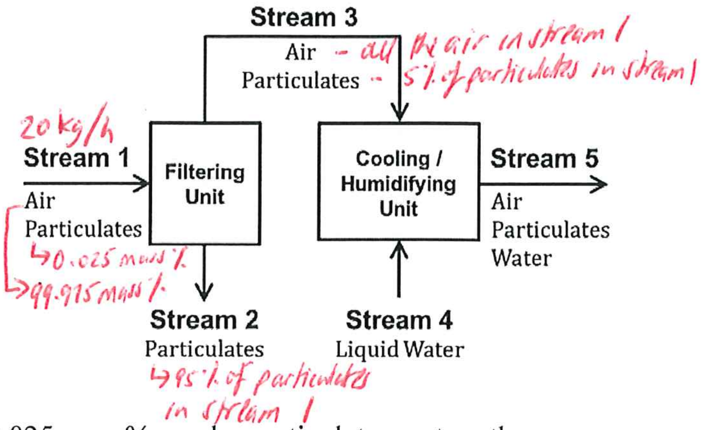
QUESTION 1 (*9 marks*)

PART A: In the summer of 2017, Calgarians experienced record high temperatures, very dry air and a high level of smoke from forest fires. Smoke is made up of microscopic airborne particles, collectively called smoke particulates. An engineering student built a home-made air conditioner that first removes most of the smoke particulates from the air in a Filtering Unit, and then cools and humidifies the filtered air in a cooling/humidifying unit as shown in the diagram.



Stream 1, which contains 99.975 mass% dry air and 0.025 mass% smoke particulates, enters the Filtering Unit at a rate of 20 kg/h. 95% of the entering smoke particulates are removed and leave the filtering unit in Stream 2. The filtered air stream and any remaining particulates exit the filtering unit in Stream 3, and then enter the Cooling/Humidifying Unit. Pure water, which is added to the Cooling/Humidifying unit in Stream 4, evaporates and the cooled, humidified air exits the Cooling/Humidifying Unit in Stream 5. Stream 5 contains 1% water by mass. The entire process operates at steady state.

(a) What is the flow rate (in g/h) of air in Stream 1?

1

$$\dot{m}_{air,1} = (20\,kg/h)(0.99975) = 19.995\,kg/h \rightarrow \boxed{19995\,g/h}$$

(b) What is the flow rate (in g/h) of Stream 3?

2

Stream 3 contains all the air in stream 1 and 5% of the particulates in stream 1

$$\dot{m}_{particulates,1} = 20000\,g/h - 19995\,g/h = 5\,g/h$$

$$\therefore \dot{m}_{particulates,3} = (0.05)(5\,g/h) = 0.25\,g/h$$

$$\therefore \dot{m}_3 = \dot{m}_{air,3} + \dot{m}_{particulates,3}$$

$$= 19995\,g/h + 0.25\,g/h$$

$$= \boxed{19995.25\,g/h}$$

QUESTION 1 ***Contd.***

(c) Assuming the process runs 24 hours per day, calculate the total amount of water (in kg) that must be added to the humidifier every single day for it to run at steady state.

3

We are told that 1mass% of stream 5 is water. That means that 99mass% of stream 5 is air + particulates. Since the system operates at steady state, all the air + particulates in stream 3 come out in stream 5.

$$\text{mass fraction of water in stream 5} = \frac{\dot{m}_{5,water}}{\dot{m}_{5,water} + \dot{m}_{5,air} + \dot{m}_{5,particulate}} = 0.01$$

$$\frac{\dot{m}_{5,water}}{\dot{m}_{5,water} + 19995.25\,g/h} = 0.01$$

$$\dot{m}_{5,water} = 201.97\,g/h.$$

Since steady state $\dot{m}_{5,water} = \dot{m}_{3,water}$

$$\therefore \text{amount of water added} = (201.97\,g/h)(24\,h/day)\left(\frac{1\,kg}{1000\,g}\right) = \boxed{4.847\,kg/day}$$

PART B: While designing the air filtration unit in part A, the engineering student estimated the pressure drop across the air filter using the Carman-Kozeny equation shown on the right.

$$\frac{\Delta P}{\Delta x} = \frac{72\,\mu\,u\,l'}{l\,d^2\,\varepsilon^2}$$

In this equation:

P is pressure [Pressure]
x is distance [L]
μ is viscosity (Pa·s) [Pressure] [t]
u is velocity [L]/[t]
l and l' are lengths [L]
d is diameter [L]
72 is dimensionless
Δ is dimensionless.

