University of Ottawa

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CVG 3116

HYDRAULICS

Final Exam **December 22th, 2009**

Dr. Majid Mohammadian Time: **3 hours**

CLOSED BOOK. Non-programmable calculators are permitted. A list of equations and parameter values is attached. One 8.5"x11" sheet of paper written on both sides is allowed.

If you do not understand a question, clearly state an assumption and proceed.

Mark distribution is provided on the exam.

At the end of the exam, when time is up:

- Stop working and turn your exam upside down.
- Remain silent.
- Do not move or speak until all exams have been picked up.

Questions (answer all questions)

Q1 (3) What is the main application of globe valves?

Q2 (3) What happens in transition from mild slope to steep slope?

Q3 (3) What is the main requirement in choosing repeating variables in Buckingham theorem?

Q4 (3) What parameter must remain constant in physical modelling of a spillway? Why?

Q5 (3) Choose the correct answer(s): What boundary conditions are possible for a one dimensional unsteady subcritical flow:

1) Upstream: both discharge and depth. Downstream: none
2) Upstream: discharge. Downstream: depth
3) Upstream: depth. Downstream: discharge
4) Upstream: none. Downstream: both discharge and depth.

Q6 (3) Which of the following statements is wrong:

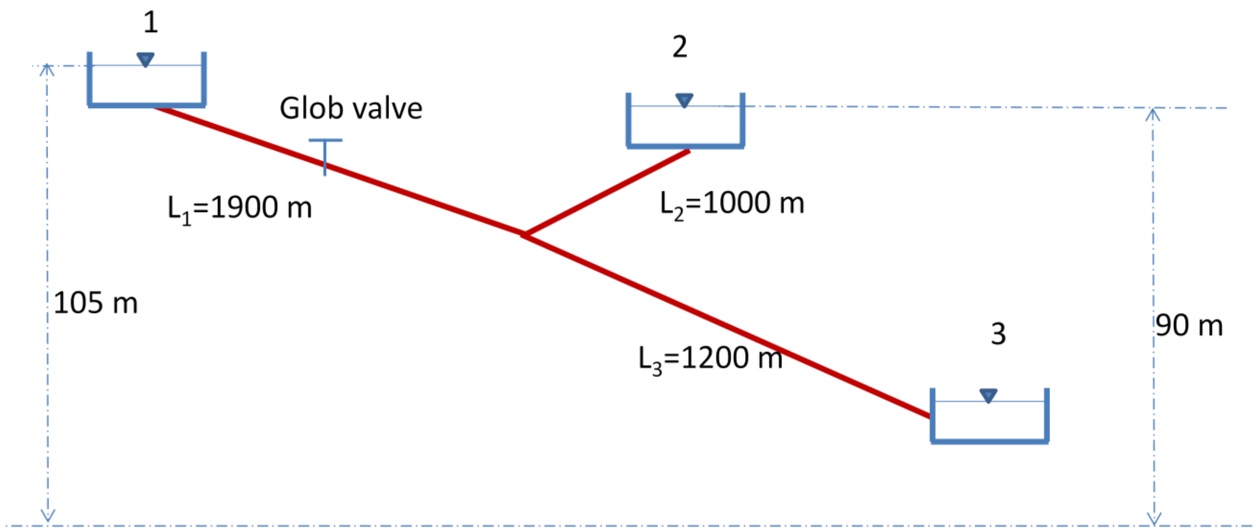
1) The discharge from a reservoir to a long steep channel may depend on the roughness of the channel.
2) Supercritical flow is always controlled from upstream.
3) In subcritical flow, water velocity is greater than the wave velocity.
4) In steep channels, the normal depth is less than the critical depth.
5) The Froude number can remain the same as the prototype in distorted river models.

Problems (solve problems 1,2,3,4, and two of 5,6,7)

P1 (22)- The normal depth in a concrete rectangular channel (n=0.013) is 0.9 m, the bed slope is 0.0004, and the channel is 7 m wide. A broad crested weir is installed in the channel in which the bed is raised by 70 cm. After the weir, the bed level returns to its value before the weir.

A) Calculate the discharge of uniform flow.
B) Show that the flow becomes critical in the weir.
C) What is the increase in water depth due to construction of the weir? What is the water depth just after the weir?
D) What is the water depth before and after the hydraulic jump?

