_1^2 dx = 1$

(ii) $d\vec{l} = dy\,\hat{y}$, $x = 2 \Rightarrow \vec{v} \cdot d\vec{l} = 2x(y+1)\,dy = 4(y+1)\,dy$

$$\Rightarrow \int \vec{v} \cdot d\vec{l} = 4\int_1^2 (y+1)\,dy = 10.$$

$\Rightarrow$ By path (1): $\int_a^b \vec{v} \cdot d\vec{l} = 1 + 10 = 11$

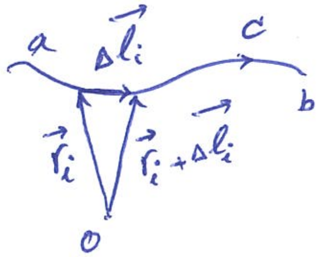
$\Rightarrow$ Path (2): $x = y$, $dx = dy$, $dz = 0$

$d\vec{l} = dx\,\hat{x} + dy\,\hat{y}$, $\vec{v} \cdot d\vec{l} = x^2\,dx + 2x(x+1)\,dx = (3x^2 + 2x)\,dx$ Just one variable!

$$\Rightarrow \int_a^b \vec{v} \cdot d\vec{l} = \int_1^2 (3x^2 + 2x)\,dx = (x^3 + x^2)\Big|_1^2 = 10$$

$\Rightarrow$ loop out (1) & back (2) $\Rightarrow \oint \vec{v} \cdot d\vec{l} = 11 - 10 = 1$

3D




f : scalar field

n segment of length vectors

$$\int_C f \, d\vec{l} = \lim_{\substack{n \to \infty \\ \vec{\Delta l_i} \to 0}} \sum_{i=1}^{n} f_i \, \vec{\Delta l_i} \quad \to \text{vector}$$

→ value of scalar f. $f(x)$ within length segment $\vec{\Delta l_i}$.

$$\int_C \vec{F} \cdot d\vec{l} = \lim_{\substack{n \to \infty \\ \vec{\Delta l_i} \to 0}} \sum_{i=1}^{n} \vec{F_i} \cdot \vec{\Delta l_i} \quad \to \text{scalar}$$

$$\int_C \vec{F} \times d\vec{l} = \lim_{\substack{n \to \infty \\ \vec{\Delta l_i} \to 0}} \sum_{i=1}^{n} \vec{F_i} \times \vec{\Delta l_i} \quad \to \text{vector}$$

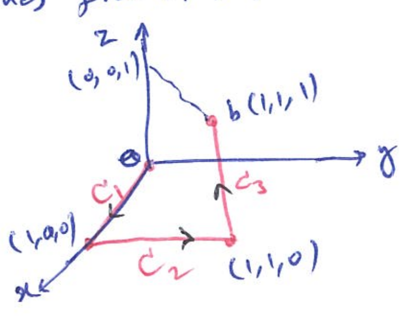
For closed path → $\oint$.

Ex: $\vec{A} = (4x + 9y)\hat{x} - 14yz\,\hat{y} + 8x^2 z\,\hat{z}$; calculate $\int_C \vec{A} \cdot d\vec{l}$ from a$(0,0,0)$ to b$(1,1,1)$ along path c:

path c: $x = t,\ y = t^2,\ z = t^3 \Rightarrow \vec{A} \cdot d\vec{l} = (4x + 9y)\,dx - 14yz\,dy + 8x^2 z\,dz$

$$\begin{cases} x = t \to dx = dt \\ y = t^2 \Rightarrow dy = 2t\,dt \\ z = t^3 \Rightarrow dz = 3t^2\,dt \end{cases} \Rightarrow \boxed{\int_C \vec{A} \cdot d\vec{l} = \int_{t=0}^{1} [4t + 9t^2 - 28t^6 + 24t^7]\,dt = 4}$$

c straight lines from a$(0,0,0) \to (1,0,0) \to (1,1,0) \to$ b$(1,1,1)$




$$\int \vec{A} \cdot d\vec{l} = \int_{C_1} \vec{A} \cdot d\vec{l} + \int_{C_2} \vec{A} \cdot d\vec{l} + \int_{C_3} \vec{A} \cdot d\vec{l}$$

$$\int_{C_1} \vec{A} \cdot d\vec{l} = \int_{x=0}^{1} 4x\,dx = \frac{4}{2} x^2 \Big|_0^1 = 2(1 - 0) = 2$$

