MATH 1300 C & D-MIDTERM # 1-2017.

Professors: Pierre-Alain Jacqmin & Rachid Bentoumi

Last NAME (In CAPITAL letters): ________________ **First Name:** ________________

Student ID# ____________________________________

INSTRUCTIONS: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 50 points.
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Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. **You may use the backs of pages.**

NO CALCULATORS. NO BOOKS. NO NOTES.

MULTIPLE CHOICE ANSWERS:

E	E	B	B
#1	#2	#3	#4

Multiple Choice Questions (1-4)

Question 1 *Find all values of x for which the following function has a tangent line of slope 0 (i.e. $f'(x) = 0$).*

$$f(x) = 3xe^{-\frac{1}{2}x^2}$$

A) $x = -\frac{3}{4}$ **B)** $x = \frac{4}{3}$ **C)** $x = -1$ **D)** $x = 1$ and $x = 3$ **E)** $x = -1$ and $x = 1$

$$f'(x) = 3e^{-\frac{1}{2}x^2} + 3xe^{-\frac{1}{2}x^2} \cdot \left(-\tfrac{1}{2} \cdot 2x\right)$$

$$= 3e^{-\frac{1}{2}x^2} - 3x^2 e^{-\frac{1}{2}x^2}$$

$$= 3e^{-\frac{1}{2}x^2}(1 - x^2)$$

$f'(x) = 0 \Rightarrow 3e^{-\frac{1}{2}x^2}(1-x^2) = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$

Question 2 *Calculate*

$$\lim_{x \to 2} \frac{x^2 + 5x - 14}{2x^2 - 3x - 2}$$

A) 2 **B)** $\frac{5}{2}$ **C)** The limit does not exist. **D)** -1 **E)** $\frac{9}{5}$

$$\frac{x^2 + 5x - 14}{2x^2 - 3x - 2} = \frac{(x-2)(x+7)}{(x-2)(2x+1)}$$

So $\displaystyle\lim_{x \to 2} \frac{x^2+5x-14}{2x^2-3x-2} = \lim_{x \to 2} \frac{x+7}{2x+1} = \frac{2+7}{2 \cdot 2+1} = \frac{9}{5}$

Question 3 *Find the value of a and b, for which the following function is continuous everywhere:*

$$f(x) = \begin{cases} |x-1| + a & \text{if } x < -3 \\ 9 - x^2 & \text{if } -3 \le x < -2 \\ \sqrt{x+2} + b & \text{if } x \ge -2 \end{cases}$$

A) $a = 11$ and $b = 10$
B) $a = -4$ and $b = 5$
C) $a = 5$ and $b = 8$
D) $a = 6$ and $b = 13$
E) $a = -4$ and $b = 6$

$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} (|x-1| + a) = |-4| + a = 4 + a$

$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} (9 - x^2) = 9 - (-3)^2 = 0$. So $4 + a = 0 \Rightarrow a = -4$

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} (9 - x^2) = 9 - (-2)^2 = 5$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (\sqrt{x+2} + b) = \sqrt{-2+2} + b = b$. So $b = 5$.

Question 4 *Solve the equation* $\log_6(x-4) + \log_6(x+1) = 1$ *for* x.

A) $x = -5$ B) $x = 5$ C) $x = 5$ and $x = -2$ D) $x = -2$ E) $x = -2$ and $x = -3$

⚠ Domains: $x - 4 > 0$ and $x + 1 > 0$

$x > 4$ $\qquad$ $x > -1$

Solution: $\log_6(x-4) + \log_6(x+1) = 1$

$\log_6((x-4)(x+1)) = 1$

$(x-4)(x+1) = 6^1 = 6$

$x^2 - 3x - 4 = 6$

$x^2 - 3x - 10 = 0$

$\Delta = 9 + 40 = 49$

$x = \frac{3 \pm \sqrt{49}}{2} = 5, -2$. But $x > 4$, so $x = 5$

Long Answer Questions (5-7)

Question 5 (10 points)

Suppose a function is defined implicitly by the equation

$$x\sqrt{y+1} + y\sqrt{x^2+5} = x^2 - 2$$

- *Find* $\frac{dy}{dx}$ *at* $(2, 0)$.
- *Find the equation of a tangent line to the curve at the point* $(2, 0)$.

