M. Golding is a manufacturer of toy sailboats used by children to play in the bathtub. He is wondering how to improve the quality of his forecasts. A management student, who knows about a lot of things, including forecasting, proposed three forecasting methods. The results of student's analysis are given in the following table:

Month	Demand	Moving Average N = 3	Exponential Smoothing α = 0.3	Linear Regression a = 1367 b = 20.5
1	1514		1514	1387.5
2	1850		1514.0	1408
3	1712		1614.8	1428.5
4	1185	1692.0	1644.0	1449
5	1250	1582.3	1506.3	1469.5
6	1283	1382.3	1429.4	1490
7	1711	1239.3	1385.5	1510.5
8	1511	1414.7	1483.1	1531
9	1425	1501.7	1491.5	1551.5
10	750	1549.0	1471.5	1572
11	875	1228.7	1255.1	1592.5
12	304	1016.7	1141.1	1613
13	2277	643.0	889.9	1633.5
14	2790	1152.0	1306.1	1654
15	1925	1790.3	1751.2	1674.5
16	1546	2330.7	1803.4	1695
17	1563	2087.0	1726.2	1715.5
18	1641	1678.0	1677.2	1736
19	2419	1583.3	1666.3	1756.5
20	1855	1874.3	1892.1	1777
21	2034	1971.7	1881.0	1797.5
22	1055	2102.7	1926.9	1818
23	1333	1648.0	1665.3	1838.5
24	405	1474.0	1565.6	1859
25	3080	931.0	1217.4	1879.5
26	3496	1606.0	1776.2	1900
27	2746	2327.0	2292.1	1920.5
28	1981	3107.3	2428.3	1941
29	2017	2741.0	2294.1	1961.5
30	2046	2248.0	2211.0	1982
31	2999	2014.7	2161.5	2002.5
32	2311	2354.0	2412.7	2023
33	2617	2452.0	2382.2	2043.5
34	1044	2642.3	2452.7	2064
35	1851	1990.7	2030.1	2084.5
36	505	1837.3	1976.3	2105.0

The coefficient of determination, R^2 for the linear regression over the 36 months was 0.082

[3] (a) Make a forecast for month 37 with the three models Show all your calculations and fill in the blanks:

Moving Average F_{37} = 1133.3 ✓

$\frac{1044 + 1851 + 505}{3}$ =

moving avg $= \frac{\Sigma \text{ demand in previous "n" periods}}{n}$

-0.25

1967.3

Exponential Smoothing F_{37} = 1437.61

$F_{37} = \cancel{1837.3} + 0.3(505 - 18$

$= 1437.61$

last period's forecasting + α (last period's demand − forecast)

4

(b) The student calculated the Sum of the Absolute Errors (ΣE_t) for the **35 months inclusive**:

	Moving Average	Exponential Smoothing	Linear Regression
Sum of the Absolute Errors (month 1 to 35)	20,990.67	18,149.89	17,757.00

Taking into consideration the forecast of period 36 showing in the table on page 2, calculate the Mean Absolute Deviation (MAD) for two of the three forecasting methods. Show all your calculations.

-0.25

Moving Average: **MAD =** Σ | actual - forecast | / n = 20990.67 + |(505 - 1837.3)| / 36

≐ 546.069

n = 33

-0.25

Linear Regression: **MAD =** Σ | actual - forecast | / n

-0.25

= 18149.89 + |(505 - 1976.3)| / 36

≐ ~~463.29~~

537.7

[2] (c) The student calculated the Sum of the Squared Errors (ΣE_t^2) for the **36 months** inclusive:

	Exponential Smoothing	Linear Regression
Sum of the Squared Errors (month 1 to 36)	20,841,369.42	18,426,019.50

Calculate the standard deviation of the errors for two of the three forecasting methods, using the above data. Show all your calculations.