→ $y = 0,\ dy = 0,\ z = 0,\ dz = 0$, $0 \le x \le 1$

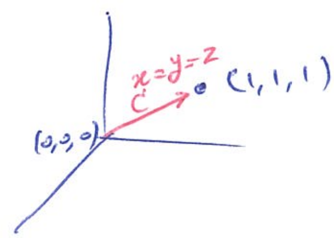
$$\int_{C_2} \vec{A}\cdot d\vec{l} = 0$$

$\searrow x=1,\ dx=0,\ z=0,\ dz=0,\ 0\le y\le 1$

$$\Rightarrow \boxed{\int_C \vec{A}\cdot d\vec{l} = 2+0+4 = 6.}$$

$$\int_{C_3} \vec{A}\cdot d\vec{l} = \int_{z=0}^{1} 8z\,dz = 4$$

$\searrow x=1,\ dx=0,\ y=1,\ dy=0,\ 0\le z\le 1$




c straight line from $P(0,0,0)$ to $Q(1,1,1)$:

$$c:\begin{cases} y = x \\ z = x \end{cases} \Rightarrow \int_c \vec{A}\cdot d\vec{l} = \int_0^1 (13x - 14x^2 + 8x^3)\,dx$$

$$\boxed{= 3.833.}$$

path-dependent!

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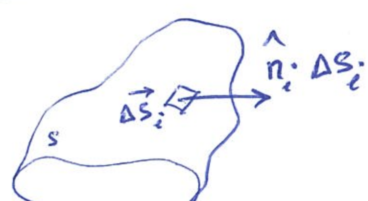
## Surface Integral:

$$\begin{cases} f = \text{scalar field } f \\ \vec{F} = \text{vector field} \end{cases}$$

we devide S into n small surfaces. $n \to \Delta S_i$

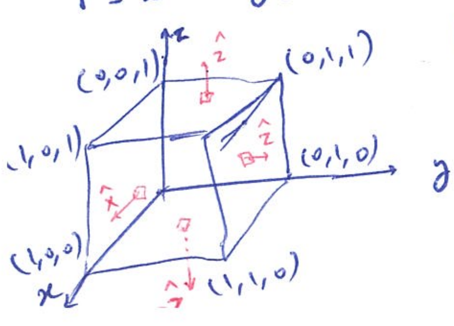



$$\int_S f\,d\vec{S} = \lim_{\substack{n\to\infty \\ \Delta\vec{S}_i \to 0}} \sum_{i=1}^{n} f_i\,\Delta\vec{S}_i \quad \to \text{vector}$$

$$\int_S \vec{F}\cdot d\vec{S} \quad ; \quad \int_S \vec{F}\times d\vec{S} = \lim_{\substack{n\to\infty \\ \Delta\vec{S}_i \to 0}} \sum_{i=1}^{n} \vec{F}_i \times \Delta\vec{S}_i$$

Ex: Evaluate $\oint \vec{r}\cdot d\vec{S}$ over the closed surface of a cube bounded by $0 \le x, y, z \le 1$, $\vec{r}$ = position vector of any point on the surface of cube:

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$




six surfaces:

$$\begin{cases} S_1: x=1 \Rightarrow d\vec{S} = dy\,dz\,\hat{x} \Rightarrow \int_{S_1} \vec{r}\cdot d\vec{S} = \int_0^1 dy \int_0^1 dz = 1 \\ S_2: x=0 \Rightarrow d\vec{S} = -dy\,dz\,\hat{x} \Rightarrow \int_{S_2} \vec{r}\cdot d\vec{S} = \int (\underset{0}{x}\hat{x} + y\hat{y} + z\hat{z})\cdot(-dy\,dz)\hat{x} = 0 \end{cases}$$

$$\begin{cases} S_3: y=1 \Rightarrow d\vec{S} = dx\,dz\,\hat{y} \Rightarrow \int_{S_3} \vec{r}\cdot d\vec{S} = 1 \\ S_4: y=0 \Rightarrow d\vec{S} = -dx\,dz\,\hat{y} \Rightarrow \int_{S_4} \vec{r}\cdot d\vec{S} = 0 \end{cases}$$

$$\begin{cases} S_5: z=1 \Rightarrow d\vec{s} = dx\,dy\,\hat{z} \rightarrow \int_{S_5} \vec{r}\cdot d\vec{s} = 1 \\ S_6: z=0 \Rightarrow d\vec{s} = -dx\,dy\,\hat{z} \rightarrow \int_{S_5} \vec{r}\cdot d\vec{s} = 0 \end{cases}$$

