Summary

Anthony Rey
ENGR251 Thermodynamics I - Winter 2017

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Here is a summary of what we have learnt so far.

First law of thermodynamics

We have three types of systems: (i) *open*, (ii) *closed*, and (ii) *isolated*. An *open system* is a system where both energy and mass can cross its boundary. A *closed system* is a system where only energy can. An *isolated system* is a system where neither energy nor mass can cross its boundary. An isolated system is a *special case* of a closed system, which is itself a special case of an open system.

That's being said, we have learned that the *first law of thermodynamics* is nothing more than an expression of the *conservation of energy principle* – the change in the total amount of energy of a system must be equal to the different forms of energy entering or leaving our system – which is given by:

$$\Delta \dot{E}_{\text{system}} = \sum_{in} \dot{m}_{\text{in}} \left(h_{\text{in}} + \frac{v_{\text{in}}^2}{2} + g z_{\text{in}} \right) - \sum_{out} \dot{m}_{\text{out}} \left(h_{\text{out}} + \frac{v_{\text{out}}^2}{2} + g z_{\text{out}} \right) + \dot{Q} - \dot{W} \tag{1}$$

So, we start by defining our system, then we apply the first law. All the terms that are not involved must be removed. If our system is isolated, everything on the right-hand side of the equation. If it is closed, every term associated with mass entering or leaving. To sum up, this is the only formula you need to remember.
We also talked about two cases: (i) *steady* and (ii) *unsteady*. First case scenario, we can remove the left-hand side of equation 1: $\Delta \dot{E} = 0$. When the process is not steady (e.g., inflation of a balloon or a pressure cooker), $\Delta E = \Delta U + \Delta KE + \Delta PE$ changes.
Sometimes, for open system, we need more information on the net change in mass of the system during the process. The *conservation of mass principle* is useful:

$$\Delta \dot{m} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out} \quad \text{and} \quad \Delta m = m_2 - m_1 = \sum_{in} m_{in} - \sum_{out} m_{out} \tag{2}$$

More simplifications can be achieved when we know the process our system has undergone:

1. isothermal ($\Delta T = 0$)
2. isobaric ($\Delta p = 0$)
3. isochoric ($\Delta V = 0$)
4. adiabatic ($Q = 0$)
5. isentropic ($\Delta S = 0$)
6. polytropic ($pV^\gamma = k = \text{constant}$)

Isothermal does not mean adiabatic (and vice versa). For example, we can receive some energy by heat and lose the same amount by work (expansion of a piston-cylinder device) which results in a constant-temperature process – where energy was exchanged by heat. Isochoric implies that we do not have boundary work: $\delta W = pdV$. Isentropic means *adiabatic and reversible* – which is characterized by $pV^\gamma = k$ (where $\gamma = c_p/c_v$ is the specific heat ratio).

Applications (ideal gases, cycles, etc.)

For ideal gases, *no matter the process*, the changes in internal energy and enthalpy are calculated as:

$$\Delta U = m\left(u_2 - u_1\right) = m\int_1^2 c_v(T)dT \approx mc_{v,avg}\Delta T \tag{3}$$

$$\Delta H = m\left(h_2 - h_1\right) = m\int_1^2 c_p(T)dT \approx mc_{p,avg}\Delta T \tag{4}$$

There are two ways: (i) by using thermodynamics tables and (ii) by using the average values of specific heats (accurate if the temperature interval is small). This is true because the *internal energy and enthalpy depend only on temperature for ideal gases.*

For *closed systems*, the right-hand side of equation 1 is reduced to $Q - W$. When we neglect the kinetic and potential energy changes, we have:

$$\Delta U = mc_v\Delta T = Q - W \tag{5}$$

Let us focus on the different processes when we have an *ideal gas* in a *closed system*[1]:

1. isothermal ($\Delta T = 0$) $\implies Q = W$
2. isobaric ($\Delta p = 0$) $\implies W_b = p\left[V_2 - V_1\right]$ or $\Delta H = Q$
3. isochoric ($\Delta V = 0$) $\implies \Delta U = Q$
4. adiabatic ($Q = 0$) $\implies \Delta U = W$

Notice that each process has its own advantage. For example, when we have an adiabatic process, we can calculate the work using the change in internal energy (with equation 3). The polytropic or isentropic processes enable us to calculate the boundary work more easily.