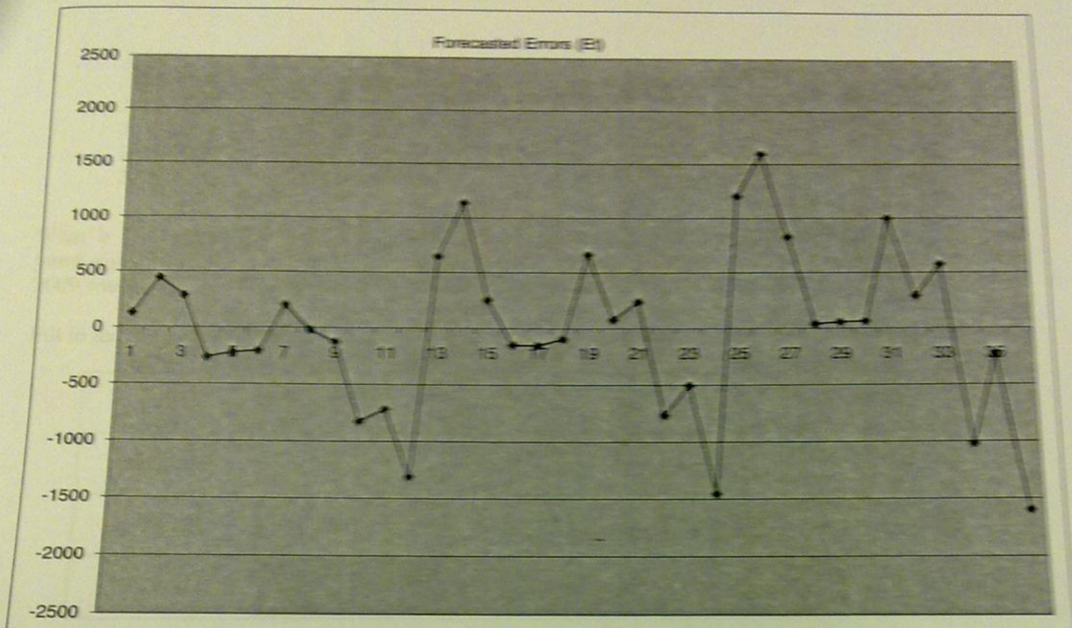
Exponential Smoothing: σ = $\sqrt{20841369.42 / 36}$

≐ 760.87

Linear Regression: σ = $\sqrt{18426019.5 / 36}$

2

Golding decides to track the evolution of the errors from the regression model through time (see graph below).



(d) Based on the graph and the measures of accuracy/adequacy for linear regression method, what are your conclusions and recommendations? (i.e. is linear regression a good method to use in this case? What forecasting method should Mr. Golding use? Should other methods be considered for this forecast? If yes, indicate which one(s) and why, but do not do any calculations. Justify each of your answers briefly.

My conclusion is that, the graph clearly shows a seasonal trend, re-occuring about every 12 months, with no relevant data to show the linear relation between the 2 sets of data graphed above. To predict the forecasting or seasonal trends, it is best to use exponential methods. to better calculate the trend, and minimize the errors.

wrong

- R^2 analysis not provided and Linear regression's not good a

- Multiplicative suggestion

exponential method can not predict

Problem 2: **Forecasting: General Independent Questions (16 marks)**

(a) The president of Province University wants to forecast student enrollments for next academic year (2010-2011) based on the following historical data:

Year	Enrollments
2005-2006	20,000
2006-2007	20,500
2007-2008	21,000
2008-2009	23,100
2009-2010	25,000

What is the forecast for 2010-2011 using double exponential smoothing (exponential smoothing with trend adjustment) if $\alpha = 0.5$ and $\beta = 0.1$? Assume that the exponentially smoothed forecast, F_t, for 2008-2009 was 21,425 and the exponentially smoothed trend estimate, T_t, for 2008-2009 was 1,027.5.

Fill in the following table: $\alpha = 0.5$ $\beta = 0.1$

Year	A_t	F_t	T_t	FIT_t
2008-2009	23,100	21,425	1,027.5	22,452.5
2009-2010	25,000	22776.25	1059.88	23836.125
2010-2011		24418.065	1118.0375	25536.[illegible]

$FIT = \alpha(A_{t-1}) + (1-\alpha)(F_{t-1} + T_{t-1}) + \beta($[illegible]

Show your calculations below:

F_t — T_t

2009-2010 $0.5(23100) + (0.5)(21425 + 1027.5)$ $= 22776.25$ $0.1($[illegible] $= 1059.88$

\+

22836.125

2010-2011 $0.5(25000) + (0.5)(22776.25 + 1059.88)$ $= 24418.065$ $0.1(24418.065 -$ [illegible] $= 1118.0375$

(b) In January 2010, Mr. Honey, a life insurance broker, wished to forecast the annual number of new life-insurance contracts for his company for 2010 using an exponential smoothing model with $\alpha = 0.2$.

