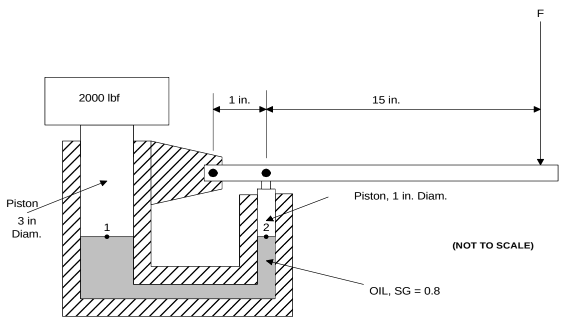
ENGINEERING 86.230A - Fluid Mechanics I
MIDTERM EXAMINATION - OCTOBER 2002

1. (a) Hydraulic jack



- the pressure in the hydraulic fluid must be the same at 1 and 2 (same elevation, continuous constant density fluid)

but $P_1 := \frac{2000}{\frac{\pi}{4}\cdot\left(\frac{3}{12}\right)^2}$ $P_1 = 4.074\times 10^4$ psig

(note that this is the gauge pressure since P_{atm} will also be pushing down on the piston, through the weight)

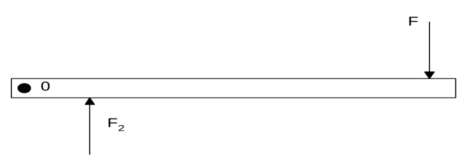
thus $P_2 := P_1$ $P_2 = 4.074\times 10^4$ psig

the net upward force on the right piston is then

$F_2 = P_2\cdot A_2$ $F_2 := P_2\cdot\frac{\pi}{4}\cdot\left(\frac{1}{12}\right)^2$ $F_2 = 222.222$ lbf

- moments about the handle pin must be in balance to hold the system in the position shown

$\Sigma M_0 = 0$

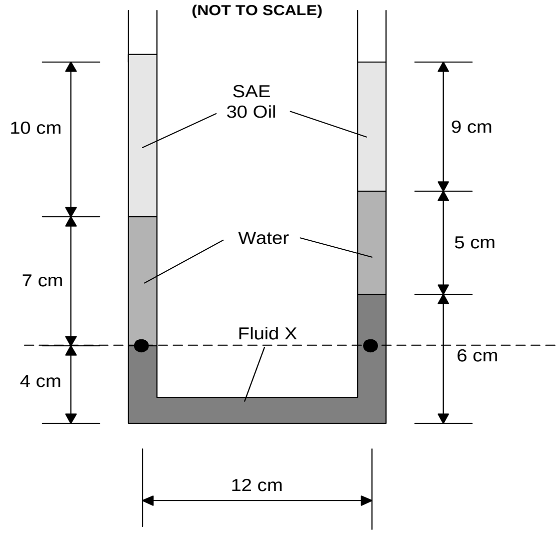


$F_2\cdot\left(\frac{1}{12}\right) - F\cdot\left(\frac{1+15}{12}\right) = 0$ (taking CCW positive)

thus $F := \frac{1}{16}\cdot F_2$ $F = 13.9$ lbf

(b) Determining SG of unknown fluid

- the pressure in fluid X must be the same at the two points shown (same elevation, continuous body of constant density fluid) - let it be P
- also, the pressure at the top of the two tubes is P_{atm}



$SG_{oil} := 0.85$

$\rho_{water} := 1000$ kg/m3

$SG_{water} := 1.0$

$\rho_{oil} := SG_{oil} \cdot \rho_{water}$

$\rho_{oil} = 850$ kg/m3

- working down the left leg of the U-tube, fluid by fluid

$$P_{atm} + \rho_{oil} \cdot g \cdot \left(\frac{10}{100}\right) + \rho_{water} \cdot g \cdot \left(\frac{7}{100}\right) = P$$

$g := 9.81$ m/s2

- similarly, for the right leg of the U-tube

$$P_{atm} + \rho_{oil} \cdot g \cdot \left(\frac{9}{100}\right) + \rho_{water} \cdot g \cdot \left(\frac{5}{100}\right) + \rho_X \cdot g \cdot \left(\frac{6-4}{100}\right) = P$$