When we do not have an ideal gas, we cannot use equations 3 and 4 *unless* we have a *constant-volume process* ($\Delta U = mc_v\Delta T$) or a *constant-pressure process* ($\Delta H = mc_p\Delta T$) – when we have a *solid* or an *incompressible fluid* the change in volume or pressure can be neglected, so we can use the same equations.

Gas power cycles require some definitions. The *compression ratio* (which is a volume ratio) is defined as:

$$r = \frac{V_{max}}{V_{min}} \tag{6}$$

The *pressure ratio* is calculated as:

$$r_p = \frac{p_{max}}{p_{min}} \tag{7}$$

The *mean effective pressure* (MEP) is given by:

$$\text{MEP} = \frac{W_{max}}{V_{max} - V_{min}} = \frac{w_{max}}{\nu_{max} - \nu_{min}} \tag{8}$$

The lager the MEP is, the more net work is produced per cycle.

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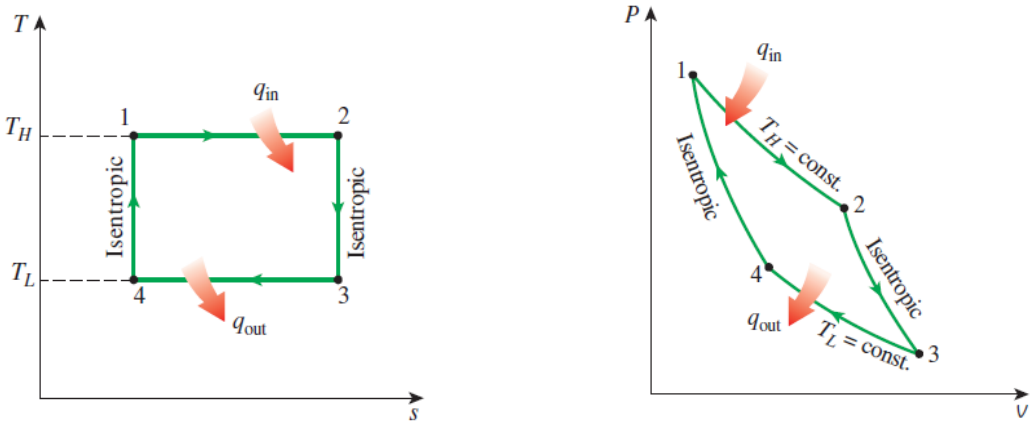
[1] We assumed that there was only some boundary work – any other forms involved should be included.

The *Carnot cycle* is composed of four processes: (i) isothermal heat addition, (ii) isentropic expansion, (iii) isothermal heat rejection, (iv) isentropic compression.



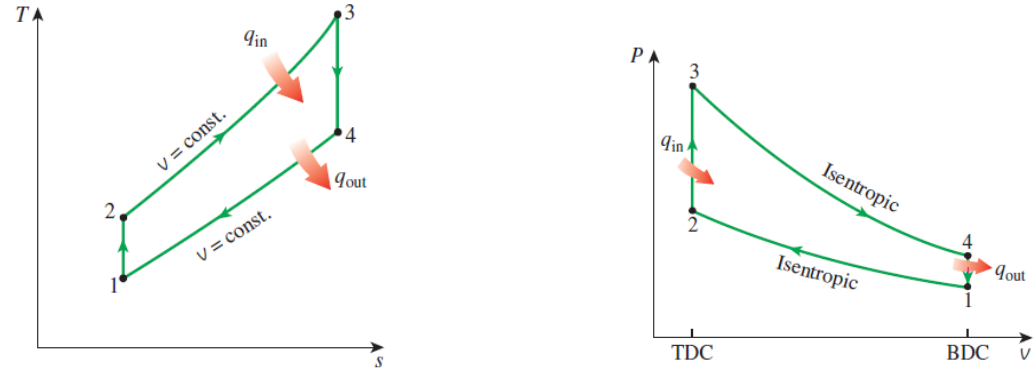
(a) $T - s$ diagram for the Carnot cycle

(b) $p - \nu$ diagram for the Carnot cycle

Figure 1: Carnot cycle [1]

Remember that we talked about that. We can find its *thermal efficiency* (see equation 12)or *work done* by *applying the first law of thermodynamics to each of the four processes*.