$$\Rightarrow \oint_S \vec{r}\cdot d\vec{s} = 3$$

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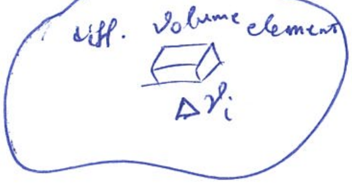
**Volume Integral:** scalar volume integral:




$$\int_v f\,dv = \lim_{\substack{n\to\infty \\ \Delta v_i \to 0}} \sum_{i=1}^{n} f_i v_i$$

volume integral of vector field $\vec{F}$:

$$\int \vec{F}\,dv = \lim_{\substack{n\to\infty \\ \Delta v_i \to 0}} \sum_{i=1}^{n} \vec{F}_i \,\Delta v_i$$

Ex: electron density distribution within a spherical volume with radius of $2^m$ is given: $n_e = \left(\frac{1000}{r}\right)\cos\left(\frac{\phi}{4}\right) \cdot \frac{\text{electron}}{m^3}$. Find the total charge enclosed in this surface.

# of electrons in that volume:

$$N = \int_v n_e\,dv = \int_v \frac{1000}{r}\cos\frac{\phi}{4}\,dv = \int_0^2 \frac{1000}{r} r^2 dr \int_0^\pi \sin\theta\,d\theta \int_0^{2\pi} \cos\frac{\phi}{4}\,d\phi = 16\times10^3 \,e$$

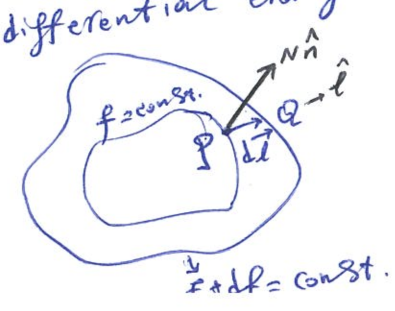
$$\Rightarrow Q = 16\times10^3\,(-1.6\times10^{-19}\,C) = -2.56\times10^{-15}\,C.$$

---

- Gradient of a scalar function: $f(x,y,z)$ - real-valued, differentiable

differential change in $f$ from $P$ to $Q$ is: $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

$$= \left[\frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}\right]\cdot\left[dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right]$$




$$df = \vec{N}\cdot d\vec{l} \Rightarrow \boxed{\frac{df}{dl} = \vec{N}\cdot\frac{d\vec{l}}{dl} = N\hat{n}\cdot\hat{l}}$$

change in the function $f$ : is max if $\hat{l}$ & $\vec{N}$ are co linear :

$$\vec{N}\cdot\hat{l} = N\cos\theta,\ \theta = 0 \Rightarrow \left.\frac{df}{dl}\right|_{max} = N$$

$\frac{df}{dl}$ is max when $\hat{l}$ is Normal to the surface $f(x,y,z) = $ Const.

$\Rightarrow \vec{N}$ is the gradient of $f(x,y,z)$. $\rightarrow \vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$

del or Nabla

$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \Rightarrow$ gradient operator in rectangular C.S.

$\Rightarrow$ differential of a function: $df = \vec{\nabla} f\cdot d\vec{l} \Rightarrow \frac{df}{dl} = \vec{\nabla} f\cdot\hat{l}$ → rate of change in a given direction.

will be used in chap 3 & 4 to find the change of a scalar function in a given direction.

directional derivative of $f$ along $\hat{l}$.

Gradient in cylindrical C.S.: $df = \frac{\partial f}{\partial \rho}d\rho + \frac{\partial f}{\partial \phi}d\phi + \frac{\partial f}{\partial z}dz$

$$\vec{\nabla} f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

" " spherical C.S.: $\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$

Ex: $\vec{r} = \rho\hat{\rho} + z\hat{z}$ ; find $\vec{\nabla} r$ where $r$ is the magnitude of the position vector in cylindrical C.S.: $r = [\rho^2 + z^2]^{1/2}$

$$\frac{\partial r}{\partial \rho} = \frac{\rho}{r},\ \frac{\partial r}{\partial \phi} = 0,\ \frac{\partial r}{\partial z} = \frac{z}{r} \Rightarrow \vec{\nabla} r = \frac{\rho}{r}\hat{\rho} + \frac{z}{r}\hat{z} = \frac{\vec{r}}{r} = \hat{r}$$

we will use this later.