- equate and solve for ρ_X

$$P_{atm} + \rho_{oil} \cdot g \cdot \left(\frac{10}{100}\right) + \rho_{water} \cdot g \cdot \left(\frac{7}{100}\right) = P_{atm} + \rho_{oil} \cdot g \cdot \left(\frac{9}{100}\right) + \rho_{water} \cdot g \cdot \left(\frac{5}{100}\right) + \rho_X \cdot g \cdot \left(\frac{6-4}{100}\right)$$

$\rho_X = \frac{1}{2} \cdot \rho_{oil} + \rho_{water}$ or $SG_X := \frac{1}{2} \cdot SG_{oil} + SG_{water}$

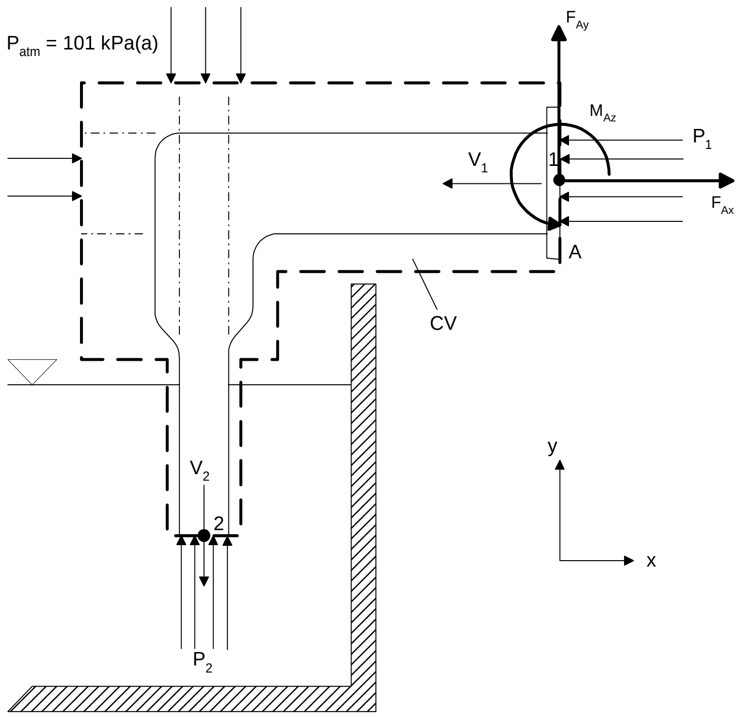
thus $SG_X = 1.425$

- if P had been worked out explicitly, would have obtained

$P := \rho_{oil} \cdot g \cdot \left(\frac{10}{100}\right) + \rho_{water} \cdot g \cdot \left(\frac{7}{100}\right)$ $P = 1.521 \times 10^3$ a(g)

2. Nozzle analysis

(a) Control volume



- forces F_{Ax} and F_{Ay} and moment M_{Az} are exposed where the control volume cuts through the flanged joint at A
- for the force and moment balances will need the pressures and velocities at 1 and 2

$Q := 0.1$ m³/s $\qquad P_{atm} := 101000$ Pa

$D_1 := 0.2$ m $\qquad A_1 := \frac{\pi}{4} \cdot D_1^2 \qquad A_1 = 0.031$ m²

$V_1 := \frac{Q}{A_1} \qquad V_1 = 3.183$ m/s

$P_{1g} := 70000$ Pa(g) or $\quad P_1 := P_{1g} + P_{atm} \qquad P_1 = 1.71 \times 10^5$ Pa(a)

$D_2 := 0.1$ m $\qquad A_2 := \frac{\pi}{4} \cdot D_2^2 \qquad A_2 = 7.854 \times 10^{-3}$ ı2

$V_2 := \frac{Q}{A_2} \qquad V_2 = 12.732$ m/s

- the nozzle discharges 1 m below the free surface
- thus

$g := 9.81$ m/s²

$SG := 0.85 \qquad \rho := 850$ kg/m³

$$P_2 := P_{atm} + \rho \cdot g \cdot 1 \qquad P_2 = 1.093 \times 10^5 \quad \text{Pa(a)}$$