The *Otto cycle* is composed of four processes: (i) isentropic compression, (ii) constant-volume heat addition, (iii) isentropic expansion, (iv) constant-volume heat rejection.



(a) $T - s$ diagram for the ideal Otto cycle

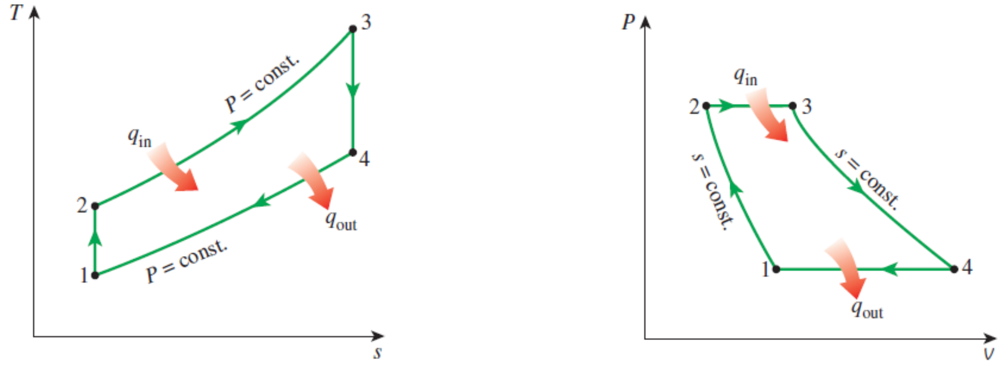
(b) $p - \nu$ diagram for the ideal Otto cycle

Figure 2: Ideal Otto cycle [1]

The thermal efficiency of the Otto cycle can also be calculated as:

$$\eta_{\text{th}} = 1 - \frac{1}{r^{\gamma-1}} \tag{9}$$

The *Brayton cycle* is composed of four processes: (i) isentropic compression in a compressor, (ii) constant-pressure heat addition, (iii) isentropic expansion in a turbine, (iv) constant-pressure heat rejection.



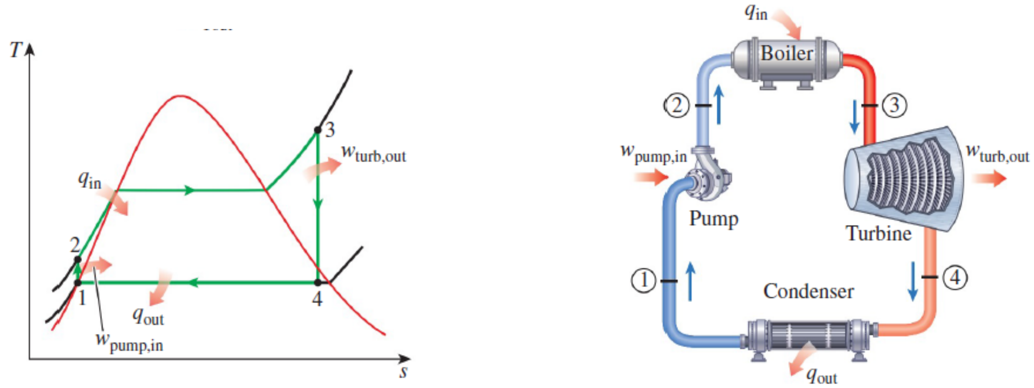
(a) $T-s$ diagram for the ideal Brayton cycle (b) $p-\nu$ diagram for the ideal Brayton cycle

Figure 3: Ideal Brayton cycle [1]

The thermal efficiency of the Brayton cycle can also be calculated as:

$$\eta_{\text{th}} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} \tag{10}$$

The *Rankine cycle* is composed of four processes: (i) isentropic compression in a pump, (ii) constant-pressure heat addition in a boiler, (iii) isentropic expansion in a turbine, (iv) constant-pressure heat rejection in a condenser.