P2 (17) Three reservoirs, are connected to a junction as shown. The water discharge in the pipe 1 (see figure) is 105 l/s and a globe valve (K_L=10) installed on this pipe. What is the water surface elevation in the third reservoir? All pipes are of diameter 40 cm with K_s=0.6 mm. Assume rough regime for pipes 2 and 3 but calculate λ in pipe 1. Ignore velocity head at the junction.

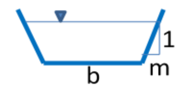


P3 (15) Water is ponded behind a vertical sluice gate to a height of 4.1 m in a rectangular channel of width 6.5 m. The gate opening is 1.1 m. Assuming the contraction coefficient C_c=0.61, calculate the water discharge in the channel if the downstream water depth is 2.5m. Is the jump submerged in this case? What is the maximum downstream water depth such that the hydraulic jump is not submerged?

P4 (15) A pump is used to supply 100 l/s water from a reservoir to an industrial complex. The pipe is made of galvanized iron (K_s=0.15 mm), is 16 m long, and has a diameter of 32 cm. The pump head and speed are respectively 21 m and 1000 rpm. Calculate the maximum elevation of the pump such that cavitation is avoided. Local head losses (entrance, bend and check valve) amount to 2.88 $V^2/2g$. Assume rough regime in the pipe.

P5 (15) A vertical venturi meter measures the flow of oil of density 720 kg/m^3 in a pipe and has an entrance of 135 mm diameter and a throat of 65 mm. The throat is 250 mm above the entrance. There are pressure gauges at the entrance and at the throat. If the coefficient for the meter is 0.96, find the discharge when the pressure difference between the entrance and at the throat is 25 kN/m^2.

P6 (15) You are designing a stable channel of trapezoidal cross section. The mean diameter of bed material, which are very rounded, is 1.1 cm with ρ=2650 kg/m^3, the bed slope is 3×10^{-4} and the required water discharge is 40 m^3/s.



A) What is the minimum stable side slope (m)?
B) If a conservative value of m=1.75 is chosen, what is the maximum allowed water depth?
C) Would the bed width of b=10 m be a good choice for this channel?

P7 (15) A circular culvert of length 26 m and diameter 75 cm, is installed at a slope of 2.5 percent with a squared edge entrance (k_L=0.5, C_d=0.6). The Manning's coefficient is n=0.014. Under the flood conditions, the maximum available headwater above the culvert inlet is 2.5 m and the tailwater depth is 1 m. Calculate the discharge capacity of the culvert.

CVG 3116, Hydraulics, final exam equation sheet

Note: some of the following equations apply only to specific circumstances.

Orifice plate $C_d = 0.65$, Venturi $C_d = 0.98$, small orifice $C_v = 0.98$, $C_d = 0.6$

large orifice $C_d = 0.8$, Contraction: $k_L = 0.44$, Sudden expansion $k_L = \left(1 - \frac{A_1}{A_2}\right)^2$

$\rho = 1000$ kg/m^3 , $\gamma = 9810$ N/m^3 , $\mu = 10^{-3}$ N·s/m^2 , $\nu = 10^{-6}$ m^2/s , g=9.81 m/s^2

$p_{atm} = 101340$ N/m^2 , $p_{vap} = 2340$ N/m^2 , $K = 2.2\times10^9$ N/m^2 , $E = 2.07\times10^{11}$ N/m^2

$$Q = VA$$

$$\frac{p_1}{\rho g} + y_1 + \alpha_1 \frac{V_1^2}{2g} + H_p = \frac{p_2}{\rho g} + y_2 + \alpha_2 \frac{V_2^2}{2g} + \sum\left(h_f + h_L\right)$$

$$\sum F_x = \beta \rho Q (V_2 - V_1)$$

$$\alpha = \frac{\int u^3 dA}{V^3 A} \quad , \quad \beta = \frac{\int u^2 dA}{V^2 A}$$

Pitot tube $u = \sqrt{2gh}$, $u = \sqrt{2gR_p(\frac{\rho_g}{\rho} - 1)}$

Venturi and Orifice place $Q_{ideal} = \frac{\pi D_1^2}{4}\left(\frac{1}{\sqrt{m^4 - 1}}\right)\sqrt{2gh^*}$, $h^* = R_p\left(\frac{\rho_g}{\rho} - 1\right)$, $m = \frac{D_1}{D_2}$, $Q_{actual} = C_d Q_{ideal}$