Divergence of a vector field: ( integration of a vector field over a surface)

$\vec{F}\cdot d\vec{S}$ ≡ outward flow of the vector field $\vec{F}$ through the surface $\vec{S}$

$\hat{s}$ Points away from the volume enclosed.

$\oint \vec{F}\cdot d\vec{S}$ = net outward flow/flux of a vector field $\vec{F}$ from volume $\Delta v$.

$$\begin{cases} \text{outward flow of } \vec{F} \text{ through the face in } +x \text{ direction} = \left(F_x + \frac{\partial F_x}{\partial x}\frac{\Delta x}{2}\right)\Delta y \Delta z \\ \text{'' '' -- -- } -x \text{ dir.} = -\left(F_x - \frac{\partial F_x}{\partial x}\frac{\Delta x}{2}\right)\Delta y \Delta z \end{cases}$$

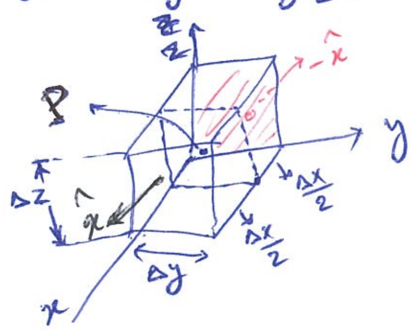
→ Taylor expansion neglecting higher order

↪ net outward flux in $x$-dir $= \frac{\partial F_x}{\partial x}\Delta x \Delta y \Delta z = \frac{\partial F_x}{\partial x}\Delta v$

→ the same in $y$, & $z$ direction: $\oint_S \vec{F}\cdot d\vec{S} = \left[\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right]\Delta v$ → scalar

⇒ define: $f = \lim_{\Delta v \to 0} \frac{1}{\Delta v}\oint_S \vec{F}\cdot d\vec{S}$

scalar field at Point P.




$$\Rightarrow f = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}.$$

$$= \left[\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right]\cdot\left[F_x\hat{x} + F_y\hat{y} + F_z\hat{z}\right]$$

scalar ← $f = \boxed{\vec{\nabla}\cdot\vec{F}}$ → divergence of $\vec{F}$. $= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$ in Rectang c.s.

* By enclosing a point P within any arbitrary small volume we can calculate the net outward flow of a vector field by its divergence at that Point.

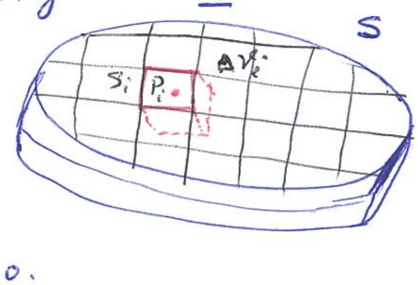
$$\vec{\nabla}\cdot\vec{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho F_\rho] + \frac{1}{\rho}\frac{\partial}{\partial\phi}[F_\phi] + \frac{\partial}{\partial z}[F_z]$$ : cylindrical c.s.

$$\vec{\nabla}\cdot\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 F_r] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta\, F_\theta] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}[F_\phi]$$ spherical c.s.

$\vec{\nabla}\cdot\vec{F} = 0 \Rightarrow \vec{F}$ is a continuous or <u>solenoidal</u> vector field.

Divergence Theorem: for elementary volume $\Delta v_i$, enclosing a point $\underline{P_i}$ bounded by surface $\underline{S_i}$

$$\vec{\nabla}\cdot\vec{F}_i = \lim_{\Delta v_i \to 0} \frac{1}{\Delta v_i}\oint_{S_i} \vec{F}\cdot d\vec{S}$$

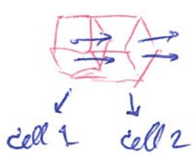



rewrite

$$\Rightarrow \oint_{S_i} \vec{F}\cdot d\vec{S} = \vec{\nabla}\cdot\vec{F}_i\,\Delta v_i + \epsilon_i\,\Delta v_i$$

$\rightarrow \epsilon_i \to 0$ as $\Delta v_i \to 0$.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\oint_{S_i}\vec{F}\cdot d\vec{S} = \lim_{n\to\infty}\sum_{i=1}^{n}\vec{\nabla}\cdot\vec{F}_i\,\Delta v_i + \lim_{n\to\infty}\sum_{i=1}^{n}\epsilon_i\,\Delta v_i$$




net flux leaving one cell cancels the net flux leaving other

$\Rightarrow$ Non-zero terms in the sum correspond to the outermost cells $\rightarrow$ surface $\underline{S}$