(d) Using dimensional analysis, determine the dimensions of ε

3

rewriting the equation in terms of dimension:

$$\frac{[Pressure]}{[L]} = \frac{[t][Pressure][L]/[t]\;[L]}{[L]\;[L]^2\;[\varepsilon]}$$

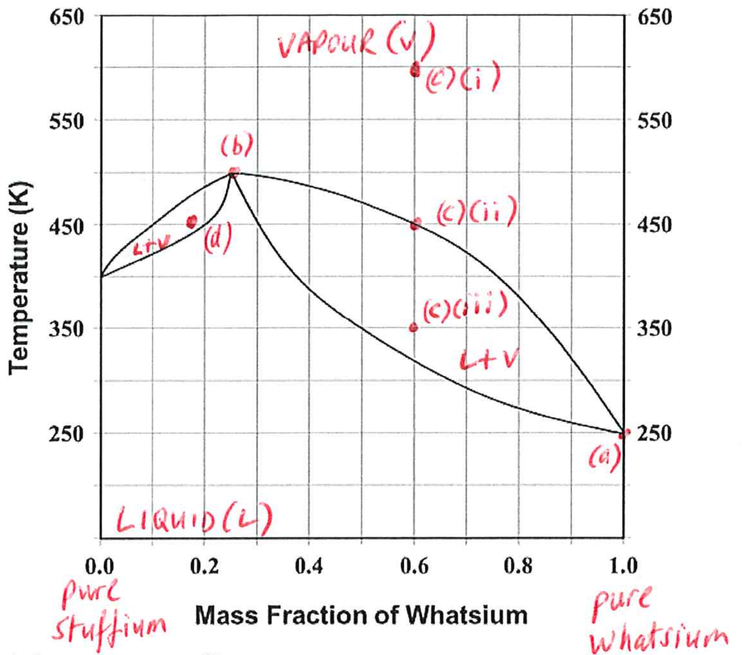
$$\frac{\cancel{[Pressure]}}{\cancel{[L]}} = \frac{\cancel{[Pressure]}}{\cancel{[L]}\;[\varepsilon]^2}$$

$$\boxed{[\varepsilon] = \text{dimensionless}}$$

Note: you could have broken down (Pressure) to fundamental dimensions $\frac{[M]}{[L][t]^2}$

QUESTION 2 (*20 Marks*)

The binary phase diagram shown on the right contains ***liquid-vapour*** equilibrium data for mixtures that contain Stuffium ($M_{Stuffium}$ = 100 kg/kmol) and Whatsium ($M_{Whatsium}$ = 150 kg/kmol) at a fixed pressure of P = 4 atm. Use the phase diagram provided to answer the following questions (*note: in order to receive full marks, you must show all of your work*).



1 **(a)** What is the boiling point (in K) of pure Whatsium?

250 K

3 **(b)** A rigid vessel containing 12.5 kg of pure Whatsium and 37.5 kg of Stuffium is maintained at 500 K. Calculate the degrees of freedom remaining at this point (*show your work*).

$$\text{mass fraction of Whatsium} = \frac{12.5\,\text{Kg}}{12.5\,\text{Kg} + 37.5\,\text{Kg}} = 0.25 \rightarrow \therefore \text{azeotrope}$$

$F = C - P + 2 = 2 - 2 + 2 = 2$ → this tells you that there are 2 degrees of freedom.

However, one degree of freedom is used up because P is fixed. A second degree of freedom is used up because the composition of the two phases are the same. Therefore, the degrees of freedom remaining is $\boxed{\text{ZERO}}$

(c) You have been given a second rigid vessel that contains 60 kg of Whatsium and 40 kg of Stuffium at a temperature of 600 K.

↳ 0.6 mass fraction Whatsium

(i) List all of the phases present under these conditions.

1 Vapour

(ii) The mixture is slowly cooled. Estimate the temperature (in K) at which at least one other phase will appear. Specify the composition (in mass %) of any new phases that appear.