Small orifice $Q_{actual} = C_v C_d A_0 \sqrt{2gh}$

Large orifice $Q = C_d \frac{2}{3} b \sqrt{2g}\left(h_2^{3/2} - h_1^{3/2}\right)$

$$\mathrm{Re} = \frac{VD}{\nu}$$

$$h_f = \frac{32 \mu L V}{\rho g D^2}$$

$$h_f = \frac{\lambda L}{D} \frac{V^2}{2g}$$

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k_s}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{\lambda}}\right)$$

Barr: $$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{k_s}{3.7D} + \frac{5.1286}{\mathrm{Re}^{0.89}}\right)$$

$$V = -2\sqrt{2gDS_f}\,\log\left(\frac{k_s}{3.7D} + \frac{2.51\nu}{D\sqrt{2gDS_f}}\right)$$

$$S_f = h_f / L$$

$$\lambda = \frac{1}{\left(2\log\left(\frac{k_s}{3.7D}\right)\right)^2}$$

$$h_L = \sum k_L \frac{V^2}{2g}$$, $$h_f + h_L = \left(\frac{\lambda L}{D} + \sum k_L\right)\frac{V^2}{2g}$$

$$\delta q = -\frac{\sum h_{fi}}{2\sum \frac{h_{fi}}{q_i}}$$

$$K = \frac{\lambda L}{2gA^2D}$$, $$h_f = Kq^2$$, $$\delta q = -\frac{\sum h_{fi}}{2\sum |K_i q_i|}$$

$$H_p = \eta \frac{2\pi r_2 n\left(2\pi r_2 n - \frac{Q}{A_2}\cot\alpha_2\right)}{g}$$, $$A_2 = 2\pi r_2 b - A_{vanes}$$

$$P_{out} = \eta P_{in} = \rho g H_p Q$$

$$P_{in}^{Total} = \frac{P_{out}}{\eta\zeta}$$

$N_s = NQ^{1/2} / H_P^{3/4}$: N (rev/min) , Q (m^3/s) , H_p (m)

10 < N_s < 70, centrifugal

70 < N_s < 165, mixed flow

110 < N_s , axial flow

$$z < \frac{P_{atm} - P_{vap}}{\rho g} - \frac{V_{inlet}^2}{2g} - \sum\left(h_L + h_f\right) - NPSH_{crit}$$

$$NPSH_{crit} = H_p \sigma_{crit}$$

$$\sigma_{crit} = \left(\frac{N_s}{191} \right)^{4/3}$$

$$\eta_t = \frac{Q_1 + Q_2}{\frac{Q_1}{\eta_1} + \frac{Q_2}{\eta_2}} \qquad \eta_t = \frac{H_{p1} + H_{p2}}{\frac{H_{p1}}{\eta_1} + \frac{H_{p2}}{\eta_2}}$$

$$\pi_1 = \frac{Q}{ND^3} = \text{constant}$$

$$\pi_2 = \frac{gH_p}{N^2 D^2} = \text{constant}$$

$$\pi_3 = \frac{P}{\rho N^3 D^5} = \text{constant}$$

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1} \qquad \frac{H_{p2}}{H_{p1}} = \left(\frac{N_2}{N_1} \right)^2 \qquad \frac{P_2}{P_1} = \left(\frac{N_2}{N_1} \right)^3$$

Scaling curve: $\frac{H_p}{H_{p_2}} = \left(\frac{Q}{Q_2} \right)^2$

$$dt = \frac{dH}{\left[\frac{Q_1}{A_1} - \frac{Q_2}{A_2} - K H^{0.5} \left(\frac{1}{A_1} + \frac{1}{A_2} \right) \right]} \quad , \quad K = \frac{A\sqrt{2g}}{\sqrt{\left(\lambda \frac{L}{D} + \sum k_{Li} \right)}} \quad , \quad Q = KH^{0.5}$$