Unfortunately, a clerk committed an error in computing the 2010 forecast: he used 1500 as the number of new contracts in **2008**, instead of 5100 contracts in that year. **Given this error, should the 2010 forecast be increased or decreased and by how much?** Give the details of your analysis.

The 2010 forecast should be increased by 7200

to accomodate for the losses in the error.

whether

$$F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1})$$

$$y = x + 0.2(1500) - 0.2F_{t-1}$$

$$= x + 3000 - 0.2F_{t-1}$$

vs. $y = x + 0.2(5100) - 0.2$

$= x + 10200 - 0.2$

$10200 - 3000 = \underline{7200}$ new contra

are not been 'inclu

in the calculation.

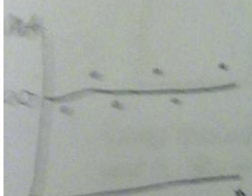
-1.5

The numerical value sets a starting point for the new forecasted contracts.

Should be increased by $\alpha(1-\alpha)(5100-1500)$ $= 576$ units.

(c) Suppose you use the multiplicative decision model (with linear regression) with 4 seasons (i.e. N = 4) on a stable time series, centered around an average value of 1,300. What would you expect the values of a (i.e. the y-intercept estimate), and b (i.e. the slope estimate), and S1, S2, S3 and S4 (i.e. the seasonal factors) to be? Justify your answer. **Hint:** plot the graph of a stable times series.

$\alpha = 0$ CMA = 1300 n = 4



$a = 1300$

$b = 0$

$S_1 = S_2 = S_3 = S_4$

$\Sigma S_i = 4$

If it is a truely stable time series, the α would equal di

to zero (0). Thus, the y-int would HAVE to be at 13

where the CMA = y-int in a perfectly stable time series data, speci

for multiplicative decision model. And the slope (b) of

the CMA to be 0, or 1 (for a upward or d

what in

4

[4] (d) The Holiday Hotel chain owns four hotels in Ottawa region. The chain collected the number of room occupancy per day in each of its hotel for the past five years. Using these data, the chain calculated the average number of room occupancy for each quarter of the year. The table below summarizes the time series data.

Average number of room occupancy per quarter

	Year				
Season	1	2	3	4	5
I	1996	2036	2182	2227	2288
II	2320	2414	2475	2552	2612
III	2844	2917	2933	3041	3102
IV	2248	2264	2338	2394	2499

← trend,

The multiplicative decomposition model is used to forecast the quarter demand. There are four seasons per year and linear regression has been used on the deseasonalized demand to estimate the trend component. The following results have been obtained using this model:

Year	Season	Period t	Demand (A_t)	Trend (T_t)	Forecast (F_t)	Error (E_t)
1	I	1	1996	2314.81	2023.14394	-27.14394
	II	2	2320	2332.57	2332.57	-12.57
	III	3	2844	2350.33	2803.94369	40.05631
	IV	4	2248	2368.09	2209.42797	38.57203
2	I	5	2036	2385.85	2085.2329	-49.2329
	II	6	2414	2403.62	2403.62	10.38
	III	7	2917	2421.38	2888.70634	28.29366
	IV	8	2264	2439.14	2275.71762	-11.71762
3	I	9	2182	2456.90	2147.3306	34.6694
	II	10	2475	2474.66	2474.66	0.34
	III	11	2933	2492.43	2973.46899	-40.46899
	IV	12	2338	2510.19	2342.00727	-4.00727
4	I	13	2227	2527.95	2209.4283	17.5717
	II	14	2552	2545.71	2545.71	6.29
	III	15	3041	2563.47	3058.21971	-17.21971
	IV	16	2394	2581.24	2408.29692	-14.29692
5	I	17	2288	2599.00	2271.526	16.474
	II	18	2612	2616.76	2616.76	-4.76
	III	19	3102	2634.52	3142.98236	-40.98236
	IV	20	2499	2652.28	2474.57724	24.42276

199

140
64
81
100
121
144
169
196
225
256
289
324
361
400

$\Sigma x = 20$

Using this multiplicative decomposition model, calculate the forecast for the four quarters of year 6. **Show your work below the table.**

Year	Season	Period	Forecast
6	I	21	4242.5
	II	22	
	III	23	
	IV	24	

233

$\Sigma y = 49681$ $\Sigma x^2 = 2870$ $\Sigma xy = 513215$

Problem 3: **Aggregate Planning (13 marks)**

Forecast demands for the Tony television sets over the next six months appear below. $2 unit/month/hold

