PROBLEM 2.83

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 43.2°$ and $\theta_z = 83.8°$. Knowing that the *y* component of the force is -50 N, determine (*a*) the angle θ_y, (*b*) the other components and the magnitude of the force.

SOLUTION

(*a*) We have $(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1$

$$(\cos\theta_y)^2 = 1 - (\cos\theta_x)^2 - (\cos\theta_z)^2$$

Since $F_y < 0$, we must have $\cos\theta_y < 0$

Thus: $\cos\theta_y = -\sqrt{1 - (\cos 43.2°)^2 - \cos(83.8°)^2}$

$$\cos\theta_y = -0.67597$$

$\theta_y = 132.5°$ ◀

(*b*) Then: $F = \dfrac{F_y}{\cos\theta_y}$

$$F = \frac{-50\text{ N}}{-0.675\,97}$$

$$F = 73.968\text{ N}$$

And: $F_x = F\cos\theta_x$

$$F_x = (73.968\text{ N})\cos 43.2°$$

$F_x = 53.9$ N ◀

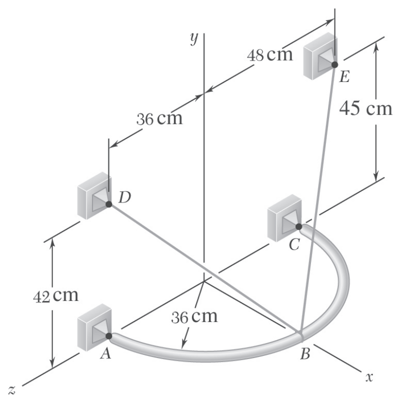
$$F_z = F\cos\theta_z$$

$$F_z = (73.968\text{ N})\cos 83.8°$$

$F_z = 7.99$ N ◀

$F = 74.0$ N ◀

PROBLEM 2.87



A steel rod is bent into a semicircular ring of radius 36 cm and is supported in part by cables *BD* and *BE* which are attached to the ring at *B*. Knowing that the tension in cable *BD* is 55 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{DB} = (36\text{ cm})\mathbf{i} - (42\text{ cm})\mathbf{j} - (36\text{ cm})\mathbf{k}$$

$$DB = \sqrt{(36\text{ cm})^2 + (-42\text{ cm})^2 + (-36\text{ cm})^2} = 66\text{ cm}$$

$$\mathbf{T}_{DB} = T_{DB}\lambda_{DB} = T_{DB}\frac{\overrightarrow{DB}}{DB}$$

$$\mathbf{T}_{DB} = \frac{55\text{ N}}{66\text{ cm}}\left[(36\text{ cm})\mathbf{i} - (42\text{ cm})\mathbf{j} - (36\text{ cm})\mathbf{k}\right]$$

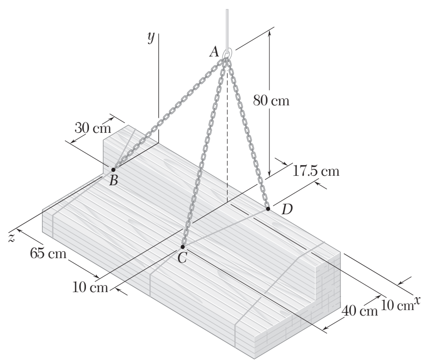
$$= (30\text{ N})\mathbf{i} - (35\text{ N})\mathbf{j} - (30\text{ N})\mathbf{k}$$

$$\therefore\ (T_{DB})_x = 30.0\text{ N} \blacktriangleleft$$

$$(T_{DB})_y = -35.0\text{ N} \blacktriangleleft$$

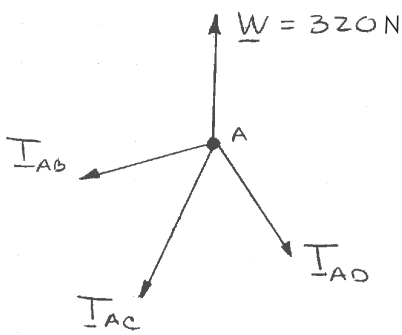
$$(T_{DB})_z = -30.0\text{ N} \blacktriangleleft$$

PROBLEM 2.109



A 320 N load of lumber is lifted using a triple leg sling. Knowing that at the instant shown the lumber is at rest, determine the tension in each leg of the sling.

