# RYERSON UNIVERSITY FALL 2011

## Discrete Mathematics MTH 110

## Final Exam 3 hours

**NAME (PRINTED)** _____

**Student Number** _____ **SIGNATURE** _____

**Circle your section:**

**Sec. 011** (Lab: Thursday 1pm) **Sec. 041** (Lab: Thursday 12pm )
**Sec. 021** (Lab: Wednesday 1pm) **Sec. 051** (Lab: Monday 11am )
**Sec. 031** (Lab: Tuesday 11am) **Sec. 061** (Lab: Thursday 9am)

| **Question** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | **Total** |
|---|---|---|---|---|---|---|---|---|
| **Value** | 8 | 12 | 8 | 10 | 10 | 12 | 10 | **70** |
| **Marks** | | | | | | | | |

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**Instructions:**

- Answer all of the following SEVEN questions. This test consists of EIGHT pages, including the title page.
- No aids allowed. Electronic devices such as calculators and cell-phones must be turned off and kept inaccessible during the test.
- Please keep your Ryerson photo ID card displayed on your desk during the test.
- In every question show all your work. The correct answer alone may be worth nothing.
- Delete all irrelevant and incorrect work because marks may be deducted for work which is misleading, irrelevant or incorrect, even if steps for a correct solution are also shown.
- Please write only in this booklet. Use of scrap paper or additional enclosures is not allowed. If you need more space continue on the back of the page, directing marker where the answer continues with a bold sign.
- **DO NOT** separate pages.

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1. [8 marks] Use the truth table below to determine whether the equivalence below is true. Be sure to show how you are determining equivalence or non-equivalence.

$$(p \wedge q) \rightarrow r \equiv p \rightarrow (\sim q \vee r)$$

| $p$ | $q$ | $r$ | |
|---|---|---|---|
| T | T | T | |
| T | T | F | |
| T | F | T | |
| T | F | F | |
| F | T | T | |
| F | T | F | |
| F | F | T | |
| F | F | F | |

2. [12 marks] Let $P$ be the statement

$$\forall\, a, b, c \in \mathbb{Z},\ (a|b \wedge a \not| c) \rightarrow a \not| (b + c).$$

(a) Give the converse of $P$ in symbolic form and give a counterexample to show that the converse of $P$ is false.

(b) Give the negation of $P$ in symbolic form.

(c) Prove $P$ by contradiction. Be sure to clearly state any assumptions you are making, justify each step and lay out your proof correctly.

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3. [8 marks] Prove that if the sum of two integers is even then either they are both even, or they are both odd.

   Use the predicates $E(x)$ and $O(x)$ to represent $x$ is even and $x$ is odd, respectively.

   Be sure to justify each step, clearly state any assumptions you are making and lay out your proof correctly. You may assume that an integer is even if and only if it is not odd.

4. [10 marks] (a) Draw a Venn diagram to show that the following set relation is false. Also give an explicit counterexample.

$$(A \cap B) \subseteq C \Rightarrow (A \subseteq C \vee B \subseteq C).$$

(b) Use the standard set identities to show that, for any sets $A, B$ and $C$,

$$A \cup (B - C) = (A \cup B) - (C - A).$$

Be sure that each rule you use is clear.

5. [10 marks]Define a binary relation $R$ on $\mathbb{Z}$ as follows:

$$xRy \Leftrightarrow 2|(x+y).$$

(a) Show that $R$ is an equivalence relation on $\mathbb{Z}$.

(b) List three elements in the equivalence class of 2, [2].

(c) Describe the distinct equivalence classes of $R$.

6. [12 marks] Define the order relation $R$ on the set $S = \{x \in \mathbb{Z}^+ \mid x \leq 7\}$ by:

$$\forall a, b \in S, \quad aRb \Leftrightarrow \text{ either } a = b \text{ or } a \bmod 3 < b \bmod 3.$$

(a) Prove that $R$ is a partial order.

(b) Is $R$ a total order? Justify your answer.

(c) Draw the Hasse diagram of $R$.

(d) Either find a greatest element or give two non-comparable maximal elements.

(e) Either find a least element or give two non-comparable minimal elements.

(f) Give a chain of length 2.

7. [10 marks] (a) Given two functions $f : A \to B$ and $g : B \to C$ prove that if both $f$ and $g$ are 1-1 (i.e. injective) functions then the composition $g \circ f$ is also 1-1 (i.e. injective).

(b) Show that the converse is false.