Month	Forecasted Demand
January	150,000
February	130,000
March	240,000
April	180,000
May	190,000
June	120,000

1000

2000

50 unit/day/worker
20 days/month
8 hours/day
16$/hour
24$/O.T
4k/hire 3.5k/fire

There are currently (end of December) 180 workers employed at the plant. Ending inventory in December is expected to be 1,000 units, and the firm would like to have 2,000 units on hand at the end of June. The number of units produced by one worker in one day is 50 units. The employee works 20 days per month, 8 hours a day, at an hourly rate of $16 per hour at regular time; and overtime costs $24 per hour. It costs $4,000 to hire an employee and $3,500 to lay off one, the cost of holding one unit in inventory for one month is $2, and backorder cost is $5/unit.

[2] (a) Given the beginning inventory of 1,000 units, determine the quantity to produce each month and the minimum number of employees required each month if Tony wishes to meet the forecasted demand and ending inventory requirements of 2,000 units on hand at the end of June by varying the workforce level only (i.e. No stockouts or back orders, no ending inventory from January to May, no overtime and idle time is allowed). Show your calculations.

Months	Demand	Production	workers needed
Jan	150 000	150 000 − 1000 = 149 000	149 000/1000 = 149
Feb	130 −	130 000	130 000/1000 = 130
Mar	240 −	240 000	240
Apr	180 −	180 000	180
May	190 −	190 000	190
Jun	120 −	120000 + 2000 = 122 000	122000/1000 = 122

50 units × 20 days = 1000 units/month/worker

* the question did NOT mention anything about calculation the cost of it all, in terms of figures / amounts.

[3] (b) Given the beginning inventory of 1,000 units and assuming that Tony wishes to maintain a safety stock corresponding to 10% exactly of the forecasted demand at the end of **each** month (this requirement is also for the month of June instead of an inventory of 2,000 units), determine ONLY the quantity to produce each month if the company wishes to meet the forecasted demand and ending inventory requirements by varying only the workforce level (i.e. a chase strategy). Show your calculations. **This question is independent of part (a).**

	Demand	10% ~~safety~~ stock	production		workers
Jan	150 000	15 000	150 000 + 15 000 − 1000 = 164000		164
Feb	130 000	13 000	~~143 000~~	128	14
Mar	240 000	24 000	~~264 000~~	151	26

Month	Forecasted Demand
January	150,000
February	130,000
March	240,000
April	180,000
May	190,000
June	120,000

[2] (c) Tony is considering a **level strategy using regular time only**, and absorbing fluctuations in demand with inventories. Given the beginning inventory of 1,000 units, what is the **minimum workforce level** that will be required at the beginning of January that will guaranty no stockouts or backlogs during any of the next six months? Since they don't wish to use any overtime, they will hire or fire as many employees as necessary at the beginning of January. Show your calculations. **This question is independent from parts (a) and (b).**

	Demand	Production	regular	additional needed
Jan	150 000		180 000	0
Feb	130 000	→		0
Mar	240 000	→		60 000 /
Apr	180 000	→		0
May	190 000	→		0
June	120 000	→	↓	0

-1

∴ there will be a total of ~~60~~ 178 workers would need to be hired additionally at the beg. of January to ensure the level of production needed for the next 6 months. HOW?

[2] (d) Forecasted demand for the last six months of the year appears below. The firm chooses to implement a level strategy: employees will produce at full capacity each month. The following table shows the end of month inventory or backorders resulting from this production plan. What is the total cost of this plan (for these six months)? Show your calculations. **This question is independent from parts (a), (b) and (c).**

Month	Forecasted Demand	Final Inventory	Backorders
July	180000	0	9,000
August	150000	11,000	0
September	170000	11,000	0
October	220000	0	39,000
November	160000	0	29,000
December	140000	1,000	0
total:		23 000	77 000

assuming that the plant originally had 180 workers!!

Inv cost 23 000 × 2 = 46000

Backorders 77 000 × 5 = 385 000

Employee 180 × 16 × 8 × 20 × 6 = ~~2 764 800~~

-0.5

(e) Find the preference thresholds for the following option.

2] (i) **Overtime versus hiring and firing**: What is the **smallest number of units** an employee can build using overtime production before it becomes advantageous to hire another employee and then fire that employee later when demands drop? Show your calculations.