SOLUTION



$$\overrightarrow{AB} = -(65\text{ cm})\mathbf{i} - (80\text{ cm})\mathbf{j} + (20\text{ cm})\mathbf{k}$$

$$AB = \sqrt{(-65\text{ cm})^2 + (-80\text{ cm})^2 + (20\text{ cm})^2} = 105\text{ cm}$$

$$\mathbf{T}_{AB} = \frac{T_{AB}}{105\text{ cm}}\left[-(65\text{ cm})\mathbf{i} - (80\text{ cm})\mathbf{j} + (20\text{ cm})\mathbf{k}\right]$$

$$= T_{AB}(-0.619\ 05\mathbf{i} - 0.7619\ 0\mathbf{j} + 0.190\ 476\mathbf{k})$$

$$\overrightarrow{AC} = (10\text{ cm})\mathbf{i} - (80\text{ cm})\mathbf{j} + (40\text{ cm})\mathbf{k}$$

$$AC = \sqrt{(10\text{ cm})^2 + (-80\text{ cm})^2 + (40\text{ cm})^2} = 90\text{ cm}$$

$$\mathbf{T}_{AC} = \frac{T_{AC}}{90\text{ cm}} \quad (10\text{ cm})\mathbf{i} - (80\text{ cm})\mathbf{j} + (40\text{ cm})\mathbf{k}$$

$$= T_{AC}(0.111\ 111\mathbf{i} - 0.888\ 89\mathbf{j} + 0.444\ 44\mathbf{k})$$

$$\overrightarrow{AD} = (17.5\text{ cm})\mathbf{i} - (80\text{ cm})\mathbf{j} - (10\text{ cm})\mathbf{k}$$

$$AD = \sqrt{(17.5\text{ cm})^2 + (-80\text{ cm})^2 + (-10\text{ cm})^2} = 82.5\text{ cm}$$

$$\mathbf{T}_{AD} = \frac{T_{AD}}{82.5\text{ cm}}\left[(17.5\text{ cm})\mathbf{i} - (80\text{ cm})\mathbf{j} - (10\text{ cm})\mathbf{k}\right]$$

$$= T_{AD}(0.212\ 12\mathbf{i} - 0.969\ 70\mathbf{j} - 0.121\ 212\mathbf{k})$$

continued

PROBLEM 2.109 CONTINUED

At A, $\Sigma\mathbf{F} = 0$

$$\Sigma F_x = 0: \quad -0.619\,05T_{AB} + 0.111\,111T_{AC} + 0.212\,12T_{AD} = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad -0.761\,90T_{AB} - 0.888\,89T_{AC} - 0.969\,70T_{AD} + W = 0 \qquad (2)$$

$$\Sigma F_z = 0: \quad 0.190\,476T_{AB} + 0.444\,44T_{AC} - 0.121\,212T_{AD} = 0 \qquad (3)$$

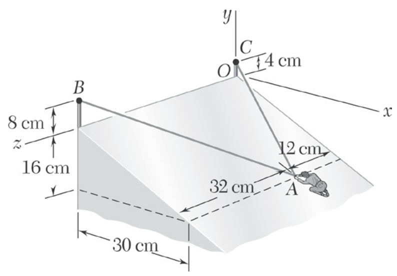
Substituting for $W = 32$ N and solving Equations (1), (2), (3) simultaneously:

$T_{AB} = 86.2$ N ◀

$T_{AC} = 27.7$ N ◀

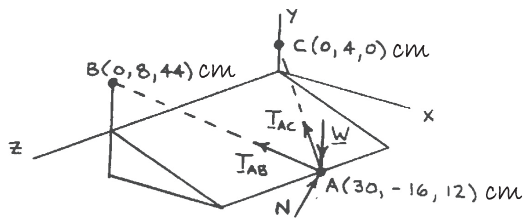
$T_{AD} = 237$ N ◀

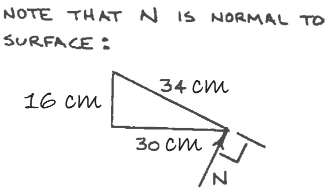
PROBLEM 2.119



In trying to move across a slippery icy surface, a 175-N man uses two ropes *AB* and *AC*. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION





$$d_{AB} = \sqrt{(-30\text{ cm})^2 + (24\text{ cm})^2 + (32\text{ cm})^2} = 50\text{ cm}$$

$$d_{AC} = \sqrt{(-30\text{ cm})^2 + (20\text{ cm})^2 + (-12\text{ cm})^2} = 38\text{ cm}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = \frac{T_{AB}}{50\text{ cm}}\left[-(30\text{ cm})\mathbf{i} + (24\text{ cm})\mathbf{j} + (32\text{ cm})\mathbf{k}\right]$$

$$= T_{AB}(-0.6\mathbf{i} + 0.48\mathbf{j} + 0.64\mathbf{k})$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = \frac{T_{AC}}{38\text{ cm}}\left[-(30\text{ cm})\mathbf{i} + (20\text{ cm})\mathbf{j} - (12\text{ cm})\mathbf{k}\right]$$

$$= T_{AC}\left(-\frac{30}{38}\mathbf{i} + \frac{20}{38}\mathbf{j} - \frac{12}{38}\mathbf{k}\right)$$

$$\mathbf{N} = \frac{16}{34}N\mathbf{i} + \frac{30}{34}N\mathbf{j}$$

$$\mathbf{W} = -(175\text{ N})\mathbf{j}$$

At point *A*, $\Sigma\mathbf{F} = 0$: $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} = 0$

i component: $-0.6T_{AB} - \frac{30}{38}T_{AC} + \frac{16}{34}N = 0$ (1)

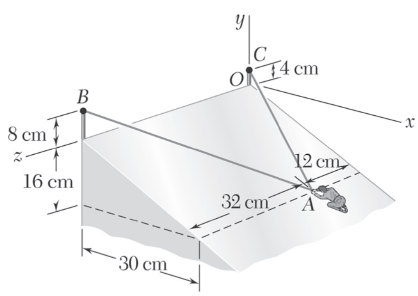
j component: $0.48T_{AB} + \frac{20}{38}T_{AC} + \frac{30}{34}N - 175\text{ N} = 0$ (2)

k component: $0.64T_{AB} - \frac{12}{38}T_{AC} = 0$ (3)

Solving Equations (1), (2), and (3) simultaneously:

$T_{AB} = 30.9\text{ N}$ ◀

$T_{AC} = 62.5\text{ N}$ ◀



PROBLEM 2.120

Solve Prob. 2.119 assuming that a friend is helping the man at *A* by pulling on him with a force $\mathbf{P} = -(45\ \text{N})\mathbf{k}$.

Problem 2.119: In trying to move across a slippery icy surface, a 175-N man uses two ropes *AB* and *AC*. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Refer to the solution of problem 2.119 and the resulting linear algebraic Equations (1), (2), (3). Include force $\mathbf{P} = -(45\ \text{N})\mathbf{k}$ with other forces of Problem 2.119.

Now at point A, $\Sigma\mathbf{F} = 0$: $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{N} + \mathbf{W} + \mathbf{P} = 0$

i component: $$-0.6T_{AB} - \frac{30}{38}T_{AC} + \frac{16}{34}\text{N} = 0 \qquad (1)$$

j component: $$0.48T_{AB} + \frac{20}{38}T_{AC} + \frac{30}{34}\text{N} - 175\ \text{N} = 0 \qquad (2)$$

k component: $$0.64T_{AB} - \frac{12}{38}T_{AC} - 45\ \text{N} = 0 \qquad (3)$$

Solving (1), (2), and (3) simultaneously:

$T_{AB} = 81.3\ \text{N}$ ◀

$T_{AC} = 22.2\ \text{N}$ ◀