(b) (i) Force in x direction at flange

- apply the linear momentum equation in the x direction, assuming 1-D flow

$$\Sigma F_x = m_{out} \cdot u_{out} - m_{in} \cdot u_{in}$$

- outside the two circles defined by the horizontal chain-dotted lines, equal pressures (either atmospheric or the hydrostatic pressures on the submerged part of the nozzle) act on equal projected areas of the CV in the positive and negative x directions
- thus, the only remaining forces on the CV in the x direction are the pressure forces on the two disks of area A_1 and the flange force F_{Ax}
- also

$$m := \rho \cdot Q \qquad m = 85 \quad \text{kg/s}$$

$$u_{out} := 0 \qquad u_{in} := -V_1 \qquad u_{in} = -3.183 \quad \text{m/s}$$

(note that u_{in} is negative since it is in the negative x direction)

- then

$$F_{Ax} + P_{atm} \cdot A_1 - P_1 \cdot A_1 = m \cdot u_{out} - m \cdot u_{in}$$

$$F_{Ax} := -m \cdot u_{in} - P_{atm} \cdot A_1 + P_1 \cdot A_1 + m \cdot u_{out}$$

$$F_{Ax} = 2.47 \times 10^3 \quad \text{N}$$

- F_{Ax} is positive and thus in the assumed direction
- the forces and moments at the flanged joint are those which the removed components apply to the control volume
- if the removed piping must exert a force in the positive x direction to hold the nozzle in place, the flanged joint must be in tension

(ii) Force in y direction at flange (neglecting the weight of the pipe and the weight of the water inside)

- apply the linear momentum equation in the y direction

$$\Sigma F_y = m \cdot v_{out} - m \cdot v_{in}$$

- outside the two circles defined by the vertical chain-dotted lines, equal pressures act on equal projected areas of the CV in the positive and negative y directions
- thus, the only remaining forces on the CV in the y direction are the pressure forces on the two disks of area A_2 and the flange force F_{Ay}
- also

$v_{out} := -V_2$ $\quad$ $v_{out} = -12.732$ m/s $\quad$ $v_{in} := 0$

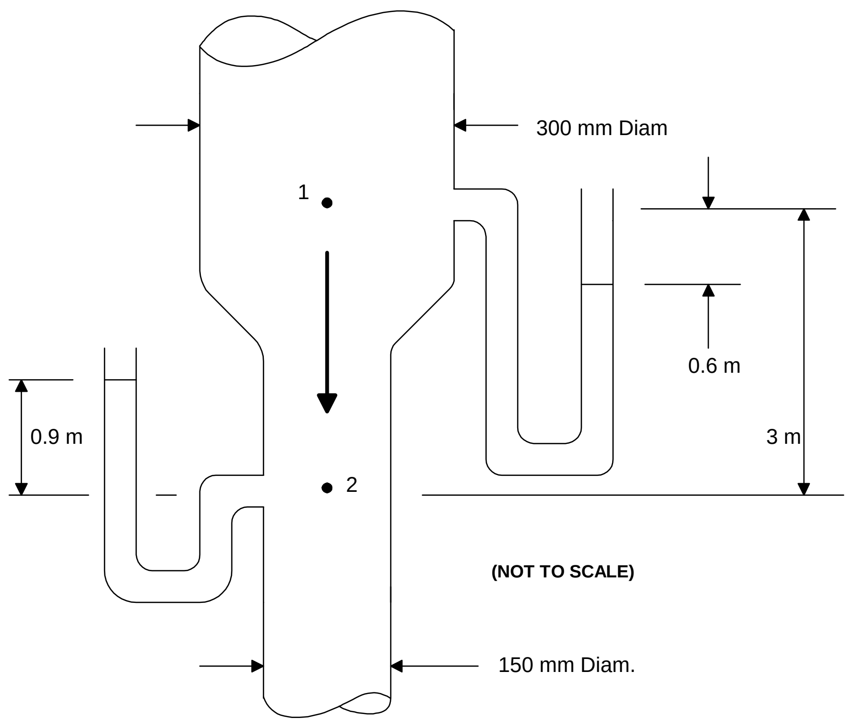
- then

$$F_{Ay} + P_2 \cdot A_2 - P_{atm} \cdot A_2 = m \cdot v_{out} - m \cdot v_{in}$$

