# SOLUTION FINAL APRIL 2015

**Q1**

a/ $4\sqrt{12} + 5\sqrt{27} - \sqrt{75} = 8\sqrt{3} + 15\sqrt{3} - 5\sqrt{3} = 18\sqrt{3}$

b/ $\frac{1}{3}\log_3 27 - \log_3(3^3 - 18) = \log_3 27^{1/3} - \log_3 9 = \log_3 (3^3)^{1/3} - \log_3 3^2 = 1 - 2 = -1$

**Q2**

a/ $\frac{5}{2\sqrt{3}} = \frac{5\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}} = \frac{5\sqrt{3}}{6}$

b/ $\frac{2-\sqrt{5}}{2+3\sqrt{5}} = \frac{2-\sqrt{5}}{2+3\sqrt{5}} \cdot \frac{2-3\sqrt{5}}{2-3\sqrt{5}} = \frac{(2-\sqrt{5})(2-3\sqrt{5})}{4-45} = \frac{(2-\sqrt{5})(2-3\sqrt{5})}{41}$

**Q3**

a/ $5x(x^3 - 5x^2) - x^2(x^2 - 7x - 5)$
$= 5x^4 - 25x^3 - x^4 + 7x^3 + 5x^2$
$= 4x^4 - 18x^3 + 5x^2$

b/ $\frac{x^3 - 8}{x^3 - 2x^2} = \frac{(x-2)(x^2+2x+4)}{x^2(x-2)} = \frac{x^2+2x+4}{x^2}$

**Q4**

a/ $4x^2 - 16x + 15 = (2x-3)(2x-5)$

b/ $1 - 8x^2 - 9x^4 = 1 - 9x^2 + x^2 - 9x^4 = (1+x^2) - 9x^2(1+x^2) = (1+x^2)(1-9x^2) = (1+x^2)(1-3x)(1+3x)$

**Q5** $\frac{4x}{x^2-4} - \frac{2}{x^2+x-6} = \frac{4x}{(x-2)(x+2)} - \frac{2}{(x+3)(x-2)} = \frac{4x(x+3)}{(x-2)(x+2)(x+3)} - \frac{2(x+2)}{(x-2)(x+2)(x+3)}$

$= \frac{4x^2 + 12x - 2x - 4}{(x-2)(x+2)(x+3)} = \frac{4x^2 + 10x - 4}{(x-2)(x+2)(x+3)}$

**Q6**

a/ $\frac{2x}{x^2-4} = \frac{4}{x^2-4} - \frac{3}{x+2}$

$\Leftrightarrow 2x = 4 - 3(x-2)$
$\Leftrightarrow 2x = 4 - 3x + 6$
$\Leftrightarrow 5x = 10$
$\Leftrightarrow x = 2$ (rejected)

b/ $\log_3(3x-1) = 2$
$\Leftrightarrow 3x - 1 = 3^2 = 9$
$\Leftrightarrow 3x = 10$
$\Leftrightarrow x = \frac{10}{3}$ (accepted)

c/ $3^{x^3} = 9^x$
$\Leftrightarrow 3^{x^3} = 3^{2x}$
$\Leftrightarrow x^3 = 2x$
$\Leftrightarrow x^3 - 2x = 0$
$\Leftrightarrow x(x^2 - 2) = 0$
$\Leftrightarrow x(x-\sqrt{2})(x+\sqrt{2}) = 0$
$\Rightarrow x = 0;\ x = \sqrt{2};\ x = -\sqrt{2}$

**Q7**

a/ $-1 \leq \frac{3-5x}{2} \leq 9$
$\Leftrightarrow -2 \leq 3 - 5x \leq 18$
$\Leftrightarrow -5 \leq -5x \leq 15$
$\Leftrightarrow 1 \geqslant x \geqslant -3$

b/ $\left|\frac{2x+3}{3} - \frac{1}{2}\right| < 1$
$\Leftrightarrow \left|\frac{2(2x+3)-3}{6}\right| < 1$
$\Leftrightarrow -1 < \frac{4x+3}{6} < 1$
$\Leftrightarrow -6 < 4x + 3 < 6$
$\Leftrightarrow -9 < 4x < 3$
$\Rightarrow \boxed{\frac{-9}{4} < x < \frac{3}{4}}$

**Q8** $\begin{cases} 4x^2+y^2=13 \\ x^2+y^2=10 \end{cases} \Rightarrow 3x^2=3 \Rightarrow x^2=1 \Rightarrow$

- $x=1 \Rightarrow y^2=10-1^2 \Rightarrow y^2=9 \Rightarrow y=\pm 3$
  Hence 2 solutions : $(1,3)$; $(1,-3)$
- $x=-1 \Rightarrow y^2=10-(-1)^2 \Rightarrow y^2=9 \Rightarrow y=\pm 3$
  Hence 2 solutions : $(-1,3)$ ; $(-1,-3)$

**Q9**

a/ $AC=\sqrt{(-1)^2+(-9)^2}=\sqrt{82}$
$BC=\sqrt{3^2+(-3)^2}=\sqrt{18}$
$\Rightarrow$ B is closer to C than A.