2 $T = \boxed{450\,K}$ → liquid will have $\boxed{\text{30 mass \% Whatsium and 70 mass \% Stuffium}}$

(iii) You now set the temperature of the mixture to 350 K. At this temperature, specify the composition (as mass fractions) of each phase present at equilibrium.

1 LIQUID: 0.5 mass fraction Whatsium, 0.5 mass fraction Stuffium

VAPOUR: 0.85 mass fraction Whatsium, 0.15 mass fraction Stuffium

QUESTION 2 ***Contd.***

(d) A third vessel with a constant volume of 50 L contains 18 kg of Whatsium and 82 kg of Stuffium at a constant temperature of 450 K

(i) Determine the specific volume (in m^3/kg) of the mixture.

1

$$V_{mix} = \frac{\text{volume of mixture}}{\text{mass of mixture}} = \frac{(50\,L)(1\,m^3/1000\,L)}{18\,kg + 82\,kg} = \boxed{5\times10^{-4}\,m^3/kg}$$

(ii) Calculate the mass (in kg) of liquid present at equilibrium.

2

Composition of mixture : 0.18 mass fraction Whatsium
0.82 mass fraction Stuffium

$$\frac{M_L}{M_{mix}} = \frac{X_{mix} - X_V}{X_L - X_V} \rightarrow M_L = (100\,kg)\left(\frac{0.18-0.1}{0.2-0.1}\right) = \boxed{80\,kg}$$

(iii) If the density of the liquid phase is 8 kg/L, determine the specific volume (in m^3/kg) of the vapour phase under these conditions.

2

volume occupied by the liquid $= (M_L)/(\rho_L) = (80\,kg)/(8\,kg/L) = 10\,L$

$\therefore$ volume occupied by the vapour $= 50\,L - 10\,L = 40\,L \rightarrow 0.04\,m^3$

mass of the vapour $= 100\,kg - 80\,kg = 20\,kg$.

$$\therefore\ V_V = \frac{\text{volume of vapour}}{\text{mass of vapour}} = \frac{0.04\,m^3}{20\,kg} = \boxed{0.002\,m^3/kg}$$

QUESTION 2 ***Contd.***

(e) A fourth vessel contains a binary mixture with 55 mol% Whatsium, with the remainder being Stuffium. The mixture is maintained at a temperature of 450 K.

3 **(i)** Determine the composition (in mass%) of the mixture.

conversion of mol% to mass%.
we are not given the amount of mixture, so assume a convenient total.
Assume we have 100 mol of mixture.

$\therefore$ Amount of Whatsium $= (0.55)(100\,\text{mol}) = 55\,\text{mol}$

Amount of Stuffium $= (0.45)(100\,\text{mol}) = 45\,\text{mol}$

mass of Whatsium $= (55\,\text{mol})(150\,\text{g/mol}) = 8250\,\text{g}$ } total $= 12750\,\text{g}$

mass of Stuffium $= (45\,\text{mol})(100\,\text{g/mol}) = 4500\,\text{g}$

$$\therefore \boxed{\begin{aligned} &\text{mass \% Whatsium} = (8250\,\text{g}/12750\,\text{g})(100\%) = 64.7\% \\ &\text{mass \% Stuffium} = 100\% - 64.7\% = 35.3\% \end{aligned}}$$

2 **(ii)** At equilibrium, determine the mass fraction of vapour in the container.

At this temperature and composition, you would only have vapour.

$\therefore$ mass fraction of vapour $= \boxed{1.0}$

2 **(iv)** If the volume of the container is 5 m^3, determine the amount (in kmol) of Whatsium present in the vessel for the overall density of the contents to be 2500 kg/m^3.

mass in the vessel $= (\rho_{mix})(V_{mix}) = (2500\,\text{kg/m}^3)(5\,\text{m}^3) = 12500\,\text{kg}$

