Name: *INSTRUCTOR SOLUTIONS*

Question 1 (5 Marks)

(a) Adding heat energy to an object causes its temperature to rise as described by the equation on the right. Use dimensional analysis to show that this equation is dimensionally homogenous.

$$\underbrace{Q}_{\text{TERM A}} = \underbrace{mc(T_1 - T_2)}_{\text{TERM B}}$$

where: m = mass (kg)
c = specific heat capacity (J/kg K)
Q = heat energy (J)
T = temperature (K)

2 $[\text{TERM A}] = [Q] = [\text{ENERGY}]$

$[\text{TERM B}] = [\cancel{M}]\dfrac{[\text{ENERGY}]}{[\cancel{M}][\cancel{T}]}[\cancel{T}] = [\text{ENERGY}]$

Since Term A and Term B have the same dimensions, this equation is dimensionally homogenous.

Alternatively, you could have broken these terms down to fundamental dimensions

$[\text{TERM A}] = [Q] = [\text{ENERGY}] = [\text{FORCE}][L] = \dfrac{[M][L]}{[t]^2}[L] = \dfrac{[M][L]^2}{[t]^2}$

$[\text{TERM B}] = [\cancel{M}]\dfrac{[\text{ENERGY}]}{[\cancel{M}][\cancel{T}]}[\cancel{T}] = [\text{FORCE}][L] = \dfrac{[M][L]}{[t]^2}[L] = \dfrac{[M][L]^2}{[t]^2}$

Again, since the two terms have the same fundamental dimensions, this equation is dimensionally homogenous.

(b) Fourier's Law of heat conduction can be written as shown in the equation to the right. Use dimensional analysis to express k in terms of fundamental dimensions, and provide typical metric units for thermal conductivity. Clearly show how you came to your answer.

$$\dot{Q} = \frac{k(T_1 - T_2)}{L}$$

where: $\dot{Q}$ is heat flux (W/m^2)
k is thermal conductivity
T is temperature (K)
L is length

3 In this case we are being asked to express k in terms of fundamental dimensions.

$[\dot{Q}] = \dfrac{[\text{POWER}]}{[L]^2} = \dfrac{[\text{ENERGY}]}{[t][L]^2} = \dfrac{[\text{FORCE}][L]}{[t][L]^2} = \dfrac{[M][L][L]}{[t]^2[t][L]^2} = \dfrac{[M]}{[t]^3}$

We also know that $[(T_1 - T_2)] = [T]$

$\therefore [\dot{Q}] = \dfrac{[k][(T_1 - T_2)]}{[L]} \rightarrow \dfrac{[M]}{[t]^3} = \dfrac{[k][T]}{[L]} \rightarrow \boxed{[k] = \dfrac{[M][L]}{[t]^3[T]}}$

typical metric units could be $\dfrac{kg\,m}{s^3\,K}$ or W/mK

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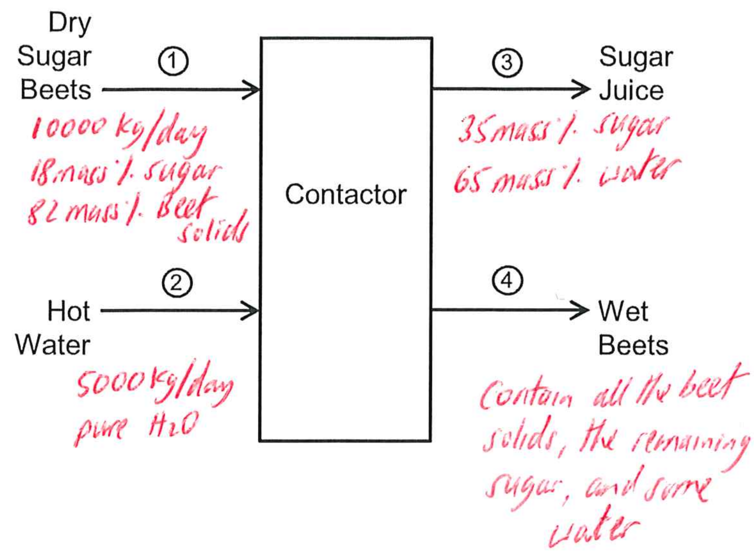
Question 2 (11 marks)

Table sugar can be produced from sugar beets. Dry sugar beets, which contain 18 mass% sugar and 82 mass% 'beet solids', are fed (Stream 1) into a vessel called a contactor at a rate of 10,000 kg/day. Pure hot water is fed (Stream 2) into the contactor at a rate of 5,000 kg/day. Within the contactor, 70% of the sugar originally in the dry sugar beets dissolves in the hot water to form sugar juice, which leaves the vessel in Stream 3. The sugar juice contains 35 mass% sugar with the balance being water (it does not contain any 'beet solids'). The dry sugar beets, which have now lost some sugar but absorbed some water, leave the contactor as wet beets in Stream 4. Assuming this process operates at steady state, and does not involve any reactions, answer the questions below.