b/ $x^2+y^2+4x+2y-20=0$

$(x^2+4x)+(y^2+2y)-20=0 \Rightarrow (x+2)^2+(y+1)^2-4-1-20=0$

$\Rightarrow (x+2)^2+(y+1)^2=25 \Rightarrow$ Center $=(-2,-1)$; $R=5$

**Q10**

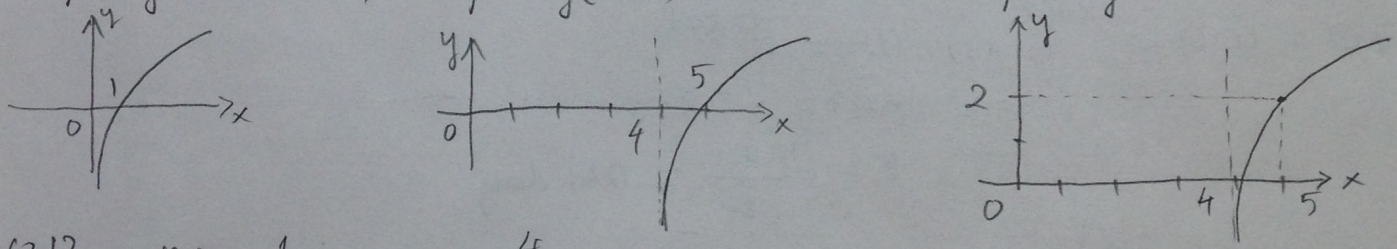
a/ $f(x)=\dfrac{2}{(2-x)^2}$
D: $x\neq 2$
Range: $(0,\infty)$

b/ $g(x)=-\sqrt{x-3x^2}$
D: $x-3x^2\geqslant 0 \Leftrightarrow x(1-3x)\geqslant 0$
$\Rightarrow$ D $=[0,\frac{1}{3}]$
Range : $(-\infty,0]$

c/ $h(x)=3|x+1|-3$
D: $\mathbb{R}$
Range : $[-3,\infty)$

**Q11**

$y=\log x \rightarrow y=\log(x-4) \rightarrow y=\log(x-4)+2$




**Q12** $f(x)=\dfrac{1}{x+2}$ $g(x)=\dfrac{4}{x-1}$

a/ $fg=\dfrac{1}{x+2}\cdot\dfrac{4}{x-1}$ b/ $\dfrac{f}{g}=\dfrac{1}{x+2}\cdot\dfrac{x-1}{4}$ c/ $fog=\dfrac{1}{\frac{4}{x-1}+2}$ d/ $gof=\dfrac{4}{\frac{1}{x+2}-1}$

**Q13** $f(x)=\dfrac{3x}{x+2}$

a/ $x=\dfrac{3y}{y+2} \Rightarrow x(y+2)=3y \Rightarrow xy+2x=3y \Rightarrow xy-3y=-2x \Rightarrow y=\dfrac{-2x}{x-3}$

b/ $\boxed{f(x)}$ HA: $y=3$
VA: $x=-2$

$\boxed{f^{-1}(x)}$ HA: $y=-2$
VA: $x=3$

14 interest at 18% = $0.18 \times 1000000 = 180000$

⇒ At 16% they need to lend: $P = \frac{180000}{0.16} = 1125000$

⇒ At 19% they need to lend: $P = \frac{180000}{0.19} = 947368.421$

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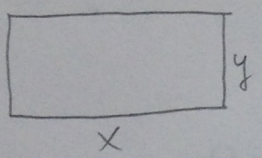


$$\left.\begin{array}{l} \text{Area} = x \cdot y \\ \text{Perimeter} = 2x + 2y \end{array}\right\} \Rightarrow \begin{cases} xy = 2(2x+2y) \\ x = 2y \end{cases}$$

$\Rightarrow 2y^2 = 2(4y + 2y)$

$\Rightarrow 2y^2 = 12y \Rightarrow 2y^2 - 12y = 0 \Rightarrow 2y(y-6) = 0 \Rightarrow \begin{cases} y = 0 \text{ (rejected)} \\ y = 6 \text{ (accepted)} \end{cases}$

$\Rightarrow x = 2 \cdot 6 = 12$

Hence: $\boxed{\begin{array}{l} x = 12 \\ y = 6 \end{array}}$

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a/ $A(9) = 100e^{-0.87 \times 9} = 0.0398(g)$

b/ $70 = 100e^{-0.87 \cdot t} \Rightarrow 0.7 = e^{-0.87t}$

$\Rightarrow \ln 0.7 = -0.87t$

$\Rightarrow t = \frac{\ln 0.7}{-0.87} = 0.41 \text{ day}$

c/ $50 = 100e^{-0.87t}$

$\Rightarrow 1 = 2e^{-0.87t}$

$\Rightarrow \frac{1}{2} = e^{-0.87t}$

$\Rightarrow \ln \frac{1}{2} = -0.87t$

$\Rightarrow t = \frac{\ln 1/2}{-0.87} = 0.797 \text{ day}$