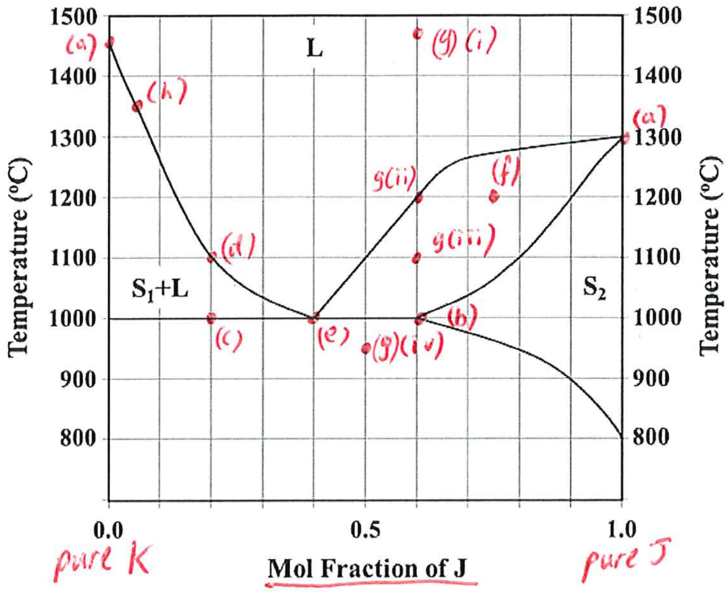
mass of Whatsium in the vessel $= (0.647)(12500\,\text{kg}) = 8087.5\,\text{kg}$

$$\therefore \text{amount of Whatsium present} = \frac{8087.5\,\text{kg}}{150\,\text{kg/kmol}} = \boxed{53.92\,\text{kmol}}$$

QUESTION 3 (*16 Marks*)

The phase diagram provided describes the ***solid-liquid*** behaviour of binary mixtures containing Component J and Component K at a constant pressure of 1 atm. The molar mass of Component J is 85 kg/kmol. The molar mass of Component K is 145 kg/kmol.

Use the phase diagram provided to answer the following questions (*note: in order to receive full marks, you must show all of your work*).



1 **(a)** Determine the melting point temperature (in °C) of each pure component

pure J → 1300°C

pure K → 1450°C

1 **(b)** Determine the maximum solubility (in mol%) of K in J.

40 mol% K in J

1 **(c)** A mixture containing 20 mol% Component J is maintained at a temperature of 1000°C. Determine the composition of each phase present at equilibrium.

three phase line → S_1 → pure K (100 mol% K, 0 mol% J)

L → 40 mol% J, 60 mol% K

S_2 → 60 mol% J, 40 mol% K

1 **(d)** For the mixture described in part (c) above, estimate the minimum temperature (in K) at which it could only exist as a single phase.

1100°C → $\boxed{1373.15\text{ K}}$

1 **(e)** Calculate the degrees of freedom remaining at the eutectic point. *Show all your work.*

$F = C - P + 2 = 2 - 3 + 2 = 1$ → but Pressure is fixed, thereby using up that degree of freedom

∴ degrees of freedom remaining = $\boxed{0}$

1 **(f)** A mixture contains an equal amount of S2 and L at 1200°C. Determine the overall composition (in mol%) of the mixture.

see graph for exact point this could occur.

$\boxed{\begin{array}{l}\text{75 mol\% Component J}\\ \text{25 mol\% Component K}\end{array}}$

QUESTION 3 ***Contd.***

(g) A mixture containing 3.9 kmol of Component J and 2.6 kmol of Component K is maintained at a temperature of 1470°C. Answer the following questions:

(i) Determine the mol fraction of Component J in the mixture.

1

$$\text{mol fraction of } J = \frac{3.9 \text{ kmol}}{3.9 \text{ kmol} + 2.6 \text{ kmol}} = \boxed{0.6}$$

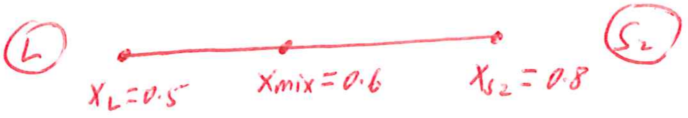
(ii) If the mixture is slowly cooled, at what temperature (in °C) will another phase appear? Determine the composition (in mol%) of that new phase.

1

1200°C ; Phase S_2 will contain 90 mol% component J
10 mol% component K

(iii) If the temperature is now rapidly dropped to a new temperature of 1100°C, determine the mass (in kg) of Component J in each phase present at equilibrium.

