Concordia University
Department of Mathematics & Statistics

Course	Number	Sections
STAT	249/2	All
Examination	**Date**	**Pages**
Final	December 2012	4
Instructors		**Course Examiner**
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Special Instructions:

- ▷ Show your work in sufficient detail and clearly identify your answer.
- ▷ Approved calculators only.
- ▷ *A table of Normal (0,1) distribution is attached at the end of this booklet.*

1. **[4+6 marks]**

(a) Write down the three axioms of probability theory and use them to prove that

$$P(A) = P(A \cap B) + P(A \cap \bar{B}),$$

where A and B are two events.

(b) Suppose that A and B are two independent events such that the probability that neither occurs is 0.1 and that the probability of B is 0.2. Find the probability of A.

2. **[4+6 marks]**

(a) A drawer contains 5 different and distinguishable pairs of socks (a total of 10 socks). If a person randomly selects four socks, what is the probability that there is no matching pair in the sample?

(b) A diagnostic test for a disease is 95% accurate, i.e., if a person has the disease, the test will detect it with probability 0.95. Also, if a person does not have the disease, the test will report that he does not have it with probability 0.99. Only 1% of the population has the disease in question. If a person is chosen at random from this population and the diagnostic test indicates that he has the disease, what is the conditional probability that he does, in fact, has the disease?

3. [5+5 marks]

(a) Define the variance of a random variable X and prove that

$$\text{Var}(X) = E(X^2) - (E(X))^2.$$

(b) Let the moment generating function of a random variable X be given by $M(t) = \exp(t + 4t^2)$. Find the first two derivatives of $M(t)$ and hence determine the mean and the variance of X.

4. [5+5 marks]

(a) Let Y denote a geometric random variable with probability of success p. Show that

$$P(Y > k) = (1-p)^k, \quad \text{for } k = 1, 2, \ldots$$

(b) A certified public accountant (CPA) has found that 9 out of 10 company audits contain substantial errors. If the CPA audits a series of company accounts, find the probability that the first account containing substantial errors is the third one to be audited.

5. [4+6 marks]

(a) An oil exploration company has targeted to finance 10 explorations. The probability that a particular exploration will be successful is 0.1. Assume that explorations are independent. Given that there is a fixed cost of \$20,000 before the exploration begins, and every successful exploration costs \$30,000 whereas every unsuccessful exploration costs \$15,000, find the expected total cost to the firm for its 10 explorations.

(b) Customers arrive at a checkout counter in department store according to a Poisson distribution at an average rate of 5 customers per hour. Find the probabilities that

(i) no more than three customers arrive in the next half an hour.

(ii) at least two customers arrive in the next hour.

6. [4+5 marks]

Let the p.d.f. of X be given by

$$f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of c.

(b) Calculate $P(X \le \frac{3}{2} \mid X > \frac{1}{2})$.

7. [10 marks]

Scores on an examination are assumed to be normally distributed with mean μ and variance σ^2. Suppose

(i) 15% of the students have scores 3 or more points above the average.

(ii) 90% of the students have scores below 85.

Find μ and σ using the Normal (0,1) table provided.

8. [6+5 marks]

The joint p.m.f. of X and Y, $p(x, y)$, is given by

$$p(1,1) = \frac{1}{9},\quad p(2,1) = \frac{1}{3},\quad p(3,1) = \frac{1}{9},$$
$$p(1,2) = \frac{1}{9},\quad p(2,2) = 0,\quad p(3,2) = \frac{1}{18},$$
$$p(1,3) = 0,\quad p(2,3) = \frac{1}{6},\quad p(3,3) = \frac{1}{9}.$$

(a) Calculate $E[Y]$ and $\text{Var}(Y)$.

(b) Calculate $P(X + Y > 3 \mid Y \leq 2)$.

9. [5+6 marks]

Suppose (X, Y) have the joint density given by

$$f(x,y) = \begin{cases} 15x^2y, & \text{if } 0 < x < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the marginal densities of X and Y. Are X and Y independent?

(b) Calculate $P(X > \frac{1}{6} \mid Y = \frac{1}{3})$.

10. [5+4 marks]

(a) Find the expected number of distinct values that appear when you throw three fair dice.

(b) Find $E[(X - Y)^2]$, where X and Y are independent binomial random variables with $n = 3$ and $p = 0.4$.