(a) Determine the mass flow rate (in kg/day) of sugar in Stream 1.

1

$$\text{mass flow rate of sugar in stream 1} = \left(\text{mass fraction of sugar in stream 1}\right)\left(\text{total flow rate of stream 1}\right)$$

$$= (0.18)(10000\ \text{kg/day})$$

$$= \boxed{1800\ \text{kg/day}}$$

(b) Determine the mass flow rate (in kg/day) of sugar in Stream 3.

1

We are told that 70% of the sugar in the dry sugar beets (i.e. stream 1) ends up in stream 3.

$$\therefore \text{mass flow rate of sugar in stream 3} = (0.7)(1800\ \text{kg/day}) = \boxed{1260\ \text{kg/day}}$$

(c) Calculate the mass flow rate (in <u>kg/h</u>) of sugar juice in Stream 3

2

$$\text{mass \% sugar in stream 3} = \frac{\text{mass flow rate of sugar in stream 3}}{\text{mass flow rate of stream 3}} \times 100\%$$

$$35\ \text{mass \%} = \frac{1260\ \text{kg/day}}{\text{mass flow rate of stream 3}} \times 100\%$$

$\therefore$ mass flow rate of stream 3 $= 3600$ kg/day ← but we are asked to provide the answer in kg/h

$$\therefore \text{mass flow rate of stream 3} = \left(3600\ \frac{\text{kg}}{\text{day}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right) = \boxed{150\ \text{kg/h}}$$

$\dot{m}$ = symbol for mass flow rate

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Question 2 *(Contd.)*

→ composed of beet solids, water and sugar

(d) What is the composition (in mass%) of wet beets in Stream 4?

4

- we are told that all beet solids in stream 1 end up in stream 4

$$\dot{m}_{beet solids,4} = (0.82)(10000 \text{ kg/day}) = 8200 \text{ kg/day}$$

- water is at steady state

$$\dot{m}_{water,4} = \dot{m}_{water,2} - \dot{m}_{water,3} = 5000 \text{ kg/day} - (3600 \text{ kg/day} - 1260 \text{ kg/day}) = 2660 \text{ kg/day}$$

- sugar at steady state

$$\dot{m}_{sugar,4} = \dot{m}_{sugar,1} - \dot{m}_{sugar,3} = 1800 \text{ kg/day} - 1260 \text{ kg/day} = 540 \text{ kg/day}$$

∴ Total mass flow rate of stream 4 = 8200 kg/day + 2660 kg/day + 540 kg/day = 11400 kg/day

∴ Composition of wet beets is:

(i) beet solids: $\frac{8200 \text{ kg/day}}{11400 \text{ kg/day}} \times 100\% =$ 71.93 mass % beet solids

(ii) sugar: $\frac{540 \text{ kg/day}}{11400 \text{ kg/day}} \times 100\% =$ 4.74 mass % sugar

(iii) water: $\frac{2660 \text{ kg/day}}{11400 \text{ kg/day}} \times 100\% =$ 23.33 mass % water

(e) Determine the mol fraction of sugar in the produced sugar juice. Note that M_{sugar} = 342.3 kg/kmol and M_{water} = 18 kg/kmol.

3

Since this question requires us to perform a mass fraction to mol fraction conversion, we can use the values given in the question, or just assume a convenient total (eg 100 kg of sugar juice).

- Assume 100 kg of sugar juice → mass of sugar = (0.35)(100 kg) = 35 kg

mass of water = (0.65)(100 kg) = 65 kg

moles of sugar = 35 kg / 342.3 kg/kmol = 0.1022 kmol

moles of water = 65 kg / 18 kg/kmol = 3.6111 kmol

$$\therefore \text{mol fraction of sugar} = \frac{0.1022 \text{ kmol}}{0.1022 \text{ kmol} + 3.6111 \text{ kmol}} = \boxed{0.0275}$$

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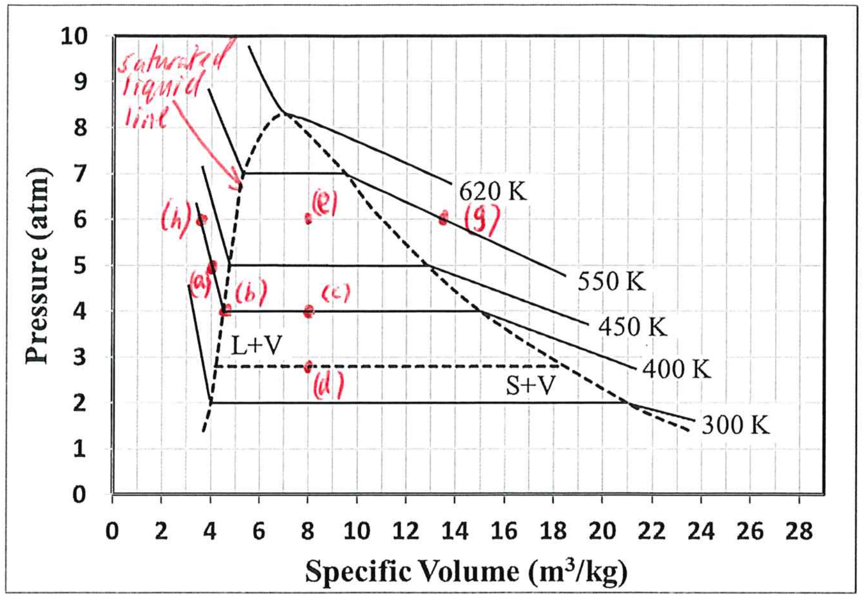
Question 3 (14 marks)