$$c = \sqrt{\frac{1/\rho}{(1/K) + (D/Ee)}} \qquad c = \sqrt{K/\rho}$$

$\Delta p = \rho c u_0$ if $t_c < \frac{2L}{c}$

$\Delta p = \frac{2L\rho u_0}{t_c}$ if $\frac{2L}{c} < t_c < \frac{20L}{c}$

$\Delta p = \frac{L\rho u_0}{t_c}$ if $t_c > \frac{20L}{c}$

$$Z_{ST_{max}} = \pm V_0 \sqrt{\frac{L}{g} \frac{A}{A_{ST}}}$$

$$\alpha = \frac{\int u^3 dA}{\overline{V}^3 A} = \frac{V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3}{\overline{V}^3 (A_1 + A_2 + A_3)} \qquad \overline{V} = \frac{Q}{A}$$

$$V = C\sqrt{RS_0}$$

$$Q = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}} S_0^{1/2}$$

$$y_1 + \alpha\frac{V_1^2}{2g} + z_1 = y_2 + \alpha\frac{V_2^2}{2g} + z_2$$

$$q = Q/b$$

$$E_s = y + \alpha\frac{q^2}{2gy^2} \quad , \quad E_s = y + \alpha\frac{V^2}{2g}$$

$$\frac{\alpha Q_{\max}^2 B_c}{gA_c^3} = 1$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} \quad , \quad V_c = \sqrt{gy_c} \quad , \quad E_c = \frac{3}{2} y_c$$

$$E_{s1} = E_{s2} + \Delta z$$

$Fr = \frac{V}{\sqrt{gD_m}}$ where D_m=A/B , $Fr = \frac{V}{\sqrt{gy}}$, $Fr^2 = \frac{q^2}{gy^3}$

$$y_1 = (y_2/2)\left(\sqrt{1+8Fr_2^2} - 1\right) \qquad y_2 = (y_1/2)\left(\sqrt{1+8Fr_1^2} - 1\right) \qquad \Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2}$$

$$\frac{1}{n}\frac{A^{5/3}}{P^{2/3}} S_c^{1/2} = \sqrt{\frac{gA^3}{B}} \quad , \quad S_c = gn^2 / y_c^{1/3} \quad , \quad S_c = \left(\frac{nQP^{2/3}}{A^{5/3}}\right)^2$$

$$\frac{\Delta y}{\Delta x} = \left(\frac{S_0 - S_f}{1 - Fr^2}\right)_{mean} \quad , \quad \Delta x = \frac{h_L + E_s^{dn} - E_s^{up}}{S_0 - S_f^{mean}}$$

Bend $h_L = k_L \frac{V^2}{2g}$, expansion / contraction $h_L = k_L \frac{\left|V_1^2 - V_2^2\right|^2}{2g}$

Culverts: $Q = C_d A\sqrt{2gh}$, $h = HW - D/2$, $HW + S_0 L = TW + \left(k_L + 1 + \frac{2gn^2}{R^{4/3}} L\right)\frac{V^2}{2g}$

Bridges: $b_{2c} = rb_4$, $r = \frac{\left(2+\frac{1}{r}\right)^3 Fr_4^4}{\left(1+2Fr_4^2\right)^3}$,

$\frac{y_1^2}{2} + \frac{q^2}{gy_1}\left(1 - \frac{\alpha C_D}{2}\right) = \frac{y_4^2}{2} + \frac{q^2}{gy_4}$, $\alpha = \frac{pier\ width}{span\ width} = \frac{b}{b_1}$, $q = \frac{Q}{n_{spans} b_1}$

Upstream positive surge $(V_1 + V)y_1 = (V_2 + V)y_2$, $V_1 + V = \sqrt{\frac{gy_2}{2}\left(\frac{y_2}{y_1} + 1\right)}$

Downstream positive surge $(V_1 - V)y_1 = (V_2 - V)y_2$, $V_1 - V = -\sqrt{\frac{gy_2}{2}\left(\frac{y_2}{y_1} + 1\right)}$

$\tau_0 = \gamma R S_0$, $F_S = \frac{\tau_{CR}}{(\rho_s - \rho) gD}$, $\begin{cases} For\ D_{gr} < 4 , & F_S = 0.24 / D_{gr} \\ 4 < D_{gr} < 10 , & F_S = 0.14 / D_{gr}^{0.64} \\ 10 < D_{gr} < 20 , & F_S = 0.04 / D_{gr}^{0.1} \\ 20 < D_{gr} < 150 , & F_S = 0.013 D_{gr}^{0.29} \\ D_{gr} > 150 , & F_S = 0.056 \end{cases}$,