(a) $T-s$ diagram for the ideal Rankine cycle (b) Schematic of the ideal Rankine cycle

Figure 4: Ideal Rankine cycle [1]

Problems with gas power cycles often require us to find: (i) the *temperature or pressure at various points in the cycle*, (ii) the *net work output*, (iii) the *thermal efficiency*, or (iv) the *mean effective pressure* (MEP) of the cycle. To do so, we need to *apply the first law of thermodynamics to each of the four processes.*

Second law of thermodynamics

The second law of thermodynamics starts with two statements:

1. Kelvin-Planck statement: *It is impossible for any system that operates in a cycle to receive heat from a single reservoir and produce a net amount of work*
2. Clausius statement: *It is impossible for any process to have as its sole result the transfer of heat from a lower-temperature reservoir to a higher-temperature reservoir*

To sum up, (i) *you cannot convert heat to work entirely* and (ii) *heat does not flow spontaneously from a cold object to a hot one.*
That's being said, we start with *heat engine* (HE) to produce work from a hot source (reservoir). Based on the first statement, we cannot achieve an efficiency 100% (since heat cannot be converted to work entirely). Remember that the *Carnot cycle* is a *reversible cycle* which gives us the *maximum achievable work possible* – since reversible. The important result from the Carnot cycle was:

$$\left(\frac{Q_H}{Q_L}\right)_{\text{rev}} = \frac{T_H}{T_L} \tag{11}$$

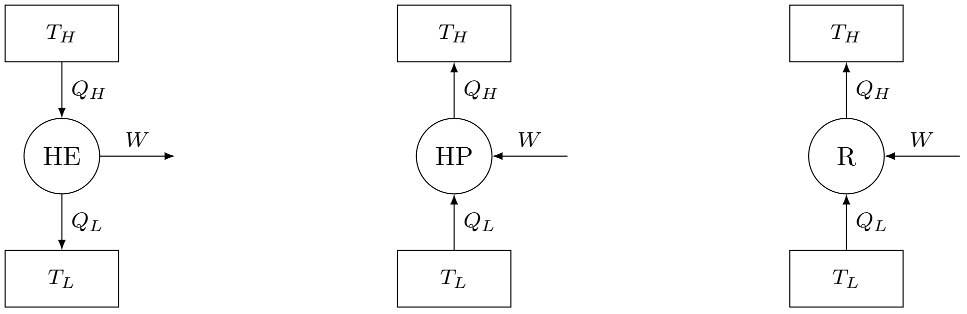
To compare heat engines, we need their respective *thermal efficiency*:

$$\eta_{\text{th}} = \frac{\text{desired}}{\text{required}} = \frac{W}{Q_H} = \frac{\dot{W}}{\dot{Q}_H} = 1 - \frac{Q_L}{Q_H} \leq 1 - \frac{T_L}{T_H} \tag{12}$$

We want to produce work (desired) by using energy from some hot source (required). We know that *energy must be conserved*, so we can use figure 5a to express W differently:

$$Q_H = W + Q_L \quad \Leftrightarrow \quad W = Q_H - Q_L \tag{13}$$

Notice that we have an inequality at the end of equation 12 which is due to reversibility. If our heat engine is reversible, we can replace *each amount of heat transfer by the temperature of the source.* No matter what, *we cannot go beyond 100% for a heat engine.*



(a) Schematic of a heat engine (b) Schematic of a heat pump (c) Schematic of a refrigerator

Figure 5: Schematic of thermodynamics engines

For heat pumps and refrigerators, we talk about *coefficient of performance* (COP) instead of thermal efficiency because the value can go beyond 100%. Let us start with a *heat pump* HP, where you provide some work (required) to heat up something (desired):

$$\text{COP}_{\text{HP}} = \frac{\text{desired}}{\text{required}} = \frac{Q_H}{W} = \frac{\dot{Q}_H}{\dot{W}} = \frac{Q_H}{Q_H - Q_L} \leq \frac{T_H}{T_H - T_L} \tag{14}$$