$\Downarrow$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\oint_{S_i}\vec{F}\cdot d\vec{S} = \oint_S \vec{F}\cdot d\vec{S}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\vec{\nabla}\cdot\vec{F}_i\,\Delta v_i = \int_v \vec{\nabla}\cdot\vec{F}\,dv$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\epsilon_i\,\Delta v_i \to 0$$

$$\Rightarrow \boxed{\int_v \vec{\nabla}\cdot\vec{F}\,dv = \oint_S \vec{F}\cdot d\vec{S}}$$ net outward flux

for a continuously differentiable vector field the <u>net outward flux</u> from a closed surface = integral of the divergence throughout the region bounded by the surface.

$\downarrow$

use alot in E&M problems.

"Verify the divergence theorem"

EX (2.20): from the book. $\vec{D} = 3x^2\hat{x} + (3y+z)\hat{y} + (3z-x)\hat{z}$ in the region bounded by cylinder $x^2+y^2=9$, and the planes $x=0, y=0, z=0$ and $z=2$.

$\vec{\nabla}\cdot\vec{D} = 6x+6$ . $\int \vec{\nabla}\cdot\vec{D}\,dv = \int_v [6x+6]\,dv = \int^3 6\rho^2 d\rho \int^{\frac{\pi}{2}} \cos\phi\, d\phi \int^2 dz +$

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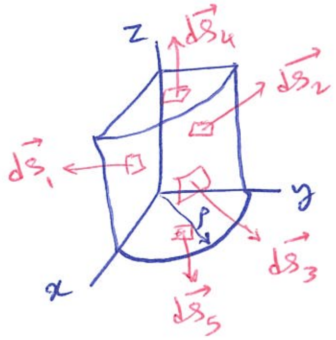
$$\cdots + \int_0^3 6\rho\, d\rho \int_0^{\pi/2} d\phi \int_0^2 dz = 192.82$$ we have to do 5 surface integrals:




$$\rightarrow y=0 \Rightarrow d\vec{S}_1 = -dx\,dz\,\hat{y} \Rightarrow \int_{S_1} \vec{D}\cdot d\vec{S}_1 = -\int_{x=0}^{3}\int_{z=0}^{2}(3y+z)\,dx\,dz = -$$

$$\rightarrow x=0 \Rightarrow d\vec{S}_2 = -dy\,dz\,\hat{x} \Rightarrow \int_{S_2} \vec{D}\cdot d\vec{S}_2 = -\int_{y=0}^{3}\int_{z=0}^{2} 3x^2\,dy\,dz = 0$$

$$\rightarrow \rho = 3 \Rightarrow d\vec{S}_3 = 3\,d\phi\,dz\,\hat{\rho} \Rightarrow \int_{S_3} \vec{D}\cdot d\vec{S}_3 = \int_{\phi=0}^{\pi/2}\int_{z=0}^{2} 3D_\rho\,d\phi\,dz$$

$$\begin{cases} D_\rho = D_x\cos\phi + D_y\sin\phi = 3x^2\cos\phi + (3y+z)\sin\phi \\ x = 3\cos\phi,\ y = 3\sin\phi \end{cases}$$

$$\Rightarrow \int_{S_3} \vec{D}\cdot d\vec{S}_3 = \int_{\phi=0}^{\pi/2}\int_{z=0}^{2}\left[3x^2\cos\phi + (3y+z)\sin\phi\right]3\,d\phi\,dz$$

$$= 156.41$$

$$\rightarrow z=2 \Rightarrow d\vec{S}_4 = \rho\,d\rho\,d\phi\,\hat{z} \Rightarrow \int_{S_4} \vec{D}\cdot d\vec{S}_4 = \int_{\rho=0}^{3}\int_{\phi=0}^{\pi/2}(6-x)\rho\,d\rho\,d\phi$$

$x = \rho\cos\phi$

$$= 33.41$$

$$\rightarrow z=0 \Rightarrow d\vec{S}_5 = -\rho\,d\rho\,d\phi\,\hat{z} \Rightarrow \int_{S_5} \vec{D}\cdot d\vec{S}_5 = \int_{\rho=0}^{3}\int_{\phi=0}^{\pi/2} x\rho\,d\rho\,d\phi = 9$$

$$\Rightarrow \oint_S \vec{D}\cdot d\vec{S} = 192.82 \checkmark$$