$D_{gr} = D\left(g[(\rho_s / \rho) - 1] / \nu^2\right)^{1/3}$

$n = 0.042 d_{50}^{1/6}$

$\tau_{s,CR} = K \tau_{b,CR}$, $K = \sqrt{1 - \frac{\sin^2 \alpha}{\sin^2 \varphi}}$, $y = \min\left(\frac{\tau_{b,CR}}{\gamma S_0}, \frac{\tau_{s,CR}}{0.75 \gamma S_0}\right)$

Du Boys $q_s = K \tau_0 (\tau_0 - \tau_{CR})$, $K = \frac{3.874 \times 10^{-8}}{D^{0.75}}$

$\text{Volume of sediment} = \frac{\text{sediment discharge} \times \text{time}}{\text{1-prosity}}$

Ackers and White $u_* = \sqrt{gR_h S_0}$

If $D_{gr} > 60$: $n = 0$, $m = 1.78$, $A_{gr} = 0.17$, $C = 0.025$

If $D_{gr} > 60$:

$$n = 1 - 0.56 \log D_{gr}$$
$$m = 1.67 + 6.83 / D_{gr}$$
$$A_{gr} = 0.14 + 0.23 / D_{gr}^{1/2}$$
$$C = 10^{2.79 \log D_{gr} - 0.98 (\log D_{gr})^2 - 3.46}$$

$$F_{gr} = \frac{u_*^n}{\sqrt{gD[(\rho_s/\rho) - 1]}} \left(\frac{V}{\sqrt{32} \log(10 D_m / D)} \right)^{1-n}$$

$$\frac{q_s D_m}{q D} \left[\frac{u_*}{V} \right]^n = C \left[\frac{F_{gr}}{A_{gr}} - 1 \right]^m$$

$$\frac{y_s}{y_1} = \left(\frac{b_1}{b_2} \right)^{3/5} - 1 \quad , \quad \frac{y_p}{d} = 1.5 K_1 K_2 \left(\frac{y_1}{d} \right)^{0.3}$$

Physical quantity	**Dimensional Form**
Length	L
Mass (m)	M
Time (t)	T
Velocity (V)	LT^{-1}
Discharge (Q)	L^3T^{-1}
Density (ρ)	ML^{-3}
Force (F)	MLT^{-2}
Pressure (P)	$ML^{-1}T^{-2}$
Energy (E)	ML^2T^{-2}
Power	ML^2T^{-3}
Dynamic viscosity (μ)	$ML^{-1}T^{-1}$
Kinematic viscosity (ν)	L^2T^{-1}

$$\lambda_L = \frac{L_m}{L_p} \quad , \quad \lambda_V = \frac{V_m}{V_p} \quad , \quad \lambda_F = \frac{F_m}{F_p} \quad , \quad \lambda_Q = \lambda_V \lambda_L^{\ 2} \quad , \quad \lambda_F = \lambda_\rho \lambda_V^{\ 2} \lambda_L^{\ 2}$$

Froude number modelling: $\lambda_V = \sqrt{\lambda_L} \quad , \quad \lambda_T = \sqrt{\lambda_L} \quad , \quad \lambda_F = \lambda_\rho \lambda_L^{\ 3} \quad , \quad \lambda_Q = \lambda_L^{\ 5/2}$

Reynolds number modelling: $\lambda_V = \frac{\lambda_\nu}{\lambda_L} \quad , \quad \lambda_F = \lambda_\rho \lambda_\nu^{\ 2} \quad , \quad \lambda_p = \lambda_\rho \lambda_V^{\ 2}$

Distorted River Models: $\lambda_V = \sqrt{\lambda_y} \quad , \quad \lambda_S = \frac{\lambda_y}{\lambda_x} \quad , \quad \lambda_Q = \lambda_x \lambda_y^{\ 3/2} \quad , \quad \lambda_n \approx \frac{\lambda_y^{\ 2/3}}{\lambda_x^{\ 1/2}}$

Movable bed models: $\alpha = \frac{\rho_s}{\rho} - 1 \quad , \quad \lambda_\alpha = \frac{\alpha_m}{\alpha_p} \quad , \quad \lambda_D = \lambda_\alpha^{-1/3} \quad , \quad \lambda_X = \lambda_\alpha^{5/3} \quad , \quad \lambda_Y = \lambda_\alpha^{7/6} \quad , \quad \lambda_t = \lambda_\alpha^{17/6}$