For a *refrigerator* (R), we provide some work (required) to cool something (desired):

$$\text{COP}_{\text{R}} = \frac{\text{desired}}{\text{required}} = \frac{Q_L}{W} = \frac{\dot{Q}_L}{\dot{W}} = \frac{Q_L}{Q_H - Q_L} \leq \frac{T_L}{T_H - T_L} \tag{15}$$

There are a couple of things to notice:

1. Heat pumps and refrigerators work in the same way but have a different purpose
2. Heat pumps/refrigerators take energy from the cold source (which makes it colder) and release energy to the hot one (which makes it hotter)
3. Air conditioners (AC) are refrigerators whose refrigerated space is a room
4. When engines (HE, HP, R, or AC) are reversible, we can replace each amount of heat transfer by the temperature of the source.

Entropy

Entropy measures *irreversibility* and is defined as:

$$dS = \left(\frac{\delta Q}{T_{\text{source}}}\right)_{rev} \tag{16}$$

The *increase of entropy principle* tells us that *the total change in entropy of an isolated system is always greater than or equal to zero*:

$$\Delta S_{\text{isolated}} \geq 0 \tag{17}$$

Remember that the universe is an isolated system – so basically everything. When we study two (or more) systems, their respective change in entropy can be negative but the total change in entropy of everything must be greater than or equal to zero:

$$\Delta S_{\text{total}} = \Delta S_{\text{isolated}} = \Delta S_{\text{systems}} + \Delta S_{\text{surroundings}} \geq 0 \tag{18}$$

Entropy can be divided into two parts:

$$\Delta S = \Delta S_{\text{exchange}} + S_{\text{generated}} \tag{19}$$

The first term on the right-hand side is due to reversible energy transfer by heat:

$$\Delta S_{\text{exchange}} = \int_1^2 \left(\frac{\delta Q}{T_{\text{source}}}\right)_{rev} \tag{20}$$

The second term is due to irreversibility – that we cannot calculate directly.
Remember that an *isentropic process* ($\Delta S = 0$) means *adiabatic and reversible.* If a process is reversible, $S_{\text{generated}} = 0$. If a process is adiabatic ($Q = 0$), $\Delta S_{\text{exchange}} = 0$. As a result, for a reversible process,

$$\Delta S_{\text{total}} = \Delta S_{\text{isolated}} = 0 \tag{21}$$

The change in entropy from one system goes to another, so the total change in entropy is equal to zero. Since entropy is a thermodynamic property, we can also find the change in entropy using the thermodynamics tables:

$$\Delta S = S_2 - S_1 = m\left(s_2 - s_1\right) = \int_1^2 \frac{\delta Q}{T_{\text{source}}} + S_{\text{gen}} \tag{22}$$

For an isothermal process ($\Delta T = 0$), the temperature can be taken out of the integral such that:

$$\Delta S_{\text{exchange}} = \frac{Q}{T_{\text{source}}} \tag{23}$$

So far, it is general. When we have an *incompressible fluid*, *no matter the process*,

$$\Delta S_{\text{exchange}} = c_{avg} \ln \frac{T_2}{T_1} \tag{24}$$

For *ideal gases*, when we constant specific heats:

$$\Delta S_{\text{exchange}} = c_{v,avg} \ln \frac{T_2}{T_1} + R_x \ln \frac{\nu_2}{\nu_1} \tag{25}$$

$$\Delta S_{\text{exchange}} = c_{p,avg} \ln \frac{T_2}{T_1} + R_x \ln \frac{p_2}{p_1} \tag{26}$$

The last three equations comes from equation 20 – where the infinitesimal amount of heat transferred to or from the system is replaced by using the first law of thermodynamics.

References

[1] Y. A. Çengel and M. A. Boles, *Thermodynamics An Engineering Approach.* New York City, NY, USA: McGraw-Hill Higher Education, 2011.