CONCORDIA UNIVERSITY

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 209 — MIDTERM TEST — JULY 29, 2014

INSTRUCTOR: Dr. Umanath Tiwari — Time: 75 minutes

Total: 40 marks.

1. Evaluate the following limits: (9 Marks)

   (A) $\lim\limits_{x\to -2} \frac{x+2}{x^2-x-6}$

   (B) $\lim\limits_{x\to -\infty} \frac{3x^2+7}{x^2-6x}$

   (C) $\lim\limits_{h\to 0} \frac{g(2+h)-g(2)}{h}$, where g(x) = x^2+1

2. Find the derivative of each of the following (do not simplify): (9 Marks)

   (A) $f(x) = x^{-2} + \frac{3}{x} + 3x^4 + e^2$

   (B) $y = [\ln(x^2+x-2)][e^{-3x^2+6x+5}]$

   (C) g(x) = $\frac{(x^2-6x+1)}{\sqrt{3x^3+1}}$, at x = 0

3. Find an equation of the tangent line at the point (1, 1) to the graph of

   $x\ln y + 2y = 2x^2$ (6 Marks)

4. The radius of a spherical balloon is increasing at the rate of 3 centimeters per minute. How fast is the volume changing when the radius is 10 centimeters? [Use $V = \frac{4}{3}\pi R^3$] (8 Marks)

5. Given the price – demand equation: $p + 0.005x = 30$ (8 Marks)
   (a) Express the demand x as a function of price p.
   (b) Find the elasticity of demand E(p).
   (c) What is the elasticity of demand when p = $10? If this price is increased by 5%, what is the approximate change in demand?

$\frac{-p f'(p)}{f(p)}$ <1 i' increase

Solution: MIDTERM TEST 209
Summer 2014.

(1) (A) $\lim_{x\to -2} \frac{(x+2)}{(x+2)(x-3)} = \lim_{x\to -2} \frac{1}{x-3} = -\frac{1}{5}$

(B) $\lim_{x\to -\infty} \frac{3x^2+7}{x^2-6x} = \lim_{x\to -\infty} \frac{3x^2}{x^2} = 3.$

(C) $g(x) = x^2+1,\ g(x+h) = (x+h)^2-1$
$= x^2+2hx+h^2-1$

$g(x+h)-g(x) = h(2x+h)$

$\frac{g(x+h)-g(x)}{h} = \frac{h(2x+h)}{h} = 2x+h$

$\lim_{h\to 0} \frac{g(2+h)-g(2)}{h} = \lim_{h\to 0} 2(2)+h = 4$

(2) (A) $f'(x) = [x^{-2}+3x^{-1}+3x^4+e^2]'$
$= -2x^{-3}-3x^{-2}+12x^3$

(B) $y' = \frac{1}{x^2+x-2}\, d(2x+1)\, e^{-3x^2+6x+5}$
$+ \ln(x^2+x-2)\cdot e^{-3x^2+6x+5} \cdot (-6x+6)$

(C) $g'(x) = \frac{(2x-6)\sqrt{3x^3+1} + (x^2-6x+1)\frac{1}{2}\frac{9x^2}{\sqrt{3x^3+1}}}{3x^3+1}$

$g'(0) = \frac{-6+0}{1} = -6$

(3) $\frac{d}{dx}[x\ln y + 2y] = \frac{d}{dx}[2x^2]$

$1\ln y + \frac{x}{y}\cdot y' + 2y' = 4x$

$(\frac{x}{y}+2)y' = 4x - \ln y,$

At (1,1) $(\frac{1}{1}+2)y' = 4 \Rightarrow y' = \frac{4}{3}$ (slope) of T-line

Equation of T-line at (1,1)

$y-1 = \frac{4}{3}(x-1) \Rightarrow y = \frac{4}{3}x - \frac{1}{3}$

(4) $V = \frac{4}{3}\pi R^3$ Diff. w.r.t. t (time)

$\frac{dV}{dt} = \frac{4}{3}\cdot 3\pi R^2 \frac{dR}{dt}$, $\frac{dR}{dt}=3$, $R=10$

$\frac{dV}{dt} = 1200\pi$ cu cm/min.

(5) (a) $x = \frac{1}{0.005}[30-p] = 200[30-p]$

$f(p) = 200[30-p],\ f'(p) = -200$

(b) $E(p) = -\frac{pf'(p)}{f(p)} = +\frac{200p}{200[30-p]} = \frac{p}{30-p}$

(c) $E(10) = \frac{10}{20} = \frac{1}{2} < 1$ inelastic.

% change in demand $= -E(p)$ % change in price

$= -\frac{5}{2}\% = -2.5\%$

Demand drops by 2.5%. However the revenue will increase.

