Math100 – Section F01

Midterm 3 – Nov 19, 2007

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Name: _____

Student Number: _____

This test has 10 questions. Circle the correct multiple-choice answer and show your workings. You have 50 minutes to complete this test.

Answers

1 H
2 C
3 J
4 F
5 C
6 I
7 E
8 A
9 A
10 J

1. Evaluate the limit: $\lim_{x\to 1^+} (x-1)^{x-1}$

A	B	C	D	E	F	G	H	I	J
-1	$1/e$	e^2	π	e	2	$-\infty$	1	0	∞

$$L = \lim_{x\to 1^+} (x-1)\ln(x-1) = \lim_{x\to 1^+} \frac{\ln(x-1)}{\frac{1}{x-1}} \quad \text{l'Hôpital} \quad = \lim_{x\to 1^+} \frac{\frac{1}{x-1}}{\frac{-1}{(x-1)^2}}$$

$$= \lim_{x\to 1^+} -(x-1) = 0$$

$$\Rightarrow \lim_{x\to 1^+} (x-1)^{(x-1)} = e^{L} = e^{0} = 1$$

2. Evaluate the integral: $\int_0^1 (e^{2x} + e^{-2x})\,dx$ (circle the approximate value closest to the exact answer)

A	B	C	D	E	F	G	H	I	J
7.38	5.01	3.63	-2.5	0	-3.08	4.78	1.89	3.14	5.43

$$\left[\frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x}\right]_0^1 = 3.63$$

3. Fred drops a coin from a balcony onto the driveway. The coin hits the driveway with a velocity of 20m/s. How high in meters is the balcony (approximate the acceleration due to the earth's gravity as $g = 10m/s^2$)?

A	B	C	D	E	F	G	H	I	J
5	4	15	3	6	7	8	30	10	20

$v(t) = gt = 10t = 20 \Rightarrow t = 2s$

$x(t) = \frac{1}{2}gt^2 = 5 \cdot 2^2 = 20\,m$

4. Given $\frac{dy}{dx} = 2x + 3$ and $y(1) = 2$, what is $y(2)$?

A	B	C	D	E	F	G	H	I	J
1	5	-5	-2	0	8	-8	10	7	4

$y(x) = x^2 + 3x + C$ use $y(1) = 2$ $\Rightarrow C = -2$

$\Rightarrow y(x) = x^2 + 3x - 2$

$y(2) = 8$

5. Find the average of $f(x) = \frac{1}{2}\sin(x)$ on $[0, \frac{\pi}{2}]$

A	B	C	D	E	F	G	H	I	J
π	$-\pi$	π^{-1}	1	0	$-\pi/4$	$\pi/2$	e	$\pi/3$	$-\pi/2$

$$f(x) = \frac{1}{\frac{\pi}{2}-0}\,\frac{1}{2}\left[-\cos x\right]_0^{\frac{\pi}{2}} = \frac{-1}{\pi}\left[0-1\right] = \frac{1}{\pi}$$

6. What is $h'(\frac{\pi}{4})$ given that $h(x) = \int_0^x t\sin(t)\,dt$

A	B	C	D	E	F	G	H	I	J
1	0	π^2	$2/\sqrt{3}$	$\sqrt{3}/2$	π	$-\pi$	$\pi/\sqrt{2}$	$\pi/(4\sqrt{2})$	2π

$$h'(x) = x\sin x$$

$$h'\left(\frac{\pi}{4}\right) = \frac{\pi}{4}\sin\frac{\pi}{4} = \frac{\pi}{4\sqrt{2}}$$

7. Evaluate $\sum_{j=1}^{41} 2 + \sum_{k=2}^{31} \frac{1}{2}k$

A	B	C	D	E	F	G	H	I	J
112.5	∞	912	189.5	329.5	64	0	1	82	632.5

$$\sum_{j=1}^{41} 2 = 2 \cdot 41 = 82$$

$$\sum_{k=2}^{31} \frac{1}{2}k = \frac{1}{2}\sum_{k=1}^{31} k - \frac{1}{2} = 8 \cdot 31 - \frac{1}{2} = 247.5$$

$$\sum_{k=1}^{31} k = \frac{31 \cdot 32}{2}$$

$$\Rightarrow 82 + 247.5 = 329.5$$

8. What is the antiderivatie of $f(x) = \frac{x^4+1}{x^2}$ evaluated at $x = 3$ (take C=0) ?

A	B	C	D	E	F	G	H	I	J
26/3	29/3	6	1	52/3	61/3	0	16/3	25/3	2

$$F(x) = \frac{1}{3}x^3 - \frac{1}{x}$$

$$F(3) = 9 - \frac{1}{3} = 8\frac{2}{3} = \frac{26}{3}$$

9. Compute the Riemann sum on the given interval partitioned into n subintervals of $f(x) = \ln(2x)$ on $[2,5]$ with $n = 6$ (for x_i^* use the right hand endpoint of the subinterval) .

A	B	C	D	E	F	G	H	I	J
5.96	9.15	8.66	6.17	5.83	4.02	3.14	2.71	1.08	5.69

$\Delta x = \frac{1}{2}$

$\frac{1}{2}\ln 5 + \frac{1}{2}\ln 6 + \frac{1}{2}\ln 7 + \frac{1}{2}\ln 8 + \frac{1}{2}\ln 9 + \frac{1}{2}\ln 10$

$= 5.96$

10. Evaluate $\int_0^1 (1-3x)^4\,dx$

A	B	C	D	E	F	G	H	I	J
2.6	1.8	3.3	2.5	$\sqrt{2}$	4.3	0.8	-1.9	0.5	2.2

$u = 1-3x \qquad du = -3dx \qquad a = 1 \qquad b = -2$

$-\frac{1}{3}\int_1^{-2} u^4\,du = \frac{1}{3}\int_{-2}^{1} u^4\,du = \frac{1}{3}\left[\frac{u^5}{5}\right]_{-2}^{1}$

$= \frac{1}{3}\left[\frac{1}{5} + \frac{32}{5}\right] = \frac{1}{3}\cdot\frac{33}{5} = \frac{11}{5} = 2.2$