4



$$\frac{\text{moles of liquid}}{\text{total moles}} = \frac{X_{S_2} - X_{mix}}{X_{S_2} - X_L}$$

$$\text{moles of liquid} = \frac{0.8 - 0.6}{0.8 - 0.5}(6.5 \text{ kmol}) = 4.33 \text{ kmol}$$

$$\therefore \text{moles of } S_2 = 6.5 \text{ kmol} - 4.33 \text{ kmol} = 2.17 \text{ kmol}$$

$$\text{Amount of component J in liquid} = (4.33 \text{ kmol})(0.5) = 2.17 \text{ kmol}$$

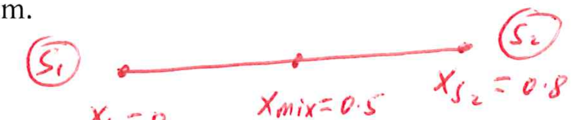
$$\text{Amount of component J in } S_2 = (2.17 \text{ kmol})(0.8) = 1.73 \text{ kmol}$$

$$\therefore \text{mass of component J in liquid} = (2.17 \text{ kmol})(85 \text{ kg/kmol}) = 184.17 \text{ kg}$$

$$\text{mass of component J in } S_2 = (1.73 \text{ kmol})(85 \text{ kg/kmol}) = 147.33 \text{ kg}$$

QUESTION 3 ***Contd.***

(iv) While maintaining a constant temperature of 1100°C, the liquid is removed from the vessel and put in another container. This liquid is then rapidly cooled to 950°C. Determine the molar ratio of the two phases present at equilibrium.



2

$$\frac{\text{moles of } S_1}{\text{moles of mix}} = \frac{X_{S_2} - X_{mix}}{X_{S_2} - X_{S_1}} \quad \text{EQN (1)}$$

$$\frac{\text{moles of } S_2}{\text{moles of mix}} = \frac{X_{mix} - X_{S_1}}{X_{S_2} - X_{S_1}} \quad \text{EQN (2)}$$

dividing EQN (1) by EQN (2)
$$\frac{\text{moles of } S_1}{\text{moles of } S_2} = \frac{X_{S_2} - X_{mix}}{X_{mix} - X_{S_1}} = \frac{0.8 - 0.5}{0.5 - 0} = \boxed{0.6}$$

(h) 42.5 kg of pure Component J is placed in a large empty vessel and heated to 1350 ~~K~~ °C. Liquid Component K is now slowly added to the vessel isothermally. Determine the amount (in kmol) of Component K that will need to be added to the vessel until a solid phase appears.

2

Initial amount of Component J = 42.5 kg / (85 kg/kmol) = 0.5 kmol

Component K is added until mol fraction of J is 0.05 (see graph)

$$\therefore \frac{\text{moles of J}}{\text{moles of J} + \text{moles of K added}} = \text{final mol fraction of J}$$

$$\frac{0.5 \text{ kmol}}{0.5 \text{ kmol} + \text{moles of K added}} = 0.05$$

$$\therefore \text{moles of K added} = \boxed{9.5 \text{ kmol}}$$

QUESTION 4 *(15 Marks)*

Ambient air is not used to pressurize airplane tires as it is susceptible to leaking, contains oxygen (M_{O2} = 32 g/mol) which is corrosive, and contains water vapour which can condense. Instead, airlines use pure, dry nitrogen (M_{N2} = 28 g/mol) for this application. Airplane tires have an internal volume of 0.4 m^3 and are pressurized with N_2 to 170 psi at 20°C. Airlines purchase large quantities of pressurized N_2 gas in rigid cylinders. Each cylinder has a height of 60 inches and an inner diameter of 12 inches (note that 1 inch = 2.5 cm). The initial pressure in a new N_2 gas cylinder is 3378 psi at 20°C. Answer the following questions assuming ideal gas behaviour in all cases:

(a) Determine the volume (in m^3) of a single gas cylinder.

2

$$v = \frac{\pi}{4} D^2 h = \frac{\pi}{4}(12\ \text{inches})^2(60\ \text{inches})\left(\frac{2.5\ \text{cm}}{1\ \text{inch}}\right)^3\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = \boxed{0.106\ \text{m}^3}$$

(b) What is the total mass (in kg) of nitrogen in a new nitrogen gas cylinder?