- $\frac{d}{dx}\left[x\sqrt{y+1} + y\sqrt{x^2+5} = x^2 - 2\right]$

$\sqrt{y+1} + x \cdot \frac{1}{2}(y+1)^{-\frac{1}{2}} \cdot \frac{dy}{dx} + \frac{dy}{dx} \cdot \sqrt{x^2+5} + y \cdot \frac{1}{2}(x^2+5)^{-\frac{1}{2}} \cdot 2x = 2x$

Plug $(2,0)$ in:

$\sqrt{0+1} + 2 \cdot \frac{1}{2} \cdot (0+1)^{-\frac{1}{2}} \frac{dy}{dx}(2,0) + \frac{dy}{dx}(2,0) \cdot \sqrt{2^2+5} + 0 \cdot \frac{1}{2}(2^2+5)^{-\frac{1}{2}} \cdot 2 \cdot 2 = 2 \cdot 2$

$1 + \frac{dy}{dx}(2,0) + 3\frac{dy}{dx}(2,0) = 4$

$4\frac{dy}{dx}(2,0) = 3$

$\frac{dy}{dx}(2,0) = \frac{3}{4}$

- $y = mx + b$. We know $m = \frac{3}{4}$

$y = \frac{3}{4}x + b$

Plug $(2,0)$ in: $0 = \frac{3}{4} \cdot 2 + b$

$b = -\frac{3}{2}$

$\boxed{y = \frac{3}{4}x - \frac{3}{2}}$

Question 6 (10 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \frac{1}{2x+1}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{2(x+\Delta x)+1} - \frac{1}{2x+1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{2x+1}{(2x+2\Delta x+1)(2x+1)} - \frac{2x+2\Delta x+1}{(2x+2\Delta x+1)(2x+1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{-2\Delta x}{(2x+2\Delta x+1)(2x+1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2\Delta x}{(2x+2\Delta x+1)(2x+1)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2}{(2x+2\Delta x+1)(2x+1)}$$

$$= \frac{-2}{(2x+1)\cdot(2x+1)}$$

$$= \frac{-2}{(2x+1)^2}$$

Question 7 (10 points)

One thousand dollars is invested at a rate of 5%,

- *How much money will be in the account in 4 years if interest is compounded every 2 months? (You do not need to simplify your answer.)*
- *How long will it take for the initial investment to double if interest is 5% and compounds continuously? (You do not need to simplify your answer.)*

- $A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$

$A(4) = 1000\left(1 + \frac{0.05}{6}\right)^{6 \cdot 4}$

$= 1000\left(1 + \frac{0.05}{6}\right)^{24}$

- $A(t) = P_0 \cdot e^{rt}$

$= 1000 \cdot e^{0.05t}$

To solve:

$2000 = A(t)$

$2000 = 1000 \cdot e^{0.05t}$

$2 = e^{0.05t}$

$\ln(2) = \ln(e^{0.05t})$

$\ln(2) = 0.05t$

$t = \frac{\ln(2)}{0.05} = 20 \ln(2)$

Space for additional work

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MULTIPLE CHOICE ANSWERS:

D	D	D	A
#1	#2	#3	#4

Multiple Choice Questions (1-4)

Question 1 *Find all values of x for which the following function has a tangent line of slope 0 (i.e. $f'(x) = 0$).*

$$f(x) = 5xe^{-\frac{1}{2}x^2}$$

A) $x = -\frac{3}{4}$ **B)** $x = \frac{4}{3}$ **C)** $x = -1$ **D)** $x = 1$ and $x = -1$ **E)** $x = -1$ and $x = -3$

$$f'(x) = 5e^{-\frac{1}{2}x^2} + 5xe^{-\frac{1}{2}x^2}\cdot\left(-\frac{1}{2}\cdot 2x\right)$$

$$= 5e^{-\frac{1}{2}x^2} - 5x^2e^{-\frac{1}{2}x^2}$$

$$= 5e^{-\frac{1}{2}x^2}(1-x^2)$$

$$f'(x) = 0 \Rightarrow 5e^{-\frac{1}{2}x^2}(1-x^2) = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Question 2 *Calculate*

$$\lim_{x\to-3}\frac{x^2 - x - 12}{2x^2 + 5x - 3}$$

A) 2 **B)** $\frac{5}{2}$ **C)** The limit does not exist. **D)** 1 **E)** $\frac{9}{5}$

$$\frac{x^2 - x - 12}{2x^2 + 5x - 3} = \frac{(x+3)(x-4)}{(x+3)(2x-1)}$$

$$\text{So } \lim_{x\to-3}\frac{(x+3)(x-4)}{(x+3)(2x-1)} = \lim_{x\to-3}\frac{x-4}{2x-1} = \frac{-3-4}{2\cdot(-3)-1} = \frac{-7}{-7} = 1$$

Question 3 *Find the value of a and b, for which the following function is continuous everywhere:*

$$f(x) = \begin{cases} |x-1| + a & if\ x < -4 \\ 9 - x^2 & if\ -4 \le x < -1 \\ \sqrt{x+1} + b & if\ x \ge -1 \end{cases}$$

A) $a = 11$ and $b = 10$
B) $a = -4$ and $b = 13$
C) $a = 5$ and $b = 8$
D) $a = -12$ and $b = 8$
E) $a = -4$ and $b = 6$

$\lim_{x \to -4^-} f(x) = \lim_{x \to -4^-} (|x-1| + a) = |-5| + a = 5 + a$

$\lim_{x \to -4^+} f(x) = \lim_{x \to -4^+} (9 - x^2) = 9 - (-4)^2 = -7$. So $5 + a = -7 \to a = -12$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (9 - x^2) = 9 - (-1)^2 = 8$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (\sqrt{x+1} + b) = \sqrt{-1+1} + b = b$. So $b = 8$.