Fire cost = $3500 Hire cost = $4 k

over time = $24 /hour

unit/hour = 50/8 = 6.25

It would take $24 × 6.25h = $150 to make one

$7500 / $150 = 50

∴ It would be benefitial to fire the employ... the more than ~~50 units~~ would ... produced on over time.

1953

(ii) **Idle time versus building extra inventory**: What is the maximum time an employee can be kept building inventory to satisfy future demand before it becomes more advantageous to keep that employee idle? Show your calculations.

Employee idle cost = $2560/worker/month reg rate: $16/hour × 6.25 = $10...

Inv cost = $2/unit $100 + $2 = $102

~~2560 / 102 ≐~~ 25

∴ It would be benefitial if the emplo... produces less than ~~25~~ for inventory/...

< 2 MONTHS

0.25

Problem 4: The Transportation Model (8 marks)

(a) The demand for a product for next four months is 600, 975, 1000 and 650 units respectively. The beginning inventory is 30 units, and no ending inventory requirements. Regular production capacity is 720 units per month and overtime production capacity is 60 units per month.

The regular production cost is a fixed cost and independent of the number of units produced. However you can opt for overtime at a cost of $25/unit. The holding cost is $5/unit per month. Backorders are accepted but cost $20/month. The raw material cost is $30/unit.

Set up the transportation table below. **Do not solve it**. If needed, you may add columns or rows to the table below. It is also possible that there are extra columns or rows which are not needed in this table.

begin	~~30~~				EXCESS CAPACITY Excess	
reg. time	0 570	5 150	10	15		30
O.T	30 —	—	—	—	—	720
B.O	55 —	45	—	—	—	~~45~~ 60
R.T	50	720				
O.T	75	60				

O15

(b) In the table below you will find a feasible solution to a Transportation Problem (not necessarily optimal). Please note that backlogs are not allowed in this problem. Based on this solution, answer the following questions:

	Month 1	Month 2	Month 3	Month 4	Excess Capacity	Capacity
Beginning Inventory	[0] 60	[25]	[50]	[75]	[0]	60
Month 1 (Regular)	[100] 440	[125] 60	[150]	[175]	[0]	500
Month 1 (Overtime)	[180]	[205] 60	[230]	[255] 20	[0]	120
Month 2 (Regular)	[ ]	[100] 500	[125]	[150]	[0]	500
Month 2 (Overtime)	[ ]	[180] 120	[205]	[230]	[0]	120
Month 3 (Regular)	[ ]	[ ]	[100] 550	[125]	[0]	550
Month 3 (Overtime)	[ ]	[ ]	[180] 100	[205] 5	[0]	125
Month 4 (Regular)	[ ]	[ ]	[ ]	[100] 550	[0]	550
Month 4 (Overtime)	[ ]	[ ]	[ ]	[180] 125	[0]	125
Demand	500	740	650	700		2650

[3] (i) State the corresponding aggregate production scheduling plan and its total cost.

-3

Production schedule	Month 1	Month 2	Month 3	Month 4	Total excess capacity over the four months
Regular production	~~15000~~ 500	~~15000~~ 500	~~16500~~ 550	~~16500~~ 550	~~$4 260~~ 0
Overtime production	~~4400~~ 80	~~6600~~ 120	~~5 715~~ 105	~~6 875~~ 125	~~$ 980~~ 60

Total Cost: ~~91 830~~ 291.025

[2] (ii) What is the inventory level at the end of each month? Show your calculations.

	Month 1	Month 2	Month 3	Month 4		
			550			550
Month 3 (Overtime)			180 100	205 5	0	125
Month 4 (Regular)				100 550	0	550
Month 4 (Overtime)				180 125	0	125
Demand	500	740	650	700		2650

[3] (i) State the corresponding aggregate production scheduling plan and its total cost.

−3

Production schedule	Month 1	Month 2	Month 3	Month 4	Total excess ca over the four m
Regular production	~~15000~~ 500	~~15000~~ 500	~~16 500~~ 550	~~16500~~ 550	~~$4 200~~
Overtime production	~~4400~~ 80	~~6600~~ 120	~~5 275~~ 105	~~6875~~ 125	~~$ 980~~

Total Cost: ~~91 830~~ 291.025

[2] (ii) What is the inventory level at the end of each month? Show your calculations.

-2

Month #	Ending Inventory
1	~~120~~ 140
2	~~0~~ 20
3	~~5~~ 25
4	~~0~~ 0