Thin plate weir: $Q_{ideal} = \frac{2}{3} b\sqrt{2g}\left[\left(h_1 + \frac{u_1^2}{2g}\right)^{3/2} - \left(\frac{u_1^2}{2g}\right)^{3/2}\right]$

Vee weir: $Q_{ideal} = \frac{8}{15}\sqrt{2g}\,\tan(\theta/2)h_1^{5/2}$

Broad crested weir: $Q_{ideal} = \sqrt{g}\,b\left(\frac{2}{3}H_1\right)^{3/2}$

Venturi flumes (flat bed): $Q_{ideal} = by_1\sqrt{\frac{2g(y_1 - y_2)}{(by_1/b_2y_2)^2 - 1}}$, $Q = C_v C_d\, Q_{ideal}$

Gravity (Ogee) spillways : $Q = CbH^{3/2}$

Stilling basins: Type II : for $Fr_2 > 4.5$, $V_2 > 20$ m/s , length: 4.3 y_3

Type III: for $Fr_2 > 4.5$, $V_2 < 20$ m/s , length: 2.7 y_3

Type IV: for $2.5 < Fr_2 < 4.5$, length: 6.1 y_3

Sluice gates: $Q_{ideal} = by_1y_2\sqrt{\frac{2g}{y_1 + y_2}}$, $y_2 = C_c y_G$

$$y_1 + \frac{Q^2}{2gb^2y_1^2} = y + \frac{Q^2}{2gb^2y_2^2} \quad , \quad \frac{y^2}{2} + \frac{Q^2}{gb^2y_2} = \frac{y_3^2}{2} + \frac{Q^2}{gb^2y_3}$$

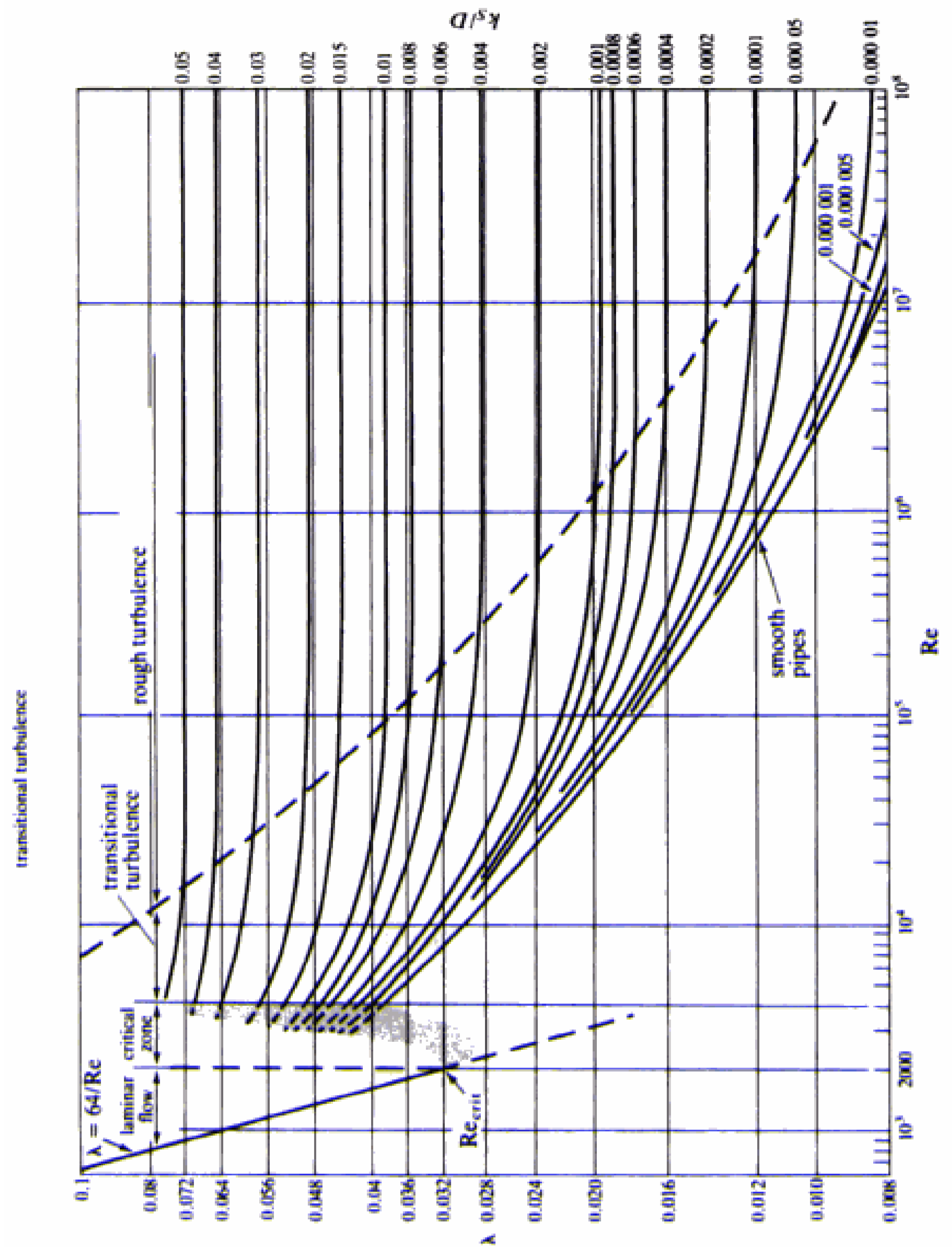
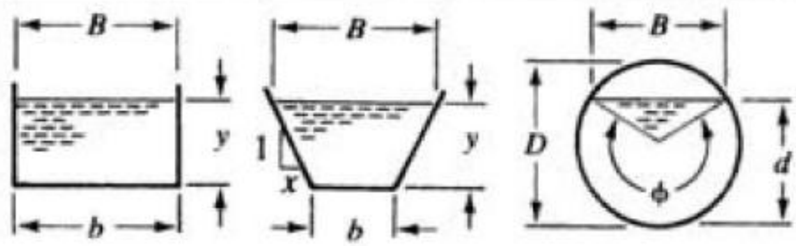


