

**CONCORDIA UNIVERSITY**  
**Department of Mathematics & Statistics**

Course	Number	Section(s)	
Mathematics	208/4	All	
Examination	Date	Time	Pages
Alternate Midterm	March 2016	1 Hour 30 minutes	2
Instructors	Course Examiner		
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**FORMULAE:**

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT \frac{(1+i)^n - 1}{i}, \quad PV = PMT \frac{1 - (1+i)^{-n}}{i}$$

**Special Instructions:**

- ▷ Answer all questions.
- ▷ Only approved calculators are allowed.

**MARKS**

- [4+6] 1. Suppose a company has fixed costs of \$28,000 and variable cost per unit of  $\frac{2}{5}x + 222$  dollars, where  $x$  is the total number of units produced. Suppose further that the selling price of its product is  $1250 - \frac{3}{5}x$  dollars per unit.

- (A) Find the break-even points.
- (B) Form the profit function from the cost and revenue functions and find the maximum profit.

- [ $2\frac{1}{2} \times 4$ ] 2. Solve for  $x$  in the following equations:

(A)  $2^{5x-x^2} = \frac{1}{64}$

(B)  $(e^2)^{2x} = (e)^{x^2-12}$

(C)  $\log_{10} 3x^2 = 2 + \log_{10} 9x$

(D)  $3 \log_b 2 + \frac{1}{2} \log_b 25 - \log_b 20 = \log_b x$

PLEASE TURN OVER

[5+5] 3.

- (A) Find the sum of all the odd numbers between 24 and 208 by only using a proper formula.
- (B) If the first term of a geometric sequence is 300 and the fourth term is 0.3, find the sum of the first 25 terms by only using a proper formula.

[10] 4. If \$2,000 is invested in an account that earns 8.25% compounded annually for 5 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.

[10] 5. A company estimates that it will have to replace a piece of equipment at a cost of \$800,000 in 5 years. To have this money available in 5 years, a sinking fund is established by making equal monthly payments into an account paying 4.8% compounded monthly.

- (A) How much should each payment be?
- (B) How much interest is earned during the last year?

[6+4] 6. A student receives a student loan for \$8,000 at 5.7% interest compounded monthly to help her finish the last 1.5 years of college. Starting 1 year after finishing college, the student must amortize the loan in the next 5 years by making equal monthly payments.

- (A) What will the payments be?
- (B) What total interest will the student pay?

Midterm Alternate March 2016

1) Cost = Variable cost + fixed cost

Variable cost = (Unit cost) x (number of items)

Cost =  $C(x) = \left(\frac{2}{5}x + 222\right)x + 28000$

Revenue =  $R(x) = (\text{Selling price/unit}) \times (\text{number of items})$   
 $= \left(1250 - \frac{3}{5}x\right)x$

A) Break even

$R(x) = C(x)$

$\left(1250 - \frac{3}{5}x\right)x = \left(\frac{2}{5}x + 222\right)x + 28000$

$x^2 - 1028x + 28000 = 0$

$(x - 1000)(x - 28) = 0$

$x = 1000, 28 \Rightarrow$  Break even at the production level of 1000 and 28

B) profit =  $R(x) - C(x)$

$= 1250x - \frac{3}{5}x^2 - \left(\frac{2}{5}x^2 + 222x + 28000\right)$

$= -x^2 + 1028x - 28000$

Profit max at the vertex (k)

$h = \frac{-b}{2a}$        $k = \frac{-(b^2 - 4ac)}{4a}$

$= \frac{-1028}{2}$

$= 514$

$= 236196$

Max profit = \$ 236196 at a production of 514 items



2a)  $2^{5x-2} = \frac{1}{64} = (64)^{-1} = 2^{-6}$

$5x-2 = -6$

$0 = x^2 - 5x - 6$

$0 = (x-6)(x+1)$

$x = 6, -1$

b)  $(e^2)^{2x} = e^{2x-12}$

$e^{4x} = e^{2x-12}$

$4x = 2x - 12$

$0 = x^2 - 4x - 12$

$= (x-6)(x+2)$

$x = 6, -2$

c)  $\log_{10} 3x^2 = 2 + \log_{10} 9x$

$\log 3x^2 - \log 9x = 2$

$\log \frac{3x^2}{9x} = 2$

$\frac{3x^2}{9x} = 10^2 = 100$

$3x^2 = 900x$

$3x^2 - 900x = 0$

$3x(x-300) = 0$

$x = 0, x = 300$

GR:

$\log 3x^2 = \log 10^2 + \log 9x$   
 $3x^2 = 100(9x) = 900x$

check

d)  $\log_b \frac{2^3 \sqrt{25}}{20} = \log_b x$  ;  $\frac{8(5)}{20} = x$

$2 = x$

3, A, Odd # bet. 24 to 208

25, 27, ... 207  $\rightarrow$  This is an AP;  $d=2, a_1=25, a_n=207$

$$a_n = a_1 + (n-1)d$$

$$207 = 25 + (n-1)2$$

$$= 25 + 2n - 2$$

$$184 = 2n$$

$$n = 92$$

$$S_{92} = \frac{n}{2} (a_1 + a_n)$$

$$= \frac{92}{2} (25 + 207)$$

$$= 46 (232)$$

$$= 10672$$

3 B) G.P. 1st term  $= a_1 = 300$   
4th = 0.3

$$n\text{th term} = a_1 r^{n-1}$$

$$0.3 = 300 (r)^3$$

$$\frac{0.3}{300} = r^3$$

$$\frac{0.1}{100} = r^3$$

$$r = \frac{1}{10}$$

$$S_{25} = \frac{a_1 (r^n - 1)}{r - 1} = \frac{a_1 (1 - r^n)}{1 - r}$$

$$= 300 \left[ \frac{1 - \left(\frac{1}{10}\right)^{25}}{1 - \frac{1}{10}} \right]$$

$$= \frac{3000}{9} \left[ 1 - (0.1)^{25} \right]$$

$$\approx \frac{3000}{9}$$

#14	Period	Interest	Amount	$r = 8\% \text{ p.a.}$ $= 0.08$
	0		2000	
	1	165.00	2165	82.5%
	2	178.61	2343.61	<u>2</u>
	3	193.34	2536.95	
	4	209.29	2746.24	
	5	226.56	2972.80	

#5 p. 151 example 2.

$$FV = 800,000, \quad r = 4.8\%, \quad i = \frac{0.048}{12} = 0.004$$

$$PMT = FV \frac{i}{(1+i)^n - 1} \quad n = 5 \times 12 = 60$$

$$= 800,000 \frac{(0.004)}{(1+0.004)^{60} - 1}$$

$$PMT = 11823.79$$

B. Interest in the 5<sup>th</sup> year = Amount in 5 yrs - amount in 4 yrs - 12 payment in 5<sup>th</sup> year.

$$FV = PMT \frac{(1+i)^n - 1}{i} \quad \text{for 4 yrs} = n = 48$$

$$= 11823.79 \left[ \frac{(1+0.004)^{48} - 1}{0.004} \right] = 624315.51$$

$$\text{Growth in 5<sup>th</sup> year} = 800000 - 624315.51 = 175684.49$$

5B interest during 5<sup>th</sup> year =  $175684.49 - 12(\text{PMT})$   
 $= 33799$

fb student loan of 8000 at 5.7% to pay in 2.5 yrs.  
 monthly annuity for 5 yrs

The compounded interest for 2.5 yrs + 8000 = A

$$A = P(1+i)^n$$

$$= 8000(1+0.057)^{30}$$

$$= 42202$$

$n = 30$ $i = 0.057$
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thus is the PV to be paid for 5 yrs.

A)  $\text{pmt} = PV \frac{i}{1-(1+i)^{-n}} = 42202 \frac{0.057}{1-(1+0.057)^{-60}}$

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