CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number		Section(s)
Mathematics	206/4		All
Examination	**Date**	**Time**	**Pages**
Final	April 2015	3 Hours	2
Instructors			**Course Examiner**
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Special Instructions

▷ **Only approved calculators are allowed.**

MARKS

[4] **1.** Simplify the expressions below. Do not use a calculator.

(a) $4\sqrt{12}+5\sqrt{27}-\sqrt{75}$ (b) $\frac{1}{3}\log_3 27-\log_3(3^3-18)$

[4] **2.** Rationalize the denominator:

(a) $\frac{5}{2\sqrt{3}}$ (b) $\frac{2-\sqrt{5}}{2+3\sqrt{5}}$

[6] **3.** Simplify the expressions:

(a) $5x(x^3-5x^2)-x^2(x^2-7x-5)$ (b) $\frac{x^3-8}{x^3-2x^2}$

[8] **4.** Factor the polynomials completely:

(a) $4x^2-16x+15$ (b) $1-8x^2-9x^4$

[4] **5.** Perform the arithmetic operations and simplify:

$$\frac{4x}{x^2-4}-\frac{2}{x^2+x-6}$$

[9] **6.** Solve the equations:

(a) $\frac{2x}{x^2-4}=\frac{4}{x^2-4}-\frac{3}{x+2}$ (b) $\log_3(3x-1)=2$ (c) $3^{x^3}=9^x$

[8] **7.** Solve the inequalities, express your answer using set notation or interval notation:

(a) $-1\le\frac{3-5x}{2}\le 9$ (b) $\left|\frac{2x+3}{3}-\frac{1}{2}\right|<1$

[4] **8.** Solve the system of equations:

$$\begin{cases} 4x^2 + y^2 = 13 \\ x^2 + y^2 = 10 \end{cases}$$

[8] **9.** (a) Which of the points $A(6,9)$, $B(2,-3)$ is closer to the point $C(5,0)$?

(b) Show that the equation $x^2 + y^2 + 4x + 2y - 20 = 0$ represents a circle. Find coordinates of the center and radius of the circle.

[6] **10.** Find the domain and range of the functions (do not graph):

(a) $f(x) = \dfrac{2}{(2-x)^2}$ (b) $g(x) = -\sqrt{x - 3x^2}$ (c) $h(x) = 3|x+1|-3$

[5] **11.** Sketch the graph of the function $f(x) = \log(x-4) + 2$, starting from the graph of the function $g(x) = \log x$ and using appropriate transformations.

[8] **12.** Let $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{4}{x-1}$. Find:

(a) fg (b) $\dfrac{f}{g}$ (c) $f \circ g$ (d) $g \circ f$

[8] **13.** (a) Find the inverse of the function $f(x) = \dfrac{3x}{x+2}$.

(b) Find the vertical and horizontal asymptotes of both f and f^{-1} above.

[5] **14.** A bank wants to lend \$1,000,000 and wants to obtain an average return of 18% per year. If they lend at 19% or at 16%, how much can they lend at 16% and 19% to meet their requirements?

[5] **15.** A rectangle has an area that is numerically twice its perimeter. If the length is twice the width, what are its dimensions?

[8] **16.** Iodine 131 is a radioactive material that decays according to the function

$$A(t) = A_0 e^{-0.87t}$$

where A_0 is the initial amount present and A is the amount present at the time t (in days). Assume that a scientist has a sample of 100 grams of iodine 131.

(a) How much iodine 131 is left after 9 days?

(b) When will 70 grams of iodine 131 be left?

(c) What is the half-life of iodine 131?