Figure 4.5 The Moody diagram.

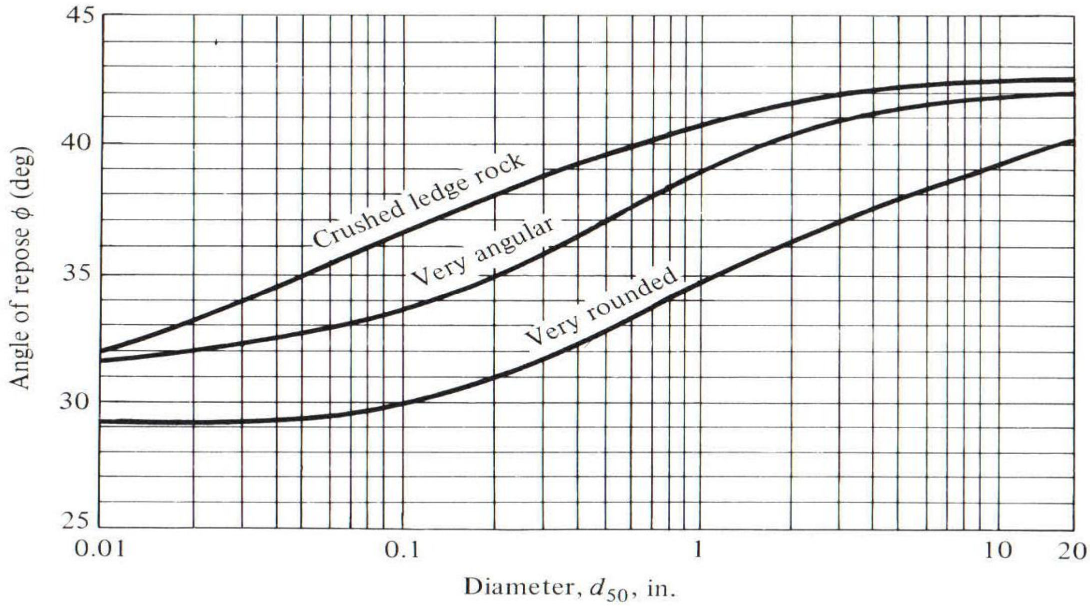
Table 4.3 Local head loss coefficients.