Shown to the right is a P-V diagram for a newly discovered substance called Stuffium ($M_{stuffium}$ = 100 kg/kmol). You conduct experiments with Stuffium using a variable volume container that is fitted with an injection port.



PART A: You inject 250 g of pure Stuffium into a completely empty variable volume container with a starting volume of 1 m³.

(a) At an equilibrium temperature of 400 K, determine the specific volume (in m³/kg) and pressure (in atm) of the contents at equilibrium.

1 $V = \frac{v}{m} = \frac{1 m^3}{0.25 kg} = 4 m^3/kg$

$P = 5$ atm (see point (a) on graph)

(b) Following from part (a), you isothermally increase the volume of the container until you have a saturated liquid. Determine the pressure (in atm) and the specific volume (in m³/kg) at equilibrium under these conditions. Isothermal → constant temperature → follow an isotherm

1 $P = 4$ atm $V = 4.5 m^3/kg$ (see point (b) on graph)

(c) Following from part (b), you continue to isothermally increase the volume until the specific volume is 8 m³/kg. Determine the phases present at equilibrium, and the specific volume (in m³/kg) of each phase present.

2 LIQUID: $V_L = 4.5 m^3/kg$ see point (c) on graph.

VAPOUR: $V_V = 15 m^3/kg$

(d) Following from part (c), you lower the temperature of the contents in the vessel at constant volume until an additional phase appears. At equilibrium under these conditions, determine the following:

(i) The pressure (in kPa). *Note that 1 atm = 101.325 kPa.*

1 $P = (2.8 atm)\left(101.325 \frac{kPa}{atm}\right) = 283.71 kPa$

(ii) The degrees of freedom present. Show all of your work. → you are on the three phase line

1 $F = C - P + 2 = 1 - 3 + 2 = 0$ (see point (d) on graph)

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Question 3 *(Contd.)*

(e) Following from part (d), you now heat the vessel contents while keeping the volume constant. Determine the density (in kg/m^3) of the liquid phase present at an equilibrium pressure of 6 atm.

1 $V_L = 5\,m^3/kg$ see point (e) on graph.

$\therefore \rho_L = \frac{1}{V_L} = \frac{1}{5\,m^3/kg} = \boxed{0.2\ kg/m^3}$

(f) Following from part (e), if you are told that exactly half of the contents in the vessel are in the vapour phase at equilibrium, calculate the volume (in m^3) occupied by the vapour phase.

2 mass of vapour $= 250g/2 = 125g = 0.125\ kg$

$\therefore$ volume of vapour $= (V_v)(\text{mass of vapour}) = (11\,m^3/kg)(0.125\,kg) = \boxed{1.375\,m^3}$

PART B: You now empty the vessel completely, and set the volume to 0.135 m^3. While keeping the volume constant, you place 10 g of Stuffium into the vessel.

(g) Determine the temperature (in K) in the vessel, and the amount (in mol) of each phase present at an equilibrium pressure of 6 atm.

2.5 $V = \frac{v}{m} = \frac{0.135\,m^3}{0.01\,kg} = 13.5\,m^3/kg \rightarrow$ only phase present is vapour (see point (g) on graph)

moles of stuffium vapour $= \frac{0.01\,kg}{100\,kg/kmol} = 1\times10^{-4}\,kmol = \boxed{0.1\,mol}$

(h) Following from part (g), you decide to isobarically inject Stuffium into the vessel while maintaining a constant volume. Determine the mass (in g) of Stuffium that you need to inject into the vessel if you want the final equilibrium temperature to be 400 K.

see point (h) on graph $\rightarrow$ this is where you will be after the injection.

2.5

$V = 3.5\,m^3/kg$

Since the volume was constant at $0.135\,m^3$

mass of stuffium in the vessel $= \frac{0.135\,m^3}{3.5\,m^3/kg} = 0.03857\,kg$

$= 38.57\,g$

However, 10g of stuffium was in the vessel when the injection started

$\therefore$ mass of stuffium added $= 38.57g - 10g = \boxed{28.57g}$