2

$$Pv = nRT \rightarrow n = \frac{Pv}{RT} = \frac{(3378\ \text{psi})\left(\frac{101325\ \text{Pa}}{14.69\ \text{psi}}\right)(0.106\ \text{m}^3)}{\left(8.314\ \frac{\text{Pa m}^3}{\text{mol K}}\right)(293.15\ \text{K})}$$

$$= 1013.35\ \text{mol}$$

∴ mass of nitrogen in a cylinder $= (1013.35\ \text{mol})\left(\frac{1\ \text{kmol}}{1000\ \text{mol}}\right)(28\ \text{kg/kmol}) = \boxed{28.37\ \text{kg}}$

→ all air is removed

(c) Assuming that each tire is evacuated prior to being filled, determine the maximum number of tires that can be completely filled using only one tank of nitrogen?

3

moles needed to fill one tire: $n = \frac{Pv}{RT} = \frac{(170\ \text{psi})\left(\frac{101325\ \text{Pa}}{14.69\ \text{psi}}\right)(0.4\ \text{m}^3)}{\left(8.314\ \frac{\text{Pa m}^3}{\text{mol K}}\right)(293.15\ \text{K})} = 192.44\ \text{mol}$

Assuming all the nitrogen in a cylinder is used $\rightarrow \frac{1013.35\ \text{mol/cylinder}}{192.44\ \text{mol/tire}} = 5.27\ \text{tires/cylinder}.$

∴ $\boxed{\text{5 tires can be completely filled}}$

QUESTION 4 ***Contd.***

(d) A new aircraft maintenance technician forgot to evacuate the air from one new tire before it was pressurized with N_2, leading to the presence of some O_2 in the tire. Initially, the tire contained 21% O_2 and 79% N_2 by volume at 20°C and 14.7 psi. Then, pure nitrogen was injected until the pressure reached 170 psi at 20°C.

(i) Determine the initial composition (in mole %) of gas in the tire.

Since these are ideal gases, the volume composition is the same as the molar composition.

1 – composition : $\boxed{\begin{array}{l} 21 \text{ mol\% } O_2 \\ 79 \text{ mol\% } N_2 \end{array}}$

(ii) Determine the partial pressure (in psi) of oxygen in the tire before nitrogen is injected.

1 $P_{O_2} = y_{O_2} P = (0.21)(14.7\,psi) = \boxed{3.09\,psi}$

(iii) Determine the final composition (in mole %) of the mixture in the tire.

4 Initial amount of gas in the tire $= n = \frac{PV}{RT} = \frac{(14.7\,psi)\left(\frac{101325\,Pa}{14.69\,psi}\right)(0.4\,m^3)}{\left(8.314\,\frac{Pa\,m^3}{mol\,K}\right)(293.15\,K)} = 16.64\,mol$

∴ Initial amount of $O_2 = (0.21)(16.64\,mol) = 3.495\,mol$

∴ initial amount of $N_2 = (0.79)(16.64\,mol) = 13.145\,mol$

Final amount of gas in a tire = 192.44 mol (from part (c))

∴ amount of N_2 added to tire = 192.44 mol − 16.64 mol = 175.8 mol

∴ final composition ; $\text{mol\% } N_2 = \left(\frac{13.145\,mol + 175.8\,mol}{13.145\,mol + 3.495\,mol + 175.8\,mol}\right)(100\%) = \boxed{98.2\,mol\%}$

$\text{mol\% } O_2 = 100\% - 98.2\% = \boxed{1.8\,mol\%}$

(iv) What is the partial pressure (in psi) of oxygen in the tire after nitrogen has been injected?

1 $P_{O_2} = y_{O_2} P = (0.018)(170\,psi) = \boxed{3.09\,psi}$

– note: you could have just written the answer from part d(ii).

(v) Determine the average molar mass (in kg/kmol) of the mixture in the pressurized tire.

1 $\overline{M} = \Sigma y_i M_i = y_{N_2} M_{N_2} + y_{O_2} M_{O_2} = (0.982)(28\,kg/kmol) + (0.018)(32\,kg/kmol)$

$\overline{M} = \boxed{28.072\,kg/kmol}$