Question 4 *Solve the equation* $\log_7(x-5) + \log_7(x+1) = 1$ *for* x.

A) $x = 6$ **B)** $x = -6$ **C)** $x = 6$ and $x = -2$ **D)** $x = -2$ **E)** $x = -2$ and $x = -3$

⚠ Domain: $x - 5 > 0$ and $x + 1 > 0$

$x > 5$ $\quad x > -1$

Solution: $\log_7(x-5) + \log_7(x+1) = 1$

$\log_7((x-5)(x+1)) = 1$

$(x-5)(x+1) = 7^1 = 7$

$x^2 - 4x - 5 = 7$

$x^2 - 4x - 12 = 0$

$\Delta = 16 + 48 = 64$

$x = \frac{4 \pm \sqrt{64}}{2} = 6,\ -2$. But $x > 5 \Rightarrow x = 6$

Long Answer Questions (5-7)

Question 5 (10 points)

Suppose a function is defined implicitly by the equation

$$x\sqrt{y+1} + y\sqrt{x^2+5} = x - 2y$$

- *Find* $\frac{dy}{dx}$ *at* $(2, 0)$.
- *Find the equation of a tangent line to the curve at the point* $(2, 0)$.

• $\frac{d}{dx}\left[x\sqrt{y+1} + y\sqrt{x^2+5} = x - 2y\right]$

$\sqrt{y+1} + x\cdot\frac{1}{2}(y+1)^{-\frac{1}{2}}\frac{dy}{dx} + \frac{dy}{dx}\cdot\sqrt{x^2+5} + y\cdot\frac{1}{2}(x^2+5)^{-\frac{1}{2}}\cdot 2x = 1 - 2\frac{dy}{dx}$

Plug (2,0) in:

$\sqrt{0+1} + 2\cdot\frac{1}{2}\cdot(0+1)^{-\frac{1}{2}}\frac{dy}{dx}(2,0) + \frac{dy}{dx}(2,0)\cdot\sqrt{2^2+5} + 0\cdot\frac{1}{2}\cdot(2^2+5)^{-\frac{1}{2}}\cdot 2\cdot 2 = 1 - 2\frac{dy}{dx}(2,0)$

$1 + \frac{dy}{dx}(2,0) + 3\frac{dy}{dx}(2,0) = 1 - 2\frac{dy}{dx}(2,0)$

$6\frac{dy}{dx}(2,0) = 0$

$\boxed{\frac{dy}{dx}(2,0) = 0}$

• $y = mx + b$. We know $m = 0$.

Plug (2,0) in:

$0 = b$.

So $\boxed{y = 0}$

Question 6 (10 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \frac{1}{3x-1}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{3(x+\Delta x)-1} - \frac{1}{3x-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{3x-1}{(3x+3\Delta x-1)(3x-1)} - \frac{3x+3\Delta x-1}{(3x+3\Delta x-1)(3x-1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{-3\Delta x}{(3x+3\Delta x-1)(3x-1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3\Delta x}{(3x+3\Delta x-1)(3x-1)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3}{(3x+3\Delta x-1)(3x-1)}$$

$$= \frac{-3}{(3x-1)(3x-1)}$$

$$= \frac{-3}{(3x-1)^2}$$

Question 7 (10 points)

One thousand dollars is invested at a rate of 7%,

- *How much money will be in the account in 3 years if interest is compounded every month? (You do not need to simplify your answer.)*
- *How long will it take for the initial investment to double if interest is 7% and compounds continuously? (You do not need to simplify your answer.)*

$$A(t) = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$

$$A(3) = 1000 \left(1 + \frac{0.07}{12}\right)^{12 * 3}$$

$$= 1000 \left(1 + \frac{0.07}{12}\right)^{36}$$

$$A(t) = P_0 e^{rt}$$

$$= 1000\, e^{0.07t}$$

To Solve: $A(t) = 2000$

$$1000\, e^{0.07t} = 2000$$

$$e^{0.07t} = \cancel{\ } 2$$

$$\ln(e^{0.07t}) = \ln(2)$$

$$0.07t = \ln(2)$$

$$\boxed{t = \frac{\ln(2)}{0.07}}$$