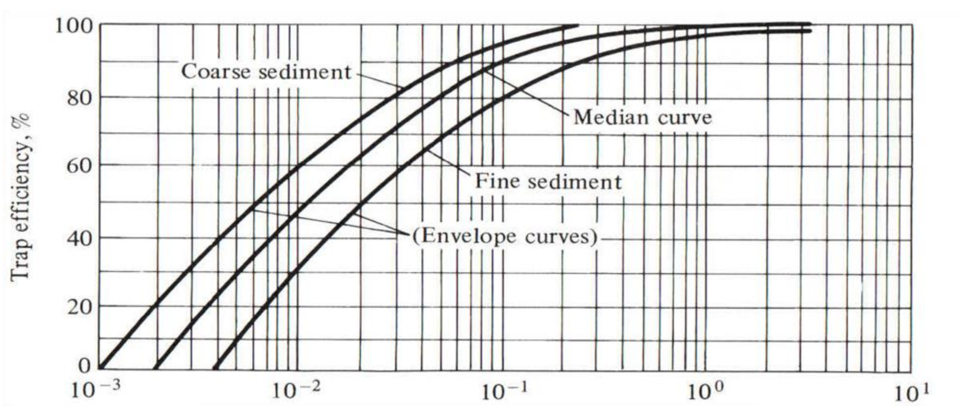
Item	k_L value		Comments
	Theoretical	Design practice	
bellmouth entrance	0.05	0.10	V = velocity in pipe
exit	0.2	0.5	
90° bend	0.4	0.5	
90° tees			
in-line flow	0.35	0.4	(for equal diameters)
branch to line	1.20	1.5	(for equal diameters)
gate valve (open)	0.12	0.25	

Table 5.1 Geometric properties of some common prismatic channels.



	Rectangle	Trapezoid	Circle
area, A	by	$(b+xy)y$	$\frac{1}{8}(\phi-\sin\phi)D^2$
wetted perimeter, P	$b+2y$	$b+2y\sqrt{1+x^2}$	$\frac{1}{2}\phi D$
top width, B	b	$b+2xy$	$\left(\sin\frac{\phi}{2}\right)D$
hydraulic radius, R	$\frac{by}{b+2y}$	$\frac{(b+xy)y}{b+2y\sqrt{1+x^2}}$	$\frac{1}{4}\left(1-\frac{\sin\phi}{\phi}\right)D$
hydraulic mean depth, D_m	y	$\frac{(b+xy)y}{b+2xy}$	$\frac{1}{8}\left(\frac{\phi-\sin\phi}{\sin(1/2\phi)}\right)D$





Capacity / annual inflow ratio

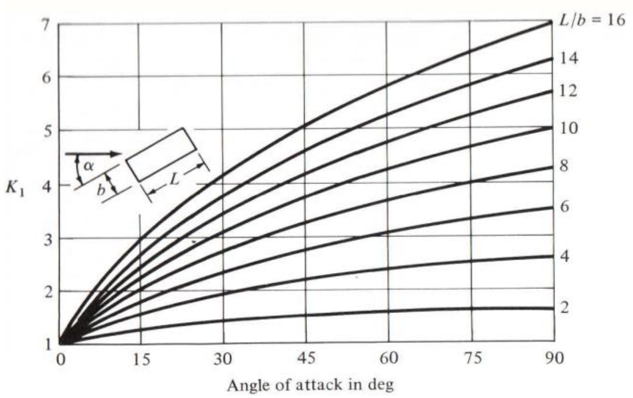


Table 8–3 Shape Coefficient K_2 for Pier Nose Forms*

Nose form	Length/width ratio	Sketch	K_2
Rectangular			1.00
Semicircular			0.90
Elliptic	2:1		0.80
	3:1		0.75
Lenticular	2:1		0.80
	3:1		0.70

*(To be used only for piers aligned with the flow.)

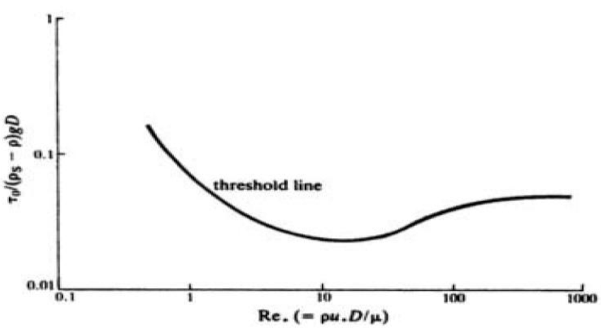


Figure 9.3 **Shields' diagram.**