Graphing strategy:

Example: sketch the graph of
$f(x) = x^4+4x^3.$

Solution: $f'(x) = 4x^3+12x^2 = 4x^2(x+3)$
$f''(x) = 12x^2+24x = 12x(x+2),$

Critical values: $f'(x)=0 \Rightarrow x=0,-3$

Possible points of inflexion: $f''(x)=0 \Rightarrow x=0,\ x=-2$

Domain of $f(x)$: $(-\infty,\infty)$

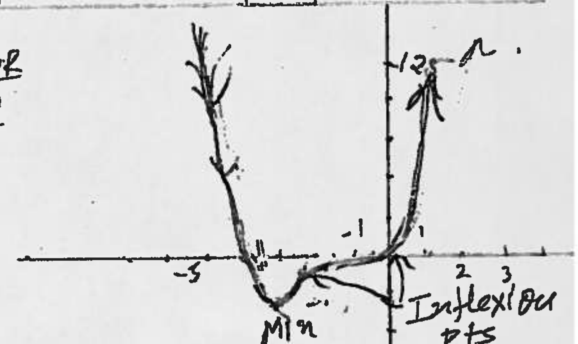
$\lim_{x\to+\infty} f(x) = +\infty$, $\lim_{x\to-\infty} f(x) = +\infty$

No Horizontal or vertical asymptotes

Range ⇒

	$-\infty$ to -3	-3 to -2	-2 to 0	0 to ∞
$f'(x) = 4x^2(x+3)$	− − − −	+ + +	+ + +	+ + + + + +
$f''(x) = 12x(x+2)$	+ + + + + +	+ +	− −	+ + + + + + +
shape of $y = x^4+4x^3$				

x	$y = x^4+4x^3$ or $y = x^3(x+4)$
−5	125
−4	0
−3	−27
−2	−16
0	0
1	5
2	48



Relative min: at $x=-3$. Rel min = −27

f(x) is decreasing on $(-\infty,-3)$
f(x) is increasing on $(-3,\infty)$
f(x) is concave up on $(-\infty,-2) \cup (0,\infty)$
f(x) is concave down on $(-3,-2)$
f(x) has inflexion points at $(-2,-8)$ and $(0,0)$.
Range $[-27,\infty)$, Domain $(-\infty,\infty)$
Relative or local min at $(-3,-27)$
No relative maximum.

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	All
Examination	**Date**	**Pages**
Final	April 22 2017	3
Instructors		**Course Examiner**
ALL		R. Raphael

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Approved calculators allowed.

MARKS

[6] **1.** (a) Find the following limits

(i) $\lim_{x\to 1} \dfrac{2x^5+7x-1}{x^2+5x+3}$

(ii) $\lim_{x\to 2} \dfrac{x^2+x-6}{2x^2-3x-2}$

(b) Prove or disprove by giving an example: there exists a function f from the real numbers to the real numbers that is discontinuous at exactly three points.

[4] **2.** Find the derivative $f'(x)$ of the functions $f(x)$: (Do not simplify)

(a) f(x)= $4x^5-9x^2+x-22$

(b) f(x)= $\dfrac{x^{-7}}{8}+\dfrac{1}{\sqrt[3]{x}}$

[10] **3.** Find $\frac{dy}{dx}$ (do not simplify):

(a) $y = \frac{7 - x^3}{e^{3x}}$

(b) $y = ln(3x^4 + 7)$

(c) $y = (4x - 5)^3(3x^2 + 4)$

(d) $y = (5 + x^3 lnx)^3$

[8] **4.** Let $f(x) = 4x^4 - x^2 - 7$

(a) Find the slope of the tangent line to the curve when $x = 1$

(b) Find the equation of the tangent line to the curve when $x = 1$

[13] **5.** Let $f(x) = x^4 - 2x^3$

Find

(a) the critical and inflection points of $f(x)$

(b) the intervals where $f(x)$ is increasing and where it is decreasing

(c) the intervals on which $f(x)$ is concave up and on which it is concave down

(d) use the above to sketch the graph

[9] **6.** If the cost of a seminar is \$400 per person 1000 people attend. For every \$5 dollar reduction in cost 20 more people will attend the seminar.
How much should be charged for the seminar to maximize revenue?

[6] **7.** Find the absolute extrema of the function $f(x) = x^3 - 12x$ on the interval $[-5, 5]$.

[4] **8.** A country has Lorenz curve $f(x) = x$. Find its Gini index. What can you conclude from the Gini index?

[10] **9.** Find the equation(s) of the tangent line(s)to the graph of $y - xy^2 + x^2 + 1 = 0$ at the point(s) with $x = 1$.

[10] **10.** Compute these antiderivatives:

(a) $\int (5x^7 - 4x^3 - 9)\, dx$

(b) $\int \frac{e^{-2x}}{4 + e^{-2x}}\, dx$

(c) $\int \frac{x^2}{\sqrt{x-5}}\, dx$

[10] **11.** Evaluate the integrals:

(a) $\int_0^1 (x^4 - 5)\, dx$

(b) $\int_6^{10} \frac{2}{x-4}\, dx$

(c) $\int_4^7 \sqrt{x-2}\, dx$

[10] **12.** Find the area bounded by the graphs of $f(x) = 5 - x^2$ and $g(x) = 2 - 2x$.