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**Part A [32 Points]** There are 8 multiple-choice questions in this part.
To choose an answer, simply draw a circle around the bullet (•) of the answer chosen (if any!). A correct choice for one question will get **4** point. An incorrect answer or marking several answers for a question will get $-1$. The minimum total mark for this part is 0.

* An Algorithm C1($n$) uses $10n \log_2 n$ operations and C2($n$) uses $n^2 + 512$ operations. Which of the following values for $n_0$ is the **smallest** such that C2($n$) $\geq$ C1($n$) for all $n \geq n_0$?

• 64 • 128 • 16 • 32

* Consider the following pseudo code:

```
a ← 0
for i ← 0 to n*n do
  for j ← 0 to i do a ← a + i
```

Which of the following characterization, in terms of $n$, of the running time of the above code is **not** correct?

• $\Omega(n^2)$ • $O(n^5)$ • $\Theta(n^4)$ • $O(n^3)$

* Assume the Sequence ADT based on a circular array implementation. Which of the following operations requires $O(n)$ steps?

- atRank
- replaceAtRank
- remove
- elemAtRank

* Consider a sequence $S$ implemented with a non-circular array. Suppose further that $S = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$, in that order, so that each element is actually equal to its rank. Which of the following statements about elemAtRank and insertAtRank is **correct**?

- $S$.remove(9) has the longest runtime
- $S$.elemAtRank(9) has the longest runtime
- $S$.insertAtRank(0,11) has the longest runtime
- $S$.insertAtRank(0,11) has the shortest runtime

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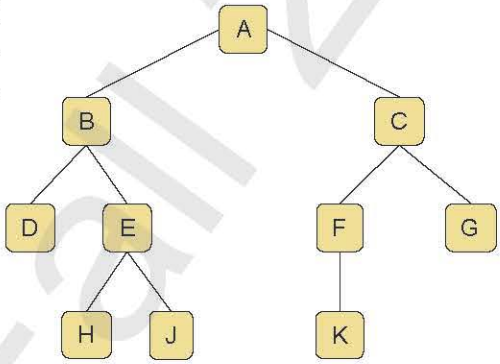
* Consider the following List ADT operations (where the $p_i$'s represent positions): $p_1$=insertFirst(8), $p_2$=insertAfter($p_1$,5), $p_3$=insertBefore($p_2$,3), $p_4$=insertFirst(9), remove($p_4$), swapElements($p_1, p_2$), replaceElement($p_3$, 7), $p_5$=insertAfter(first(),2). Which of the following options describes the final list in a **correct** way?

  - (8,5,3,9)
  - (5,7,3,2)
  - (5,2,7,8)
  - (3,7,5,9)

* Which of the following chronological sequences of node names does **not** correctly represents a traversal of the tree to the right that corresponds to a preorder or inorder or postorder traversal?

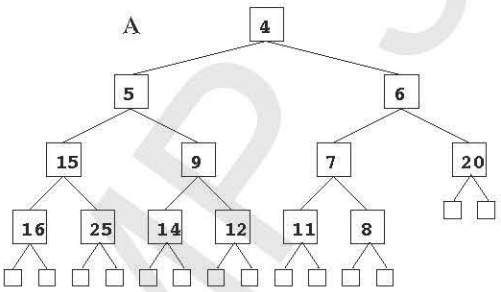


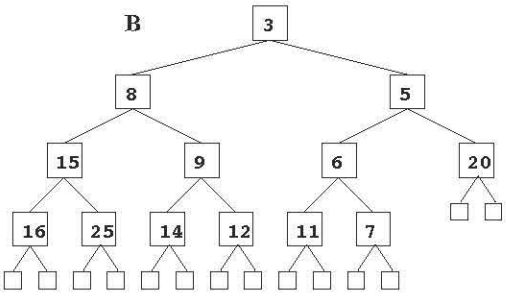

  - A,B,D,E.H,J,C,F,K,G
  - D,H,J,E,B,K,F,G,C,A
  - A,B,E,H,J,K,D,C,F,G
  - D,B,H,E,J,A,K,F,C,G

* Which of the following options list the functions in **correct** non-decreasing order if they are compared by asymptotic growth?

  - $\log n, n, n\log n, n^2 + \log n, n - n^3 + 7n^5, 2^n, n^n, n!$
  - $\log n, n, n\log n, n^2 + \log n, n - n^3 + 7n^5, 2^n, n!, n^n$
  - $\log n, n, n\log n, n^2 + \log n, n - n^3 + 7n^5, n!, 2^n, n^n$
  - $\log n, n, n\log n, n - n^3 + 7n^5, n^2 + \log n, 2^n, n^n, n!$

* What is the **correct** sequence of `insert` and/or `removeMin` operations on heap A that will transform it into heap B (as shown below)?




  - `insert(7),removeMin()`
  - `removeMin(), removeMin(), insert(3)`
  - `removeMin(), insert(3)`
  - `removeMin(), insert(3), insert(5)`

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**Part B [44 Points]**

**B.1** [**20 Points**] Write in pseudo code a tail-recursive algorithm "reverse" using the Sequence ADT that creates from a given sequence a new copy in reverse order. For instance, if S1 represents the sequence (1,2,3), then S2 := reverse(S1) returns a sequence that represents (3,2,1) but leaves S1 unchanged.

1. [**8 Points**] Describe in pseudo-code a tail-recursive version of reverse.

2. [**8 Points**] Describe in pseudo-code an iterative version of reverse.

3. [**4 Points**] What are the worst-case runtimes of your above algorithms (use the big-Oh notation). Justify your answers.

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**B.2** **[24 Points]** The *Towers of Providence* is a variation of the classical Towers of Hanoi problem. There are four pegs, denoted $A$, $B$, $C$, and $D$, and $n$ disks of different sizes. Originally, all the disks are on peg $A$, stacked in decreasing size from bottom to top. Our goal is to transfer all the disks to peg $D$, and the rules are that we can only move one disk at a time, and no disk can be moved onto a smaller one. Write a recursive algorithm in **pseudo code** that solves the *Towers of Providence* problem.
**Hint**: use the operator '&' to concatenate elements and lists, the functions first(L) which returns the first element of a list L, rest(L) which return the list L without its first element, and the predicate empty(L) which is true if L does not contain an element.

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**Part C [24 Points]** For each of the 4 questions in this part, mark **T** if the given statement is **ALWAYS** true. Otherwise mark **F** and **justify** your answer. If you do not justify the FALSE case you will lose marks. There is **no penalty** for selecting the wrong answer.
**Hint:** A correct counter example or the correct specification will give you the full mark.

- If $f(n)$ and $g(n)$ are both $O(h(n))$, then $f(n) + g(n)$ is **not** $O(h(n))$. ☐ **T** ☐ **F**

- If $x$ and $y$ are real numbers such that $0 < x < y$ then $n^x$ is $O(n^y)$ and $n^y$ is $O(n^x)$.
  ☐ **T** ☐ **F**

- A preorder traversal of a heap yields the keys in nondecreasing order. ☐ **T** ☐ **F**

- if $f(n)$ is $\Omega(n)$ and $O(n)$, then $f(n)$ is $\Theta(n)$. ☐ **T** ☐ **F**