$$F_{Ay} := -m \cdot v_{in} - P_2 \cdot A_2 + P_{atm} \cdot A_2 + m \cdot v_{out}$$

$$F_{Ay} = -1.148 \times 10^3 \text{ N}$$

3. Duct flow



(a) Gauge pressures sensed by manometers

$\rho := 1000$ kg/m3 $g := 9.81$ m/s2

- upper manometer

$P_1 = P_{atm} - \rho \cdot g \cdot (0.6)$ or $P_{1g} := \rho \cdot g \cdot (-0.6)$

$P_{1g} = -5.886 \times 10^3$ Pa(g)

- lower manometer

$P_2 = P_{atm} + \rho \cdot g \cdot (0.9)$ or $P_{2g} := \rho \cdot g \cdot (0.9)$

$P_{2g} = 8.829 \times 10^3$ Pa(g)

- these are the static pressures in the pipe at levels 1 and 2 respectively

(b) Mass flow rate in the duct

- neglecting friction, can apply Bernoulli between 1 and 2

$$P_1 + \frac{1}{2}\cdot\rho\cdot V_1^2 + \rho\cdot g\cdot y_1 = P_2 + \frac{1}{2}\cdot\rho\cdot V_2^2 + \rho\cdot g\cdot y_2$$

or

$$P_{1g} + P_{atm} + \frac{1}{2}\cdot\rho\cdot V_1^2 + \rho\cdot g\cdot y_1 = P_{2g} + P_{atm} + \frac{1}{2}\cdot\rho\cdot V_2^2 + \rho\cdot g\cdot y_2 \qquad (1)$$

choosing level 2 as the datum, then $y_2 := 0$ $y_1 := 3$ m

- since we know the pressures, eqn. (1) is an equation in two unknowns: V_1 and V_2

- but the velocities are also related by continuity

$$\rho\cdot A_1\cdot V_1 = \rho\cdot A_2\cdot V_2$$

$$\rho\cdot\left(\frac{\pi}{4}\cdot D_1^2\right)\cdot V_1 = \rho\cdot\left(\frac{\pi}{4}\cdot D_2^2\right)\cdot V_2$$

where $D_1 := 0.3$ m $D_2 := 0.15$ m

thus $V_1 = D_2^2\cdot\frac{V_2}{D_1^2}$ $\left(\frac{D_2}{D_1}\right)^2 = 0.25$ $V_1 = 0.25\cdot V_2$

- substituting into (1) and solving for V_2

$$P_{1g} + \frac{1}{2}\cdot\rho\cdot\left(0.25\cdot V_2\right)^2 + \rho\cdot g\cdot y_1 = \left(P_{2g} + \frac{1}{2}\cdot\rho\cdot V_2^2\right)$$

$$V_2 := \frac{4}{(15\cdot\rho)}\cdot\sqrt{30}\cdot\sqrt{\left[\rho\cdot\left(P_{1g} + \rho\cdot g\cdot y_1 - P_{2g}\right)\right]}$$

$V_2 = 5.603$ m/s

and $m := \rho\cdot\frac{\pi}{4}\cdot D_2^2\cdot V_2$ $m = 99$ kg/s

- a couple of variations on this are possible:

- could solve for V_1 instead
- this would have given

$$V_1 := 0.25 \cdot V_2 \qquad V_1 = 1.401 \quad \text{m/s}$$

- could have written the velocities in terms of the unknown volume flow rate, Q

$$V_1 = \frac{Q}{A_1} \qquad V_2 = \frac{Q}{A_2}$$

and then solve (1) for Q

- this would have given $Q := \frac{m}{\rho}$ $\quad Q = 0.099$ $\quad$ m^3/s